



SIRT-TV 3D image reconstruction for simulated muon tomography of the QinShiHuang tomb

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Abstract

Cosmic-ray muons are suitable for the nondestructive imaging of large-scale objects. However, owing to the low statistics of cosmic-ray muons and the complicated surroundings of transmission muography experiments, the applicability of transmission muography is limited by challenges such as noise and incomplete data, which hinder 3D image reconstruction. In this study, we applied the SIRT-TV algorithm to muon tomography to perform 3D image reconstruction simulations based on the QinShiHuang tomb phantom. The results indicate that compared with the conventional SIRT algorithm, the SIRT-TV algorithm can effectively suppress artifacts in the reconstructed image, allowing for a more accurate reconstruction of the underground palace. The geometric similarity of the walls reconstructed by SIRT-TV is nearly three times that of the results reconstructed by SIRT. This paper further discusses the impact of the TV-minimization algorithm parameters, measurement configurations, and durations on the quality of the reconstructed image, providing a reference point for future experiments.

Keywords Muography · Monte Carlo simulation · Image reconstruction

1 Introduction

Transmission muography, which utilizes naturally existing cosmic-ray muons, images the internal structures of objects penetrated by muons. Because of their high average energy

(~4 GeV at sea level [1]), cosmic-ray muons have strong penetrating power and nearly straight-line trajectories. Additionally, they are ubiquitous on Earth. These characteristics make cosmic-ray muons particularly suitable for the non-destructive imaging of large-scale objects such as volcanoes [2–4], faults [5–7], mineral deposits [8–11], cultural heritage [12–16], and nuclear reactors [17–19].

The flux attenuation due to the energy loss of muons interacting with matter relates to both the thickness and density of the material penetrated by the muons. Analyzing the cosmic-ray muon flux detected from a single view and combining it with prior topographic information yield the average density along the line of sight, allowing for the identification of density anomalies within the object. This 2D transmission muography (“muon radiography”) has a limitation: The density map is only a 2D projection. This introduces ambiguities when internal structure is complex, as density anomalies may overlap along the measurement direction, creating degenerate structures. To reduce this ambiguity, obtaining 3D information requires at least two different views of cosmic-ray muon flux measurements to locate density anomalies. In the ScanPyramids experiment of the Khufu Pyramid in 2017 [12], three independent

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muon measurements were conducted, each using two distinct observation positions for triangulation. The resulting locations were consistent across all three analyses, providing high-confidence confirmation of the hidden chamber. With muon flux data from multiple views, reconstructing the 3D density distribution becomes possible—this approach is called “muon tomography.”

Owing to the characteristics of cosmic-ray muons, muon tomography presents two major challenges. First, because of the relatively low flux of cosmic-ray muons, which is approximately $1 \text{ cm}^{-2} \text{ min}^{-1}$ at sea level [1], the number of views is limited by both the experimental time and the availability of muon detectors. Additionally, the choice of detector sites is constrained given that the structure under test in transmission muography is often a large-scale object located in the field, and the surroundings are complicated. Consequently, the image reconstruction problem in muon tomography is often under-constrained, potentially leading to ambiguous results. Second, fluctuations in cosmic-ray muon flux data can disturb image reconstruction in muon tomography, including statistical fluctuations caused by the low statistics of cosmic-ray muons and directional fluctuations caused by multiple Coulomb scattering of muons penetrating objects [20].

To address the two challenges mentioned above, efforts can be made from two perspectives: combining cosmic-ray muon data with other types of geophysical data for joint inversion and improving image reconstruction algorithms. From the perspective of joint inversion, both transmission muography and gravimetry are sensitive to the density of objects, making them naturally suitable for joint inversion. Thus, muon data are often combined with gravity data for such inversions. Several studies have reported inversion results that combine cosmic-ray muon data with gravity data. For instance, Nishiyama et al. combined single-view cosmic-ray muon data with gravity data obtained from 30 gravity stations to reconstruct the 3D density structure of the Showa-Shinzan lava dome. They also demonstrated (through the inversion of synthesized data) that the results of joint inversion using both gravity and cosmic-ray muon data had better resolution than that obtained using only gravity data [21]. However, Jourde et al. noted in their study that when an object is covered by cosmic-ray muon data from more than two viewpoints, the improvement in image quality from adding gravity data is insignificant [22].

For enhancing image reconstruction algorithms, establishing robust algorithms with strong noise resistance is crucial, as muon tomography involves reconstructing images from noisy and incomplete data. The algorithms for 3D image reconstruction in muon tomography fall into two main categories. The first is the linear inversion method based on Bayesian principles, widely used in geophysical model inversion and applicable to muon tomography. Nagahara

et al. used linear inversion to reconstruct the 3D density distribution of the Omuroyama scoria cone from cosmic-ray muon data from 10 viewpoints [23]. Barnoud et al. used this method for joint inversion of single-view cosmic-ray muon data and gravimetric data to reconstruct the Puy de Dôme volcano’s 3D density structure [24]. However, this method requires careful tuning of multiple a priori parameters that affect inversion results, and density smoothing constraints may blur density anomaly boundaries [25]. The second category is algebraic reconstruction techniques, commonly used in medical CT image reconstruction and suitable for incomplete and noisy data. Erlandson et al. used the algebraic reconstruction technique (ART) algorithm for cosmic-ray muon tomography reconstruction of the ZED-2 research reactor [26]. S. Procureur reconstructed images of a simulated concrete cube with cavities using the simultaneous ART algorithm [27]. Hartling et al. simulated a 12-view muon tomography for a cylindrical model containing a uranium rod and compared various iterative reconstruction algorithms, concluding that the simultaneous iterative reconstruction technique (SIRT) is a suitable algorithm for muon tomography reconstruction [28]. Compared to the classic ART algorithm, the SIRT algorithm can better suppress noise.

