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RESEARCH HIGHLIGHT

Beyond axial symmetry: high-energy collisions unveil the ground-state shape of ^{238}U

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How does the strong force shape the structure of atomic nuclei? The STAR collaboration at the BNL Relativistic Heavy Ion Collider (RHIC) demonstrate that ultra-relativistic collision experiments give key insights into this fundamental question. From dedicated measurements in ^{238}U+^{238}U collisions at 100 GeV/nucleon energy, the STAR collaboration determine the deformed shape of the ^{238}U nucleus, showing in particular that the experimental observables probe the elusive ground-state *triaxiality* of this isotope. These results pave the way to systematic characterizations of ground-state nuclear properties at high-energy colliders.

How do we reveal the microscopic structure of a quantum many-body system? With electromagnetic probes, the method is as old as nuclear physics itself: photons energetic enough are shot through a target to take snapshots of *frozen* configurations of its inner content. Attosecond laser pulses, for which the Nobel prize in physics was awarded in 2023, enable us today to resolve the motion of electrons at the atomic scale (10^{-10} m). Going down to nuclear or sub-nuclear scales (10^{-15} m), things become more challenging, as direct imaging via electromagnetic probes becomes unfeasible. Over many decades, low-energy nuclear structure experiments have characterized the electromagnetic properties of nuclei (charge radii, transition probabilities, ...) with exquisite precision for thousands of states, which can then be used to benchmark theoretical models of nuclear structure. However, a more direct imaging of the nuclear ground state requires gluon-mediated strong interactions, acting on time scales faster than any internal nuclear dynamics: 1 yoctosecond, or 10^{-24}s, to be compared with the typical time scale of few-Mev nuclear excitations, on the order of 10^{-21}s. A new analysis by the STAR collaboration [1] demonstrates in particular that ion-ion collisions at the highest energies achieved at RHIC provide us with a tool to image ground-state nuclear geometries, with an application to the ^{238}U nucleus.

A ultra-relativistic nucleus-nucleus collision occurs among the protons and the neutrons, collectively referred to as *nucleons*, within the colliding nuclei. Nucleon-nucleon interactions (between two different nuclei) shape, thus, the density of matter formed in the interaction region, or quark-gluon plasma (QGP). Consider now a head-on collision where the two ions collide via the full overlap of their geometries, such that most of the nucleons are involved in the process. In quantum mechanical terms, this implies that the collisions probe in full the complexity of the wave functions of the incoming nuclei. After averaging over millions of events, the measured observables probe indeed non-trivial expectation values computed with respect to these nuclear states [2]. Due to the fermionic nature of the constituent nucleons, and to the complexity of the strong nuclear force, i.e., the residual of quantum chromodynamics (QCD) through which such nucleons (within a nucleus) interact, nuclei are in general strongly-correlated systems characterized by emergent collective phenomena at various energy scales, including notable long-range many-nucleon correlations. These phenomena impact the *geometry* of the QGP formed in the high-energy collisions, with important observable implications.

Remarkably, low-energy nuclear structure physics tells us that we can visualize collective correlations of nucleons within a nucleus through the notion of the *nuclear shape*. A nucleus can indeed be modeled as a density of matter in a fictitious intrinsic nuclear frame which is randomly oriented with respect to the lab frame. Observables measured in the laboratory are then obtained via rigid rotations of the intrinsic density, which effectively correlate nucleons in space. Typically, the shape corresponds to some radial profile associated with a surface deformed through the Bohr expansion:

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$$R(\theta,\phi) = R_0\left[1 + \beta_2\left(Y_2^0(\theta)\cos\gamma + Y_2^2(\theta,\phi)\sin\gamma\right)\right], \quad (1)$$

where $Y_\ell^m(\theta,\phi)$ is a real-form spherical harmonic, and $R_0 \approx 1.2A^{1/3}$. The surface in Eq. (1) has a *quadrupole*, ellipsoidal deformation. The value of β_2 determines the overall elongation of the ellipsoid along the symmetry axis, with well-deformed nuclei having $\beta_2 \approx 0.3$. The parameter $0 \leq \gamma \leq 60°$ determines instead the relative imbalance between the three principal axes. The ellipsoid is axially symmetric for γ=0 (prolate ellipsoid) and γ=60° (oblate ellipsoid), and *triaxial* otherwise. The STAR collaboration has achieved a determination of the structure of ^{238}U [1]. Figure 1 displays the intrinsic shape of a uranium nucleus with parameters β_2 and γ consistent with the STAR results.

