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Features of dual-purpose structure for heavy ion and light particles

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Abstract
We propose a dual-purpose magneto-optical lattice to accelerate both heavy ions and light particles. Dispersion modulation allows for the control of the transition energy for light particles, whereas minimal modulation is optimal for heavy ions to reduce intrabeam scattering. Our results demonstrate that both particle types achieve stable acceleration with minimal structural modifications, thereby ensuring efficient beam dynamics and luminosity.

Keywords Transition energy · Intrabeam scattering · Stochastic cooling · Resonant lattice · Dual-purpose structure

1 Introduction

Regardless of the purpose of the ring, always in the case of two modes, when multiply charged heavy particles and one or two charged light particles are accelerated, the problem arises of what the magneto-optical structure should look like to satisfy all the conditions of stable motion for both types of particles. Multiply charged particles have a prevailing heating effect due to intrabeam scattering, and light particles have a greater chance of crossing through the transition energy. All of these effects are of great importance for colliders, where luminosity plays a decisive role. When developing a lattice that meets all requirements for differently charged particles, it is fundamentally important to have a retunable structure without introducing design differences. This structure is called dual-purpose or simple, dual.

In the NICA collider the dual magneto-optical lattice opens up the prospect of accelerating both heavy ions, such as gold, and light particles like protons and deuterons. The design of this lattice requires a different approach, owing to the varying charge-to-mass ratios involved.

2 Light particles

In a classical regular lattice, the transition energy is approximately equal to the betatron tune, $\gamma_{\mathrm{tr}} \simeq \nu_x$ [1]. For the same magnetic rigidity $B\rho$, the maximum energy for light particles is greater than that for heavy ions owing to their charge-to-mass ratio. This means that the lattice structure for heavy ions optimized for operating up to a certain transition energy would require overcoming that energy to operate with light particles. Therefore, lattices with varying transition energies can be considered.

2.1 Transition energy

In general, the transition energy is determined by the momentum compaction factor

$$\alpha = \frac{1}{{\gamma_{\mathrm{tr}}}^2} = \frac{1}{C}\int_0^C \frac{D(s)}{\rho(s)}\mathrm{d}s, \tag{1}$$

where C is the orbit length, $D(s)$ is the dispersion function, and $\rho(s)$ is the radius of orbit curvature, and s is the longitudinal coordinate. This is a characteristic of the lattice and remains constant regardless of the particle type. In the first order, the slip factor $\eta = \eta_0 = 1/\gamma_{\mathrm{tr}}^2 - 1/\gamma^2$; thus, the frequency of the synchrotron oscillations $\omega_s \sim \eta$ tends to zero

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when the beam energy approaches the transition value. In this case, the adiabaticity of the longitudinal phase motion is violated, which leads to instabilities as well as the influence of nonlinear effects of higher orders of momentum spread δ. The introduction of modulation into the $D(s)$ or $\rho(s)$ function leads to variations in the momentum compaction factor and, consequently, the transition energy.

2.2 Superperiodic modulation

The equation for the dispersion function with biperiodic variable focusing [2]

$$\frac{\mathrm{d}^2 D}{\mathrm{d}s^2} + [K(s) + \varepsilon k(s)]D = \frac{1}{\rho(s)}, \tag{2}$$

where $K(s) = \frac{e}{p}G(s)$, $\varepsilon k(s) = \frac{e}{p}\Delta G(s)$, $G(s)$ is the gradient of magneto-optical lenses, $\Delta G(s)$ is the superperiodic gradient modulation. Here, is considered an additional perturbation to the regular one $\varepsilon k(s) = \sum_{k=0}^{\infty} g_k \cos(k\phi)$, where g_k is the k-th harmonic of the gradient modulation in the Fourier series expansion of the function. The solution for the superperiod momentum compaction factor is as follows for gradient modulation only:

$$\alpha_{\mathrm{s}} = \frac{1}{\nu^2}\left\{1 + \frac{1}{4(1-kS/\nu)}\left(\frac{\bar{R}}{\nu}\right)^4 \frac{g_k^2}{\left[1-(1-kS/\nu)^2\right]^2}\right\}, \tag{3}$$

where $\overline{R}_{\mathrm{arc}}$ is the average value of the curvature, $\nu = \nu_x$ is the betatron tune in horizontal plane on arc, S is the number of superperiods per arc length. Equation (3) considered without introducing curvature modulation, because of the possibility of introducing a variation in the transition energy into a stationary lattice. To increase the transition energy, it is necessary to reduce $\alpha_{\mathrm{s}} = 1/\gamma^2_{\mathrm{tr,\,arc}}$, meaning that the expression under the sum sign must be negative, which is realizable under the condition $kS/\nu_{x,\mathrm{arc}} > 1$.