Additional information can be incorporated into the SIRT algorithm to further improve the reconstructed image quality. Total-variation (TV) minimization is an effective constraint. Signals or images with transform sparsity can be reconstructed from a small number of samples by minimizing the L1-Norm of the transform coefficients [29–31]. For piecewise constant images, their gradient-magnitude images meet the transform sparsity condition, and TV is the L1-Norm of gradient-magnitude image. Therefore, TV-minimization-based image reconstruction algorithms may achieve high-precision reconstruction from a limited number of views [32, 33]. The density distribution of large cultural heritage structures, such as the Khufu Pyramid, is piecewise constant, making them suitable for TV minimization. Therefore, combining SIRT with TV minimization could enhance the quality of muon tomography reconstruction images for such cases.

In 2022, our group reported the preliminary results of a cultural heritage muon radiography simulation study based on the archeological data of the QinShiHuang tomb. This study demonstrated that muon radiography could identify the inner structures of the underground palace, including the tomb chamber, walls, and rammed earth. The length, width, and burial depth of the tomb chamber were estimated from the muon radiography simulation results obtained from two viewpoints [34]. Based on previous research, in this study, we performed 3D image reconstruction for muon tomography of the QinShiHuang tomb. A digital phantom built based on the archeological data of the QinShiHuang tomb was used for the

transmission muography simulation in Geant4 and for testing the image reconstruction algorithm. The SIRT-TV algorithm, which combined SIRT and TV minimization, was applied to the image reconstruction of 12-view muon tomography images. The remainder of this paper is organized as follows. Section 2 introduces the principles of transmission muography and the SIRT-TV algorithm. Section 3 describes the simulation setup and presents the 2D slices of the reconstructed images obtained using the SIRT-TV and SIRT algorithms. The slices reconstructed using the two algorithms, with the same number of iterations, are compared visually. Section 4 evaluates the reconstructed image quality based on the geometric-similarity index for the structures of the underground palace, providing a quantitative comparison between SIRT and SIRT-TV. This section also discusses some factors that may influence the reconstructed image quality, including the value of the TV-minimization algorithm parameters, the number of views in muon tomography, and the statistics of the muon data. The final section concludes the study.

2 Methods

2.1 Principles of muon radiography

When passing through an object, muons interact with the matter and lose energy through ionization, bremsstrahlung, pair production, and photonuclear reactions. The energy loss behavior can generally be described as

$$-\frac{dE}{dX} = a + bE, \quad (1)$$

where a represents the ionization term, and b is the sum of the other three radiation loss terms. Both a and b depend on the muon energy and composition of the material. Their values can be obtained from the tables provided by Groom et al. [35]. $X = \int \rho dl$ is the integral of the density of the object along the muon path, which is referred to as the density length of the object. If a muon with energy $E_{\min}$ loses its energy after penetrating an object with density length X_1 , X_1 can be expressed as

$$X_1 = \int_{E_\mu}^{E_{\min}} -\frac{dX}{dE} dE, \quad (2)$$

where E_μ is the muon rest-mass energy. Combining Eqs. (1) and (2), the value of X_1 can be numerically calculated. Alternatively, Monte Carlo simulations can be used to determine the maximum density lengths at which muons of different energies can penetrate. The simulation results are fitted to derive a formula describing the relationship between X_1 and $E_{\min}$. $E_{\min}$ can be determined from the measured cosmic-ray

muon flux I , which can be expressed as an integral of the differential energy spectrum $\phi(E)$:

$$I = \int_{E_{\min}}^{+\infty} \phi(E) dE. \quad (3)$$

Unlike traditional imaging techniques that use artificial radiation sources, transmission muography uses naturally existing radiation sources whose energy spectra $\phi(E)$ cannot be manually modulated. Moreover, because the targets in transmission muography experiments are usually large, using detectors to measure the cosmic-ray muon energy spectrum $\phi(E)$ in real time, before the muons penetrate the target, is impractical. Nonetheless, theoretical studies and experimental measurements show that the sea-level cosmic-ray muon energy spectrum is relatively stable and can be reliably modeled. The angular distribution of sea-level cosmic-ray muons is approximately $\propto \cos^2 \theta$, where θ is the zenith angle of the cosmic-ray muons. Their energy spectra follow a power law with a negative exponent. Therefore, the transmitted muon flux mainly consists of muons with energies near $E_{\min}$. Classic formulas such as Gaisser/Tang and Reyna are commonly used to describe the cosmic-ray muon energy spectrum at sea level because they provide reliable descriptions in the energy range most relevant to transmission muography [36].

Combining Eqs. (1)–(3) and the sea-level cosmic-ray muon energy spectrum model, the density length of the object in different directions can be obtained, resulting in a 2D projection map of the object's density length X .

2.2 3D image reconstruction of muon tomography

Based on a 2D projection map of X obtained from Sect. 2.1, the 3D density distribution can be solved for. Let m be the number of directions. By uniformly dividing the 3D region of interest (ROI) into n voxels, the integral $X = \int \rho dl$ can be discretized as

$$x_i = \sum_{j=1}^n l_{ij} \rho_j, i = 1, \dots, m, \quad (4)$$

where x_i represents the density length along the i th ray, ρ_j is the density of the j th voxel, and l_{ij} is the intersection length of the i th ray within the j th voxel. The relationship between the density lengths along different directions and density distribution can be expressed as a system of linear equations:

$$X = L\rho, \quad (5)$$

where $X = (x_1, x_2, \dots, x_m)^T$ represents the density lengths of the object along different directions, $L = \{l_{ij}\}$ represents the intersection length of rays within the voxels along different

directions, and $\rho = (\rho_1, \rho_2, \dots, \rho_n)^T$ represents the densities of the voxels.

Because the density length data are obtained from cosmic-ray muon flux data, which are usually noisy and incomplete, the linear equations in Eq. (5) face issues such as being underdetermined and inconsistent and having a large sparse system matrix. Algebraic reconstruction techniques are commonly employed to address these problems. The most basic algebraic reconstruction algorithm is the ART algorithm [37], which is based on the Kaczmarz optimization algorithm. The ART principle can be understood as the gradual projection of a trial solution onto the hyperplanes determined by each equation to obtain a new solution. If the system of equations has a unique solution, then the trial solution converges iteratively. In every iteration of the ART algorithm, each equation of Eq. (5) updates ρ once. This equation-by-equation update method is sensitive to noise and can cause divergence if the noise is severe. The SIRT algorithm was developed based on the basic ART algorithm [38]. In each iteration of the SIRT algorithm, the contribution of each equation to ρ is first calculated without updating ρ . After going through all the equations, their contributions are averaged and ρ is updated accordingly. The updated equation for SIRT is

$$\rho_{\text{SIRT},j}^k = \rho_{\text{SIRT},j}^{k-1} + \lambda \frac{\sum_{i=1}^m L_{i,j} \frac{X_i - L_i \rho_{\text{SIRT}}^{k-1}}{\sum_{j=1}^n L_{i,j}}}{\sum_{i=1}^m L_{i,j}}, \quad (6)$$

where k represents the number of iterations and λ is used to control the iteration step size, which is usually set to 1.