The collisions obliterate the incoming nuclei: We only observe hadrons emitted to the final states. How do we reconstruct the nuclear shape from the measured particle distributions? We use the fact that the QGP behaves like a relativistic fluid, such that its motion is controlled by *deterministic* macroscopic laws. Local conservation of momentum leads in particular to the Euler equation, $\mathbf{F} \propto \nabla P$, stating that the flow build-up is driven by pressure-gradient forces. As the latter depend on the geometry of the QGP's energy density, one can infer straightforward connections between geometric properties of the initial QGP and bulk properties of the final particle distributions in momentum space. We follow the cartoon made by the STAR collaboration, reported in Fig. 2, for head-on collisions. A fluid that expands in vacuum from an elliptical shape, such as that formed when the deformed nuclei interact in a body-body configuration (upper panels), is driven by pressure gradients presenting a quadrupole modulation in space. This leads to *elliptic flow*, or v_2, in the final-state distributions. Tip-tip configurations (lower panels) lead instead to a minimal elliptic flow, but they maximize the average transverse momentum, $[p_T]$, due to higher gradients that form in the more compact interaction region. From Fig. 2 we learn that we can discern body-body and tip-tip geometries by analyzing the interplay between v_2 and $[p_T]$: large v_2 implies small $[p_T]$, and vice versa. The anti-correlation between these variables due to the deformation of ^{238}U was first predicted in Ref. [3]. It was later realized that the same mechanism probes as well the value of γ [4].

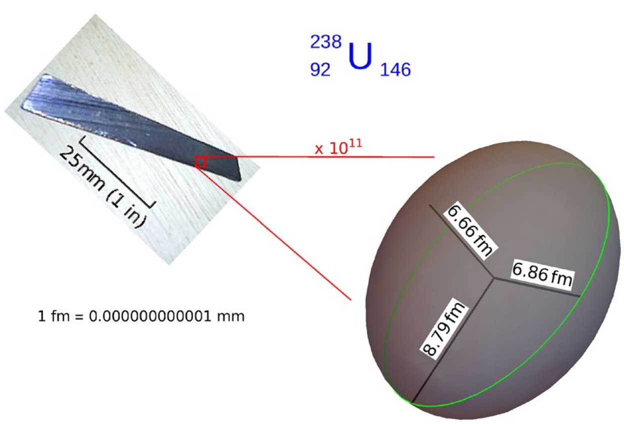

Fig. 1 (Color online) A sample of Uranium-238 is made of atoms whose nuclei can be visualized as femtometer-sized [$\mathcal{O}(10^{-15})$ m] ellipsoids with random orientations in space. In their lowest-energy state, the shape of these ellipsoids can be determined with precision in experiments on ultra-relativistic ^{238} U+^{238}U collisions, as done by the STAR collaboration at the BNL RHIC [1]. The ellipsoid shown here corresponds to the parametrization of Eq. (1) with β_2=0.29 and γ=5°. These values are consistent with the extractions of the STAR collaboration. The structure of the ellipsoid reflects quantum many-body correlation effects within the ground state of the ^{238}U nucleus

The STAR collaboration analyzes two observables to determine the structure of ^{238}U, namely, the variance of the average transverse momentum, denoted by $\left\langle(\delta p_T)^2\right\rangle$, and the covariance of the average transverse momentum with the elliptic flow, denoted by $\left\langle v_2^2 \delta p_T\right\rangle$, where angular brackets mean averages over events in the limit of head-on collisions (in jargon, close to 0% centrality). As shown by Jiangyong Jia [5], one of the leaders of the STAR analysis, these observables have a simple leading dependence on the shape parameters:

$$\begin{aligned} \left\langle(\delta p_T)^2\right\rangle &= a_0 + a_1\beta_2^2, & a_0, a_1 > 0\,, \\ \left\langle v_2^2\delta p_T\right\rangle &= a_0' - a_1'\beta_2^3\cos(3\gamma), & a_0', a_1' > 0. \end{aligned} \quad (2)$$

The value of $\left\langle(\delta p_T)^2\right\rangle$ reflects fluctuations in the overall size of the QGP medium at a given centrality, which are enhanced by the quadrupole deformation. The value of $\left\langle v_2^2\delta p_T\right\rangle$ quantifies instead the interplay between the ellipticity and the size of the QGP. According to the explanations in Fig. 2, it should be negative when β_2 is high enough. To facilitate the extraction of the shape parameters, the STAR collaboration compares ^{238}U+^{238}U data to ^{197}Au+^{197}Au data, at the same energy, via ratios of observables. As the structure of ^{197}Au is robustly understood in low-energy nuclear physics, taking such ratios eliminates a number of uncertainties in the interpretation of the measurements, more specifically, the model uncertainties on the parameters $a=0,a=0',a_1$, a_1' in Eq. (2). This technique for the minimization of uncertainties has been pioneered by the STAR collaboration in the context of so-called *isobar collisions* [6], and its effectiveness has been demonstrated in several theoretical studies, in particular by Jiangyong Jia and Chunjian Zhang [7] (a STAR physicist and professor at Fudan University), as well as by other groups worldwide [8, 9]. The STAR results are shown in Fig. 3. Based on high-precision hydrodynamic computations, the STAR collaboration achieves an extraction of both the large β_2 and the small γ of ^{238}U. For β_2, the result