First harmonic $k = 1$ has a dominant influence, the condition is implemented for $S = 4$, $\nu_{x,\mathrm{arc}} = 3$. Figure 1 shows 12 FODO cells per arc, 3 FODO cells are combined into one superperiod with complex transition energy $\gamma_{\mathrm{tr,\,arc}} = i8$ [3]. Thus, an integer number of betatron oscillations on the arc forms tune, which is a multiple of 2π, so the arc has the property of a first-order achromat. Straight sections can correct tuning for an entire ring to avoid betatron resonances. Moreover, by choosing the ratio of the superperiod and tuning for the arc, second-order achromat properties can be achieved. Such a structure was first implemented at the KAON factory [4], and later at J-PARC [5], neutrino factory [6].

For structure where the missing magnet technique is used, for one reason or another it can be also implemented (Fig. 2),

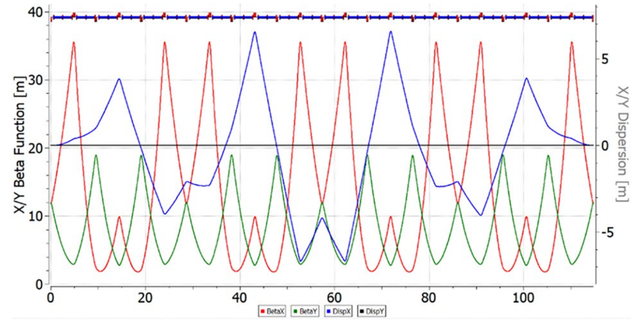

Fig. 1 (Color online) "Resonant" magneto-optic lattice with dispersion modulation and increased transition energy

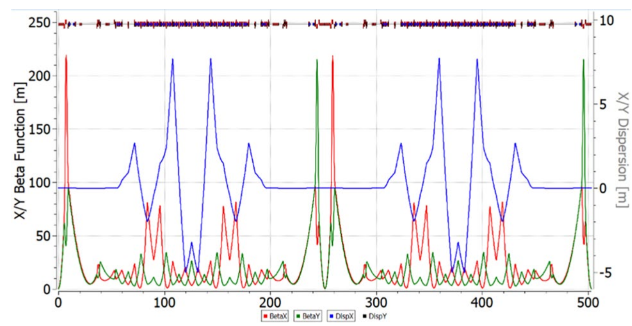

Fig. 2 (Color online) "Resonant" NICA magneto-optical adapted lattice with increased transition energy and missing magnet

but the dispersion at the edge of the arc must be suppressed [7]. The transition energy for the entire ring with straight sections is achieve $\gamma_{\mathrm{tr}} = 15$.

3 Heavy-ion mode

The lifetime of the beam luminosity in a collider experiment is achieved through the reduction of intrabeam scattering effects coupled with the application of stochastic and electron beam cooling techniques. This approach is particularly important for high-intensity ion beams. The temporal evolution of the emittance and momentum spread in the presence of cooling processes is governed by the following set of equations:

$$\begin{aligned} \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} &= \underbrace{-\frac{1}{\tau_{\mathrm{tr}}}\cdot\varepsilon}_{\text{cooling}} + \underbrace{\left(\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right)_{\mathrm{IBS}}}_{\text{heating}}, \\ \frac{\mathrm{d}\delta^2}{\mathrm{d}t} &= \underbrace{-\frac{1}{\tau_{\,\mathrm{long}}}\cdot\delta^2}_{\text{cooling}} + \underbrace{\left(\frac{\mathrm{d}\delta^2}{\mathrm{d}t}\right)_{\mathrm{IBS}}}_{\text{heating}}, \end{aligned} \tag{4}$$

where ε is the transverse emittance, τ_{tr} is the transverse cooling time, $\delta = \frac{\Delta p}{p}$ is the momentum spread, and τ_{long} is the longitudinal cooling time. For time-independent, stationary values, the time derivatives become zero, then

$$\varepsilon_{st} = \tau_{tr} \cdot \left(\frac{d\varepsilon}{dt}\right)_{IBS}\Bigg|_{\varepsilon=\varepsilon_{st}}, \quad \delta_{st}^2 = \tau_{long} \cdot \left(\frac{d\delta^2}{dt}\right)_{IBS}\Bigg|_{\delta^2=\delta_{st}^2}. \tag{5}$$