Both ART and SIRT solve the reconstruction problem based solely on the projection data X , without introducing additional constraints. However, muon tomography often lacks sufficient views, resulting in the ill-posedness of the image reconstruction. To improve the quality of the reconstructed image further, additional constraints must be introduced during the reconstruction procedure. We observed that the density distribution of the targets in the muon tomography analysis typically exhibited piecewise constancy, indicating that its gradient-magnitude image was sparse. This feature allowed using TV minimization as a constraint.

Let the voxel density at the coordinates (r, s, t) be $\rho_{r,s,t}$. Then, the TV in the density image is

$$\begin{aligned} \|\rho\|_{\text{TV}} &= \sum_{r,s,t} |\nabla \rho_{r,s,t}| \\ &= \sum_{r,s,t} \sqrt{(\rho_{r,s,t} - \rho_{r-1,s,t})^2 + (\rho_{r,s,t} - \rho_{r,s-1,t})^2 + (\rho_{r,s,t} - \rho_{r,s,t-1})^2}. \end{aligned} \quad (7)$$

Using the gradient descent algorithm to solve the TV-minimization problem, we let:

$$\rho_{\text{TV}}^0 = \rho_{\text{SIRT}}^k, \quad (8)$$

$$d\rho = \|\rho_{\text{SIRT}}^k - \rho_{\text{SIRT}}^{k-1}\|_2, \quad (9)$$

$$v^{k'} = \nabla_{\rho} \|\rho_{\text{TV}}^{k'}\|_{\text{TV}}, \quad (10)$$

and

$$\hat{v}^{k'} = v^{k'} / |v^{k'}|, \quad (11)$$

for $k' = 1, 2, \dots, N_{\text{TV}}$, and perform TV gradient descent:

$$\rho_{\text{TV}}^{k'} = \rho_{\text{TV}}^{k'-1} - \alpha \cdot d\rho \cdot \hat{v}^{k'-1}. \quad (12)$$

where α controls the TV gradient descent step size, and the total number of gradient descent iterations is controlled by N_{TV} . In Ref. [32] TV minimization is combined with ART; however, considering that data noise is more significant in muon tomography, we combined SIRT with TV minimization for image reconstruction. The SIRT-TV procedure terminates when the number of iterations reaches N_{max} . The overall process of the muon tomography image reconstruction is illustrated in Fig. 1. Notably, after the SIRT-step in each iteration k , a non-negative constraint is applied to ρ_{SIRT}^k . Any additional prior information related to ρ can also be incorporated in this step.

3 Simulation and reconstruction

3.1 Muography simulation

The simulation is performed using Geant4, which is a Monte Carlo toolkit developed by CERN based on C++ [39]. Geant4 can simulate the transport and interactions of particles in matter and is a commonly used and reliable simulation platform for transmission muography. The simulation model can be divided into three parts: the QinShiHuang tomb phantom, muon track detector, and cosmic-ray muon source.

The QinShiHuang tomb was studied using 20 geophysical approaches as part of the National 863 Hi-tech Project. Archeological results confirmed the depth of the underground palace, as well as the size and location of the tomb chamber and surrounding walls [40]. Based on these archeological data, we built a phantom model of the QinShiHuang tomb for our simulation, which included the mound, rammed earth, loam wall, stone wall, burial chamber, and surrounding land, as shown in Fig. 2.

The cosmic-ray muon source is sampled using the Reyna formula [41]:

Fig. 1 (Color online) Flowchart of muon tomography image reconstruction using the SIRT-TV algorithm

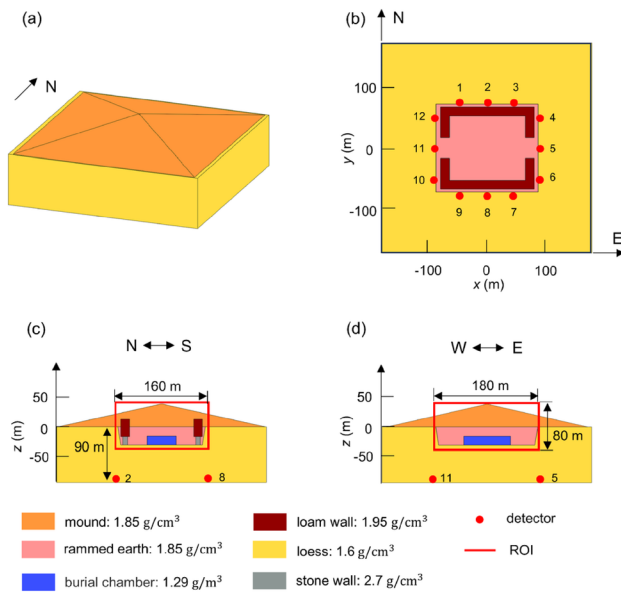
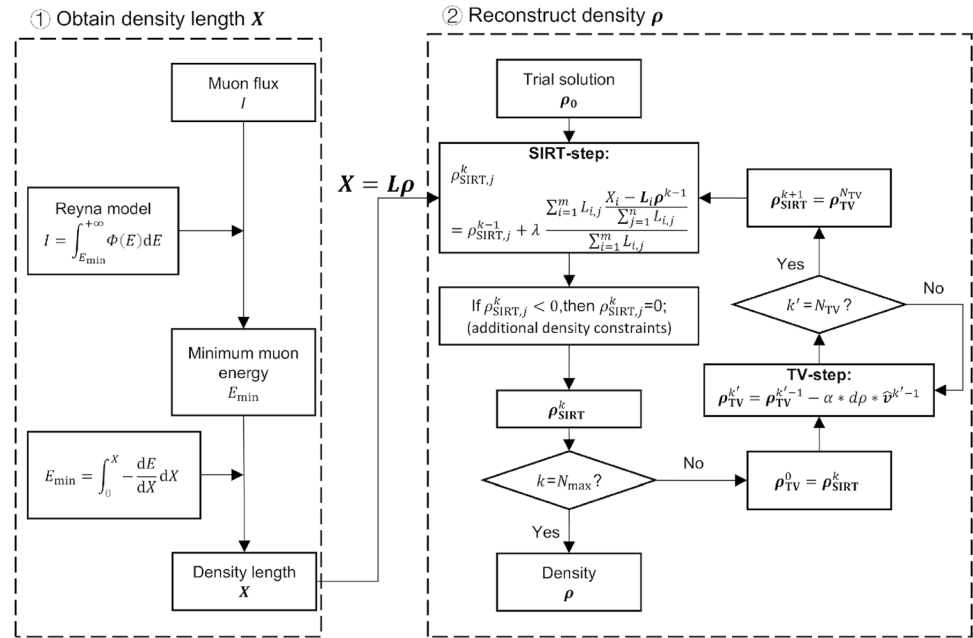


Fig. 2 (Color online) Schematic of the QinShiHuang tomb phantom. **a** 3D view of the phantom. **b** Cross-sectional view of the phantom at $z = 0$, with red dots indicating the projections of the detector positions on the cross section. **c** Cross-sectional view of the phantom at $x = 0$. **d** Cross-sectional view of the phantom at $y = 0$. The rectangles in (c) and (d) represent the ROI for 3D image reconstruction

$$\phi(p, \theta) = c_1 \cos^3 \theta (p^*)^{-[c_2 + c_3 \log_{10}(p^*) + c_4 \log_{10}^2(p^*) + c_5 \log_{10}^3(p^*)]}, \quad (13)$$

where θ is the zenith angle of cosmic-ray muons, p is the momentum of cosmic-ray muons in GeV/c, $p^* = p \cos \theta$, $c_1 = 0.00253$, $c_2 = 0.2455$, $c_3 = 1.288$, $c_4 = -0.2555$, and

$c_5 = 0.0209$. This formula is suggested for a momentum range of $1 \text{ GeV}/c < p < 2000/\cos \theta \text{ GeV}/c$ and zenith angle range of $0^\circ \leq \theta \leq 90^\circ$.

The coordinate system is defined as shown in Fig. 2b–d. The muon track detector is a $1 \text{ m} \times 1 \text{ m}$ ideal detector placed horizontally, recording cosmic-ray muons' direction. The detector positions are 5 m away from the underground palace boundaries. Notably, 12 detector locations are evenly spaced at 30° surrounding the palace center, numbered 1 to 12, as shown in Fig. 2b. Because cosmic-ray muons travel downward, detectors must be positioned below the palace level to ensure that the field of view covers the ROI. The detector depth affects the angular size of the ROI, overburden thickness, and muon zenith angles. We set the detectors' burial depth to 90 m, as shown in Fig. 2c, where the maximum zenith angle of muons crossing the ROI is 75° . Following Ref. [42], we determined the minimum muon energy required to reach the detector through the phantom, sampling only muons above this threshold. The simulated cosmic-ray muon statistics per view equal a 180-d physical measurement.

3.2 Image reconstruction

Since the tomb phantom is much larger than the detectors, they are treated as points for calculating muon flux and density length maps. Simulated cosmic-ray muon fluxes after penetrating the QinShiHuang tomb phantom, detected by each detector, are divided into 75×180 directions by zenith and azimuth angles, as shown in Fig. 3. Each pixel's direction in Fig. 3 is represented by its center coordinate. The minimum energy $E_{\min}$ of cosmic-ray muons penetrating

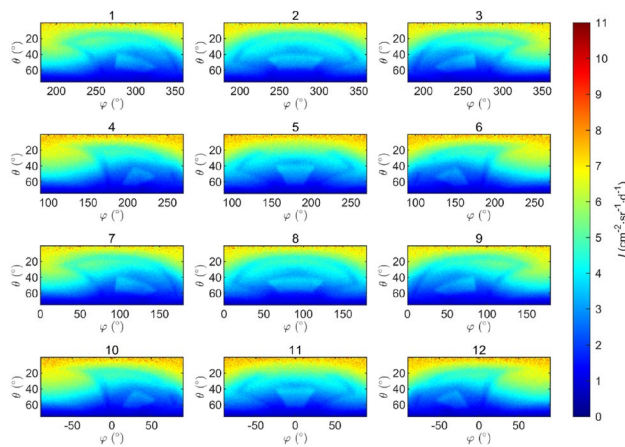


Fig. 3 (Color online) Simulated cosmic-ray muon fluxes penetrating the QinShiHuang tomb phantom, measured by 12 detectors using Geant4. The numbers above the figures correspond to the detector IDs in Fig. 2. θ and φ represent the zenith and azimuth angles of the incident muons in the horizontal coordinate system

the phantom is calculated using Eq. (3). For density length conversion, Geant4 simulates the average muon penetration range in loess, with results in Fig. 4. A second-order polynomial is fitted to the curve in Fig. 4.

Since we are interested in the underground palace structure, we define the ROI as shown in Fig. 2c, d and perform 3D reconstruction for density distribution within the ROI. The density length data from different directions include contributions from inside and outside the ROI. For the tomb phantom, knowing the density distribution outside the ROI allows us to calculate its density length. Subtracting this from the obtained data gives density length within the ROI, as shown in Fig. 5. Some directions have negative density length values due to cosmic-ray muon flux fluctuations. Only non-negative values are used in reconstruction.