The benchmark for evaluating the effectiveness of a cooling technique can be determined by comparing the timescales of stochastic or electron cooling processes with the beam lifetime owing to IBS over the entire energy spectrum.

3.1 Stochastic cooling

Let's consider stochastic cooling using the approximate theory developed by D. Mohl [8, 9]. Based on the main findings, the cooling rate can be determined using the following expression:

$$\frac{1}{\tau_{tr,1}} = \frac{W}{N}[\underbrace{2g\cos\theta\left(1-1/M_{pk}^2\right)}_{\text{coherent effect(cooling)}} - \underbrace{g^2\left(M_{kp}+U\right)}_{\text{incoherent effect(heating)}}], \tag{6}$$

where $W = f_{max} - f_{min}$ is the system bandwidth, N is the effective number of particles recalculated based on the ratio of orbit length to the beam length, along with the particle distribution, g is the fraction of observed sample error corrected per turn, U is the ratio of noise to signal, M_{pk} and M_{kp} are the mixing factors between the pickup-kicker and the kicker-pickup, respectively. Equation (6) in the absence of noise at $g = g_0 = \frac{1-M_{pk}{}^2}{M_{kp}}$ reaches the maximum

$$\frac{1}{\tau_{tr}} = \frac{W}{N}\frac{\left(1-1/M_{pk}^2\right)^2}{M_{kp}}, \quad \frac{1}{\tau_l} = 2\frac{W}{N}\frac{\left(1-1/M_{pk}^2\right)^2}{M_{kp}}. \tag{7}$$

The mixing coefficients are defined as

$$M_{pk} = \frac{1}{2\left(f_{max}+f_{min}\right)\eta_{pk}T_{pk}\frac{\Delta p}{p}}, \quad M_{kp} = \frac{1}{2\left(f_{max}-f_{min}\right)\eta_{kp}T_{kp}\frac{\Delta p}{p}}, \tag{8}$$

where $\eta_{pk}T_{pk}\delta$ and $\eta_{kp}T_{kp}\delta$ are the relative particle displacement times (mixing), η_{pk} and η_{kp} are the slip factor, as a first approximation $\eta_{pk} = \alpha_{pk} - 1/\gamma^2$, $\eta_{kp} = \alpha_{kp} - 1/\gamma^2$, α_{pk} and α_{kp} are the first order of local momentum compaction factors, T_{pk} and T_{kp} are the absolute times between the pickup-kicker and kicker-pickup, respectively. The stochastic cooling times in Eq. (7) depend on the ratio of the effective particle density to the cooling system bandwidth and the properties of the magneto-optics and local momentum compaction factors α_{pk}, α_{kp}.

The maximum value of the frequency band is determined by the requirement that the "Schottky" beam bands do not overlap. In the simplest case, this can be expressed as

$$f_{max} < \frac{1}{\eta_{pk}T_{pk}\frac{\Delta p}{p}}, \tag{9}$$

thus, a mixing factor $M_{pk} > 1$. Otherwise, the cooling efficiency would become zero. Thus, it is desirable to achieve the highest possible frequency band for a given number of particles. From an electronic point of view, modern technologies allow for the implementation of a 10 GHz frequency band [10]; however, their use is not always feasible owing to the large magnitude of the slip factor η_{pk} and momentum spread δ.

Equation (6) is derived for the coasted beam. The particle density of a single harmonic RF resonator is described by a Gaussian distribution:

$$\rho(s) = \frac{N_{bunch}}{\sigma_{bunch}\sqrt{2\pi}} \cdot e^{-\frac{s^2}{2\sigma_{bunch}^2}}, \tag{10}$$

where s is the distance from the beam center, σ_{bunch} is the dispersion of the particle distribution, and N_{bunch} is the number of particles. Assuming that the cooling is at its minimum at the center ($s = 0$), the effective particle number at orbit length C_{orb} can be calculated as follows:

$$N = \frac{N_{bunch}}{4\sigma_{bunch}} \cdot C_{orb}. \tag{11}$$

For a beam generated by a multi-harmonic barrier-type RF system, so-called Barrier Bucket, the particle distribution in the beam can be considered approximately uniform along its entire length. The effective particle number is determined by the simple ratio of the beam length to the total orbit length:

To summarize, the effective number of particles depends on their distribution and is determined by their form factor $F_{bunch} = \sqrt{2\pi} \div 4$

$$N = N_{bunch} \cdot \frac{C_{orb}}{F_{bunch} \cdot \sigma_{bunch}}. \tag{12}$$

For example, let us consider the case of NICA with maximal form factor $F_{bunch} = 4$ with $C_{orb} = 503.04$ m, $\sigma_{bunch} = 0.6$ m, $N_{bunch} = 2.2 \times 10^9$. Considering the accumulated FNAL

[11] experience, the realistic values for the frequency band are $f_{max} = 8$ GHz and $f_{min} = 2$ GHz. For the NICA, $f_{max} = 4$ GHz and $f_{min} = 2$ GHz. With these parameters, the maximum achievable cooling rate was $1/\tau_{tr} = 1/230\,\text{s}^{-1}$.

Based on Eq. (8), asymptotic growth may occur in two scenarios:

1. slip factor approaches the value $\eta \rightarrow \frac{1}{2(f_{max}+f_{min})T_{pk}\delta}$, the beam Schottky spectrum becomes continuous and $M_{pk} \rightarrow 1$;
2. slip factor approaches zero, mixing between the kicker to the pickup does not occur and $M_{kp} \rightarrow \infty$.

The efficiency of stochastic cooling depends on the properties of the magneto-optical structure. In classical "regular" lattices, transition energy is acquired through the horizontal frequency $\gamma_{tr} \approx \nu_x$ and slip factor $\eta = 1/\gamma_{tr}^2 - 1/\gamma^2$ can achieve zero. To avoid asymptotic growth, it is necessary to vary the slip factor, that is, γ_{tr}. This is possible in "resonant" lattice, where transition energy can be increased or even reach complex value. In more exotic case, can be used "combined" lattice then η_{pk} (pickup-kicker) with real transition energy at one arc

$$\eta_{pk} = 1/\gamma_{tr}^2 - 1/\gamma^2 \tag{13}$$

compensated by η_{kp} (kicker-pickup) with complex transition energy at another

$$\eta_{kp} = -1/\gamma_{tr}^2 - 1/\gamma^2 \tag{14}$$

for the whole ring. This structure achieves the required ratio of mixing factors for a maximum cooling rate close to the ideal [12]. Let us delve deeper into the declared lattice in greater detail.

The behavior of the β-functions and D the dispersion across the entire "regular" ring is illustrated in Fig. 3 with $\gamma_{tr} = 7$. Straight sections, which remain constant in all lattices, are essential for analyzing the resonant characteristics of the entire structure. Their arrangement did not affect intrabeam scattering or transition energy. To suppress dispersion in the "regular" lattice, missing magnets technique implemented on both sides of the arc.

The "resonant" lattice is based on the principle of resonant modulation of the dispersion function and can be obtained from a "regular" one by introducing additional family of focusing quadrupoles. To suppress dispersion, two edge focusing quadrupoles on both sides of the arc or only two families of focusing quadrupoles on the arc can be used, when an integer number of betatron oscillations is reached.

The case of a "combined" lattice, one arc operates in a regular mode, while the other employs resonant modulation (Fig. 4). This choice was based on the principle of compensation, as described by Eqs. 13 and 14, which requires a greater modulation depth of the quadrupoles than in purely "resonant" lattice with increased transition energy.

As illustrated in Fig. 5, "resonant" optics with increased transition energy up to $\gamma_{tr} = 15$, the second asymptotic is at higher energy compared to the "regular" lattice. In "combined" magneto-optics, the cooling efficiency is closer to the ideal value in a large energy range from 2.5 to 4.5 GeV/u, while in "regular" optics the cooling rate is almost two times lower at the most optimal point ~ 3 GeV/u. This behavior is explained by the absence of a second point of asymptotic growth.