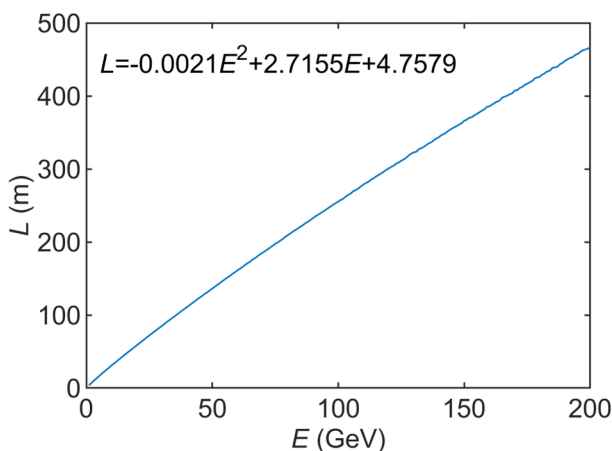


Fig. 4 (Color online) The average penetration range of muons in loess as a function of muon energy obtained from Geant4 simulations

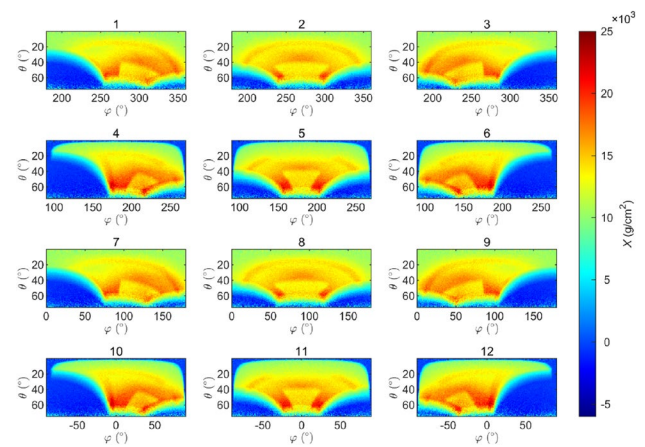


Fig. 5 (Color online) Density length maps of the tomb phantom ROI converted from the 12-view simulated muon fluxes

In 3D image reconstruction, the ROI is divided into $60 \times 50 \times 20$ voxels, with a voxel size of $3 \text{ m} \times 3.2 \text{ m} \times 4 \text{ m}$. This voxelization ensures appropriate voxel numbers for solving linear equations in Eq. (5) while maintaining resolution for fine structure reconstruction. SIRT and SIRT-TV reconstruct images from the same density length data for comparison. Following Ref. [32], we set $\lambda = 1$, $\alpha = 0.2$, and $N_{\text{TV}} = 20$. Initial density values are set to $\rho_0 = 1.6 \text{ g/cm}^3$, matching the loess density in the phantom. Both algorithms iterate $N_{\text{max}} = 50$ times. An additional constraint in each iteration restricts voxel density outside the burial mound to 0.00129 g/cm^3 , matching the phantom's air density. Slices at three representative locations are selected for comparison with phantom slices, as shown in Fig. 6.

A comparison of the reconstructed image slices, obtained by the two algorithms, with those of the phantom reveals that the shapes and positions of the tomb chamber and walls in the SIRT-TV reconstruction results match those of the phantom. By contrast, the slices obtained using the SIRT algorithm exhibit significant salt-and-pepper artifacts, particularly at the bottom of the reconstructed image, around the tomb chamber, and at the top of the burial mound. These artifacts can potentially lead to misinterpretation of the underground palace's structure. The reconstruction results obtained using SIRT-TV do not show such significant artifacts, preliminarily verifying that the inclusion of TV minimization can effectively suppress artifacts.

4 Discussion

4.1 Reconstructed image quality assessment

Figure 6 shows a visual comparison of the reconstructed images obtained using SIRT and SIRT-TV. However, visual

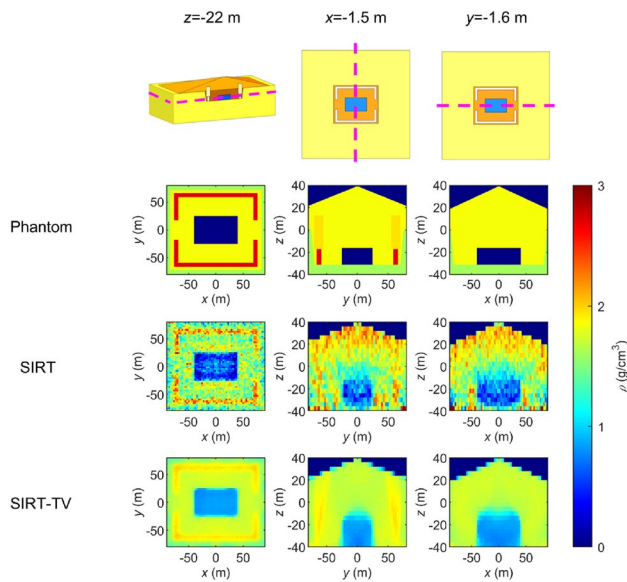


Fig. 6 (Color online) Comparison of the slices of 3D density distribution between the phantom and the reconstruction results of SIRT and SIRT-TV. The dashed lines indicate the locations of the slices

comparison lacks quantitative information and is noncomprehensive because the reconstructed images are 3D. To quantitatively compare the reconstruction results of SIRT and SIRT-TV, using appropriate image quality assessment metrics is crucial. Classic metrics include mean squared error (MSE), peak signal-to-noise ratio (PSNR), and structural similarity (SSIM) [43]. These metrics can evaluate the overall similarity between the reconstructed image and phantom. However, this study is primarily focused on the feasibility of accurately reconstructing the structures of the underground palace, specifically the tomb chamber and walls. Referring to the image quality assessment metrics used in Ref. [28], we use the Jaccard similarity index J to assess the geometric similarity of the reconstructed images to the phantom for the tomb chamber and walls. J is defined as follows:

$$J(P, R) = \frac{|P \cap R|}{|P \cup R|}, \quad (14)$$

where P represents the set of voxels belonging to the tomb chamber or walls in the phantom and R represents the set of voxels belonging to the same structure in the reconstructed image. The term $|P \cap R|$ denotes the number of elements at the intersection of P and R , and $|P \cup R|$ denotes the number of elements in the union of P and R . A larger $J(P, R)$ indicates a higher geometric similarity, and when $J(P, R) = 1$, the geometric structures of the phantom and reconstructed image are identical. When calculating $J(P, R)$ for the images reconstructed from Sect. 3, we encounter the issue

of inconsistent voxel segmentation between the phantom and reconstructed images because the phantom established in Geant4 is not voxelized. To address this problem, we modified Eq. (14). We define a vector $\mathbf{p}$, which represents the proportion of overlap between the voxels defined in the reconstructed images and the corresponding structure in the phantom. Consequently, element p_i corresponding to the i th voxel can take three types of values:

$$p_i = \begin{cases} 0 & \text{voxel outside the corresponding structure,} \\ 1 & \text{voxel inside the corresponding structure,} \\ q & \text{voxel partially overlaps with the corresponding} \\ & \text{structure with a proportion of } q. \end{cases} \quad (15)$$