3.2 Intrabeam scattering

Intrabeam scattering represents a fundamental limitation on the beam lifetime in the collider. Consequently, the selection of an appropriate cooling technique depends on comparing its characteristic timescales with the rate at which the beam is heated owing to intrabeam scattering. This is derived from the fundamental principles that govern the process

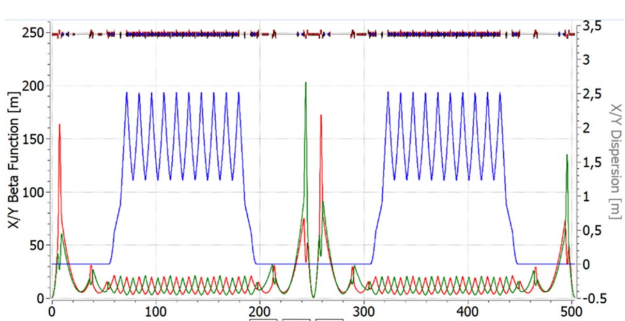

Fig. 3 (Color online) "Regular" FODO NICA magneto-optical lattice with missing magnets

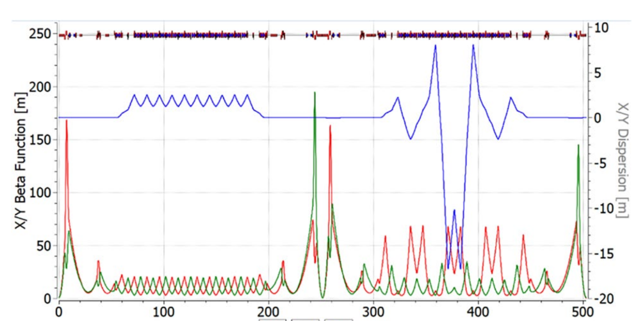

Fig. 4 (Color online) "Combined" NICA magneto-optical lattice with real and complex transition energies in arcs

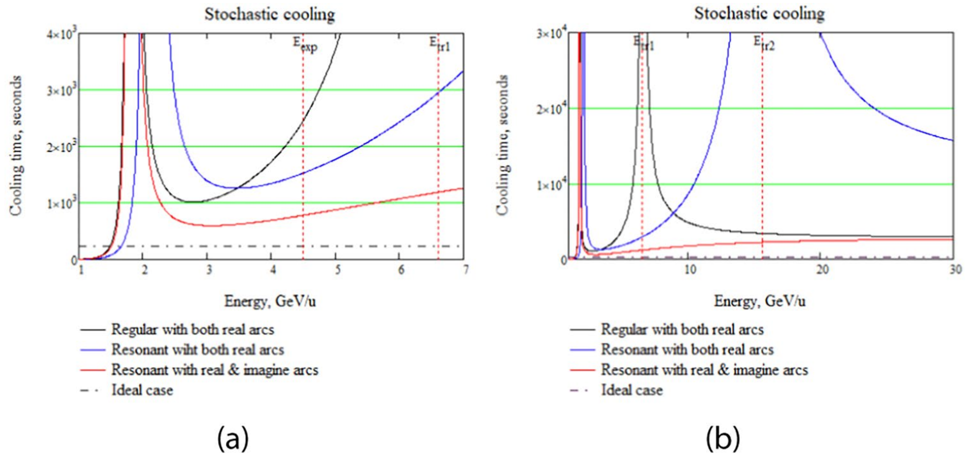

Fig. 5 (Color online) The dependence of stochastic cooling time on the energy for various lattices. Energy range **a** 1 – 7, **b** 0–30 GeV per nucleon

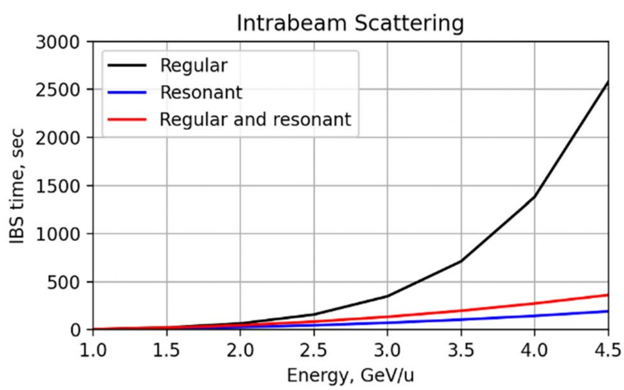

Fig. 6 (Color online) The dependence of the beam lifetime due to intrabeam scattering in “regular”, “resonant” and “combined” lattices on the beam energy for heavy-ion beam