We define a vector $\mathbf{r}$ to determine whether each voxel belongs to a structure in a reconstructed image. Element r_i corresponding to the i th voxel can take two values:

$$r_i = \begin{cases} 0 & \text{voxel outside the corresponding structure,} \\ 1 & \text{voxel inside the corresponding structure.} \end{cases} \quad (16)$$

Therefore, Eq. (14) can be redefined as:

$$J(\mathbf{p}, \mathbf{r}) = \frac{\sum_{i=1}^n p_i \cdot r_i}{\sum_{i=1}^n p_i + \sum_{i=1}^n r_i - \sum_{i=1}^n p_i \cdot r_i}. \quad (17)$$

In this phantom study, partial overlaps exist between voxels and structures; therefore, $J(\mathbf{p}, \mathbf{r})$ cannot reach 1. When calculating $\mathbf{r}$, image segmentation extracts tomb chamber and wall structures from reconstructed images. Since tomb chamber and wall densities correspond to minimum and maximum densities respectively, segmentation uses density thresholds to extract these structures based on voxel density values. For the tomb chamber:

$$r_i^{\text{chamber}} = 1, \text{ if } \rho_i < \text{threshold of chamber and } z < 0. \quad (18)$$

And for the walls:

$$r_i^{\text{wall}} = 1, \text{ if } \rho_i > \text{threshold of wall.} \quad (19)$$

Ideally, thresholds should be set according to structure densities in the phantom. Due to differences between reconstructed image densities and the phantom—especially higher tomb chamber densities compared to that of air in the phantom—an optimization approach finds suitable thresholds for segmentation. Because chamber density is lower than loess, while wall density is higher, the chamber threshold is searched within $(0, 1.6 \text{ g/cm}^3]$ and wall threshold within $(1.6 \text{ g/cm}^3, 2.7 \text{ g/cm}^3]$, using 0.1 g/cm^3 steps. For each threshold, J is calculated, with highest J taken as geometric similarity, denoted as J_{chamber} for the chamber and J_{wall} for walls. Thresholds are searched for reconstructed images from Sect. 3. Voxel sets for chamber and walls are extracted using

thresholds with highest J , and positions from segmentation are shown in Fig. 7.

Both the wall and tomb chamber geometries in the SIRT reconstruction result show noticeable overflow compared to the phantom. In contrast, the counterparts extracted from the SIRT-TV reconstructed image align better with the phantom. Table 1 lists the J -calculation results for the structures in the reconstructed images obtained using the two algorithms. Both J_{chamber} and J_{wall} in the SIRT-TV reconstruction results are superior to those obtained using SIRT. In particular, J_{wall} is 2.8 times that of SIRT.

4.2 Iterative convergence

Convergence speed is an important metric for evaluating iterative image reconstruction algorithms. In Sect. 4.1, J_{chamber} and J_{wall} of the reconstruction results are compared after the same number of iterations for both algorithms. However, the number of iterations required for each algorithm to converge may differ. To compare the convergence speeds of the two algorithms, they were iterated 150 times, and the curves showing the changes in J_{chamber} and J_{wall} with the number of iterations are plotted in Fig. 8. J_{chamber} and J_{wall} of SIRT-TV generally increased with the number of iterations until convergence, whereas those for SIRT initially increased but then decreased with additional iterations before convergence.

To quantitatively evaluate the convergence speed, we defined a convergence metric as follows:

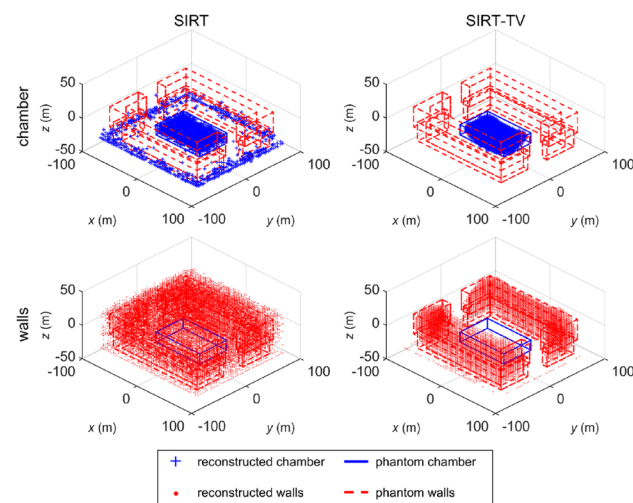


Fig. 7 (Color online) Comparison between the structures extracted from reconstructed images obtained using two algorithms and the structures in the phantom, based on the best density thresholds. The blue crosses and red dots represent the voxel positions of the chamber and the walls in the reconstructed images, respectively. The blue box outlines the boundaries of the tomb chamber in the phantom, and the red box outlines those of the walls

Table 1 J_{chamber} and J_{wall} of the reconstructed images obtained using SIRT and SIRT-TV

Algorithm	J_{chamber}	J_{wall}
SIRT-TV	0.5929	0.5498
SIRT	0.4611	0.1978

$$\Delta J/J = \frac{|J^{k+1} - J^k|}{J^k} < \varepsilon, \quad (20)$$

where k represents the number of iterations and ε is the threshold for convergence. The $\Delta J/J$ values for both algorithms are plotted in Fig. 8 as functions of the number of iterations. $\Delta J/J$ for both algorithms fluctuate while converging due to structure thresholds and voxelization discreteness. With ε set to 0.002, J_{chamber} and J_{wall} for SIRT-TV converge after 36 and 24 iterations, while SIRT has not yet converged. SIRT-TV reaches convergence faster due to TV-minimization constraint. In SIRT, J_{chamber} and J_{wall} converge at different speeds because the tomb chamber is a large cubic volume, while the walls are complex and thinner.