Table 1 Main parameters of lattices

Lattice	Regular	Resonant	Combined
Energy, per nucleon (GeV/u)	4.5	12.6	12.6
Transition energy, γ_{tr}	7	15	$i50$
Modulation depth	–	25%	45%
Cooling time at 4.5 GeV/u (s)	2500	1500	800
Heavy ions IBS time at 4.5 GeV/u (s)	2500	400	250
Protons IBS time at 12.6 GeV (s)	1.8×10^4	4.5×10^3	7.9×10^3
Tunes	9.44/9.44	9.44/9.44	9.44/9.44

$$\frac{1}{\tau_{\mathrm{IBS}}} = \frac{\sqrt{\pi}}{4} \frac{cZ^2 r_{\mathrm{p}}^2 L_{\mathrm{C}}}{A} \cdot \frac{N}{C_{\mathrm{orb}}} \cdot \frac{\langle \beta_x \rangle}{\beta^3 \gamma^3 \varepsilon_x^{5/2} \left\langle \sqrt{\beta_x} \right\rangle} \times \times \left(\left\langle \frac{D_x^2 + \dot{D}_x^2}{\beta_x^2} \right\rangle - \frac{1}{\gamma^2} \right) \tag{15}$$

Unlike stochastic cooling, the IBS rate increases as decreasing energy $1/\gamma^3$. In addition, the expressions in parentheses are proportional to the slip factor η. Therefore, it is expected that in optics with a value η close to zero, the heating rate should decrease. Figure 6 shows the dependences of the heating time constant in the three above-mentioned lattices calculated using MADX programs [13] for the parameters of the heavy-ion beam $^{197}_{79}$Au of the NICA collider with maximum luminosity 10^{27} cm^{-2}s^{-1}. In the context of light nuclei, such as protons and deuterons, the IBS time significantly increases as the charge decreases. The corresponding IBS times for heavy and light beams are presented in Table 1 for beam intensities $N_{\mathrm{heavy}} = 2.2 \times 10^9$ ppb and $N_{\mathrm{light}} = 1 \times 10^{12}$ ppb with a number of bunches $n_{\mathrm{bunch}} = 22$. Consequently, at the experimental energy, IBS times differ by approximately 10 times, so the issue of intrabeam scattering becomes critical for heavy-ion beams.

From the comparison of the IBS lifetime with the cooling time it can be concluded that in a regular lattice, stochastic cooling is able to balance intrabeam scattering in the energy range $W \geq 4.5$ GeV/u. To apply stochastic cooling over the entire energy range, it is obvious that the luminosity of the beam at low energies must be sacrificed by increasing the emittance. In the resonant lattices, the IBS time was notably reduced. This is explained by the fact that the structure has a greater ratio $\left\langle \frac{D_x^2+\dot{D}_x^2}{\beta_x^2} \right\rangle$ between the dispersion and the beam β function than in the case of a regular function. Thus, for heavy ions, the configuration should be regular and minimally modulated. Electron cooling was used in the regular lattice to reduce the beam by 4.5 GeV/u [14, 15].

4 Summary

The dual magneto-optical structure is proposed for accelerating both heavy ion and light particle beams, exemplified by the NICA facility. For light particles, owing to their charge-to-mass ratio, the experimental energy can rise above the lattice transition energy, which is optimal for heavy ions. Using dispersion modulation, transition energy increases or even reaches a complex value in a "resonant" lattice. However, owing to the modulation of β-function and D dispersion, the intrabeam scattering time decreases, which is crucial for multiply charged heavy particles. For this reason, a "regular" lattice with minimally modulated dispersion and β-function is optimal in the heavy-ion mode. Despite the fact that stochastic cooling in "regular" lattices is significantly weaker than in "resonant" and "combined" ones, it can compensate IBS effect.

No special changes are required to convert the "regular" lattice into a "resonant" one. This is sufficient to introduce only a separate family of quadrupoles.

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