4.3 Impact of TV parameters on reconstruction

The parameter values in the TV-minimization algorithm may affect the quality of the reconstructed images. To assess this impact, we varied the maximum iteration number N_{TV} and the TV gradient descent step size α . J_{chamber} and J_{wall} are shown as functions of the number of iterations for different parameter values in Fig. 9.

For most (N_{TV}, α) combinations, convergence occurs within 50 iterations, indicating that choosing a maximum iteration number $N_{\text{max}} = 50$ as the termination criterion for SIRT-TV is reasonable. For J_{wall} , when $N_{\text{TV}} = 10$ and $\alpha = 0.1$, the trend resembles that of SIRT, and a continuous downward trend is visible after 150 iterations, showing the ineffectiveness of TV minimization. With different α values, J_{wall} is always the lowest when $N_{\text{TV}} = 10$. Unlike J_{wall} , for different α values (except $\alpha = 0.1$), J_{chamber} is always the

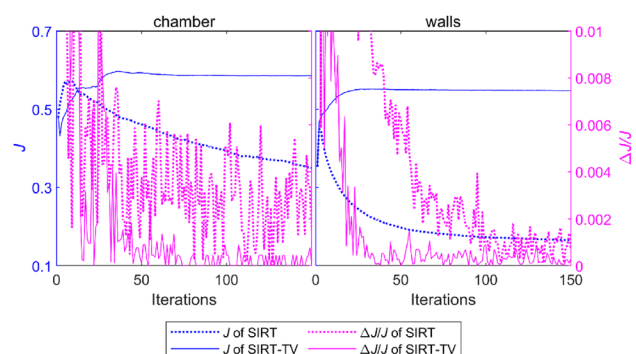


Fig. 8 (Color online) J and $\Delta J/J$ of the reconstructed images obtained by SIRT and SIRT-TV as functions of iterations

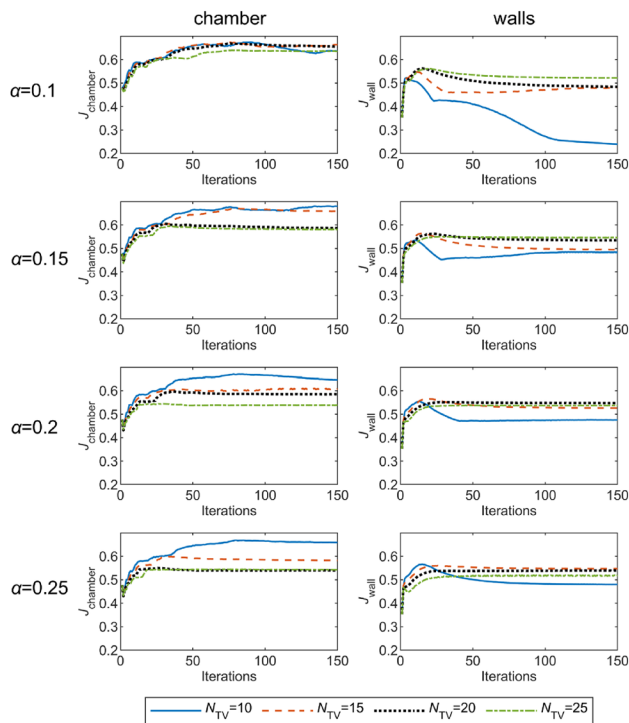


Fig. 9 (Color online) J_{chamber} and J_{wall} of reconstructed images obtained under different TV parameters, as functions of iterations

highest when $N_{\text{TV}} = 10$, and remains above 0.5 irrespective of the parameter values. This result suggests that compared to the walls, the reconstruction of the tomb chamber does not require a stringent TV constraint.

Overall, the different parameter values in Fig. 9 (except $N_{\text{TV}} = 10$ or $\alpha = 0.1$) do not significantly affect the reconstruction results, indicating that the SIRT-TV algorithm can reliably produce high-quality images.

4.4 Impact of measurement configuration and duration

As mentioned in Sect. 1, the main challenges in 3D reconstruction of muon tomography are the presence of noise and incomplete data. In Sect. 3, we simulated relatively ideal conditions to test the feasibility of the algorithm. This section discusses the impact of less-ideal measurement configurations and durations (i.e., fewer views and lower muon statistics) on reconstruction with SIRT-TV.

4.4.1 Measurement configuration

Using the same simulation data from Sect. 3, the number of views is reduced to four, and three different detector configurations are selected for reconstruction. The parameters of the image reconstruction algorithm were the same as those used in Sect. 3. Figure 10 shows slices of the phantom and

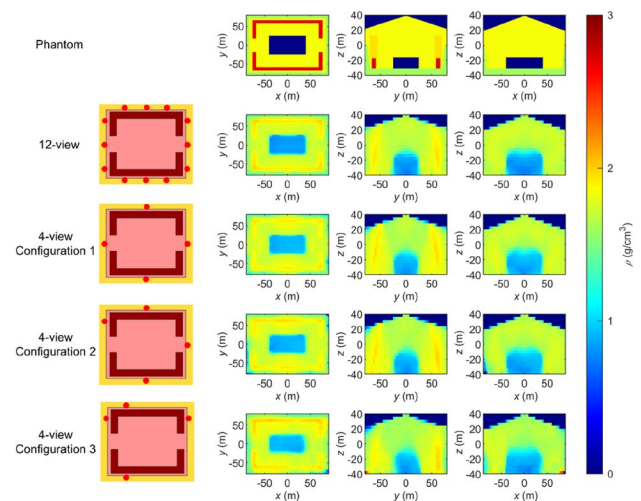


Fig. 10 (Color online) Slices of the phantom, and reconstructed images for the 12-view configuration and three 4-view configurations

the reconstructed images for the 12-view and three 4-view configurations. Reducing the number of views leads to more severe artifacts. Among these 4-view configurations, the symmetrical 4-view configuration shows better geometric symmetry in its reconstruction results, whereas the reconstruction results for the other two 4-view configurations exhibit distortions in the shapes of the tomb chamber and walls. The calculation results of J_{chamber} and J_{wall} in Table 2 also indicate that, among the three different 4-view configurations, the reconstructed image quality is the highest for the symmetrical 4-view configuration. These artifacts can be explained by the ray densities in the voxels. Given that the voxelization for each reconstruction is the same, more views lead to more rays, higher ray densities in the voxels, and more information regarding voxel density. When voxelization and the number of views were the same for each reconstruction, asymmetrical measurement configurations caused greater variation in ray densities across voxels. This implies that some voxels are crossed by only a few rays, making it difficult to reconstruct their densities. A symmetrical measurement configuration results in better reconstruction quality.

Table 2 J_{chamber} and J_{wall} of the images obtained from the 12-view reconstruction and three 4-view reconstructions

Measurement configuration	J_{chamber}	J_{wall}
12-view	0.5929	0.5498
4-view configuration 1	0.5161	0.4552
4-view configuration 2	0.4818	0.4452
4-view configuration 3	0.4576	0.3777

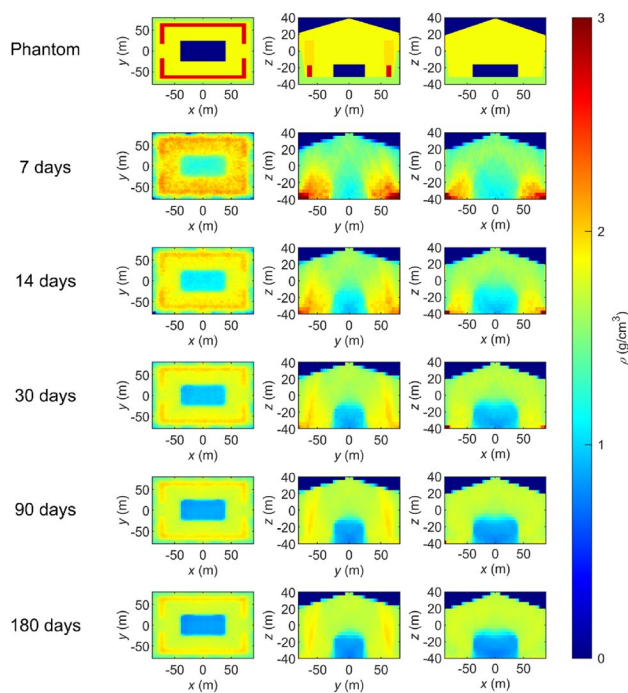


Fig. 11 (Color online) Slices of the phantom and SIRT-TV reconstructed images for different equivalent measurement durations

Table 3 J_{chamber} and J_{wall} of reconstructed images obtained using SIRT-TV under different equivalent measurement durations

Time (day)	J_{chamber}	J_{wall}
7	0.2615	0.2050
14	0.4514	0.2465
30	0.5123	0.3383
90	0.5525	0.4994
180	0.5929	0.5498

4.4.2 Measurement duration

The number of cosmic-ray muons detected by the detectors is proportional to the measurement duration. This affects the statistical fluctuations, which, in turn, affects the quality of the reconstructed image. To investigate the reconstructed image quality for different measurement durations, we select datasets from the beginning of the simulation data in Sect. 3, corresponding to 7, 14, 30, 90, and 180 d of equivalent measurement time. The algorithm parameters are the same as those used in Sect. 3. Slices of the reconstructed images are shown in Fig. 11.

In the reconstructed image slices for 7-d and 14-d measurement durations, severe artifacts appear between the chamber and walls, especially at the bottom. For measurement durations beyond 30 d, artifacts are suppressed, and the chamber-earth boundary becomes clearer. The 90-d and 180-d reconstructed image slices show similar results, with J_{chamber} and J_{wall} both ≈ 0.5 , as shown in Table 3.

5 Conclusion

In this study, we simulated 3D image reconstruction of muon tomography for the QinShiHuang tomb phantom using the SIRT-TV algorithm. The results show that compared to SIRT, the SIRT-TV algorithm effectively suppresses salt-and-pepper artifacts, improves geometric similarity J_{chamber} by nearly one-third and raises J_{wall} to nearly three times that of SIRT, and converges faster. This result indicates that SIRT-TV is more suitable for 3D image reconstruction of large-scale cultural heritage muon tomography. Using this algorithm with 12-view cosmic-ray muon data, locations and shapes of the tomb chamber and walls are reconstructed.

We investigated TV-minimization algorithm parameters' impact on image quality by varying maximum iteration number N_{TV} and TV gradient descent step size α . While optimal TV parameters differ between walls and tomb chamber reconstruction, selecting parameters within a reasonable range achieves high-quality reconstructions. Considering the potential constraints in experiments, this paper also discusses the impact of measurement configurations and durations on the reconstruction results. The results show that reducing the number of views from 12 to 4 decreases J . Among the 4-view configurations, J values are highest when detectors are positioned along ROI symmetry axes. For 12-view configuration, J values increase with equivalent measurement duration, with both J_{chamber} and $J_{\text{wall}} \approx 0.5$ after 90 d.

Although SIRT-TV shows improved reconstruction quality compared to SIRT, enhancement opportunities remain: Walls with different densities remain indistinguishable, and geometric similarity J can be improved. Future work could enhance image quality through machine learning techniques, using reconstructed images from varying viewpoints as training inputs.

While the QinShiHuang tomb phantom is simplified, it retains essential structural features of the underground palace. As a well-known imperial tomb, its structure is likely representative, and this simulation would provide insights for potential transmission muography experiments on other imperial tombs.

Author Contributions All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Ning Su, Yuan-Yuan Liu, Li Wang, Jian-Jie Zhang, Zhi Zhou, Xin Zhao, Wen-Wan Ding, Peng Zheng, and Jian-Ping Cheng. The first draft of the manuscript was written by Ning Su, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Data Availability The data that support the findings of this study are openly available in Science Data Bank at <https://cstr.cn/31253.11.sciedb.j00186.01010> and <https://doi.org/10.57760/sciedb.j00186.01010>.

Declarations

Conflict of interest Yuan-Yuan Liu is an editorial board member for Nuclear Science and Techniques and was not involved in the editorial review, or the decision to publish this article. All authors declare that there are no conflict of interest.

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