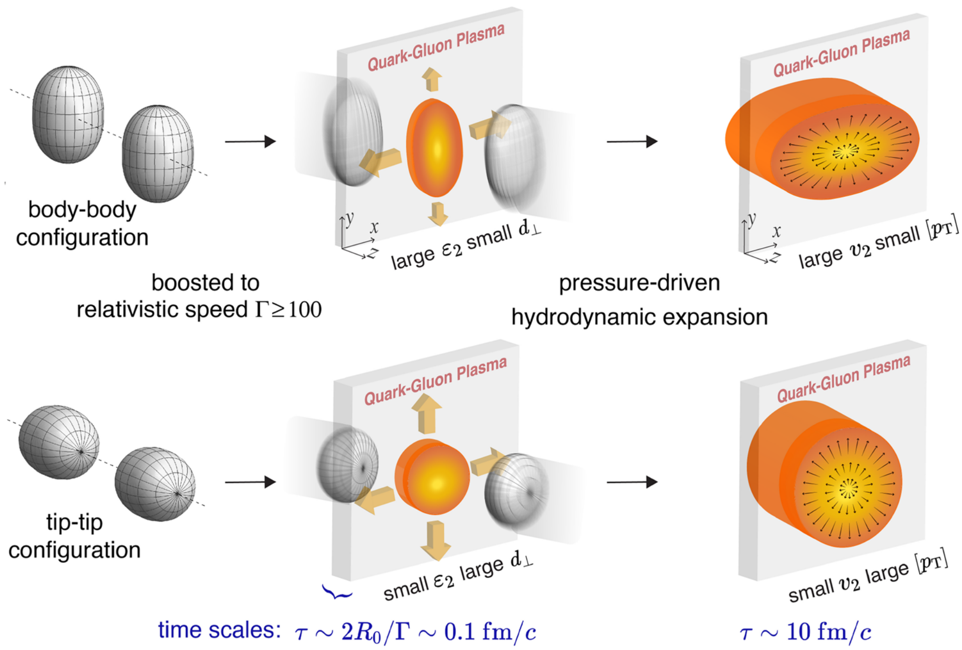

Fig. 2 (Color online) Method for imaging the nuclear quadrupole deformation in high-energy collisions. *Left:* in the limit of ultra-central collisions with the two nuclei fully overlapping in the transverse plane, the geometry of the interaction region can range from strongly elliptical (body-body collisions) to nearly circular (tip-tip collisions). *Middle:* body-body collisions thus maximize both the ellipticity, ε_2, and the overall transverse area, $d_\perp$, of the created QGP. Tip-Tip collisions, on the other hand, yield minimal ε_2 and $d_\perp$ values. *Right:* in terms of final-state observables resulting from the expansion of the QGP, which is driven by pressure-gradient forces, we shall observe that body-body collisions lead to the highest elliptic flow, v_2, and the lowest average transverse momentum, $[p_T]$, while tip-tip collisions have the lowest v_2 and the highest $[p_T]$. This leads to an observable anti-correlation between v_2 and $[p_T]$, driven by the large deformation of the ^{238}U nucleus, in the limit of ultra-central collisions. Figure adapted from Ref. [1]

is broadly consistent with the expectations of low-energy nuclear structure physics [10], where little is instead known about the value of γ.

In summary, the STAR collaboration demonstrate that high-energy nuclear smash-ups enable us to infer precise information about the ground-state structure of the collided species. Many applications are expected to follow. One example pertains to the triaxiality of well-deformed nuclei. This information is hardly accessible in low-energy experiments. Particularly in the region of rare-earth species, knowledge of the values of γ should yield much insight into the nuclear force and the origin of the nuclear shapes [11]. Further, accessing many-body correlations in the ground state of stable isotopes will aid in the search for neutrinoless double beta decay ($0\nu\beta\beta$), a conjectured lepton-number-violating process which, if observed, would shake our understanding of fundamental physics. This decay occurs between two isobaric nuclei. Theoretical calculations have demonstrated that the nuclear matrix elements (NME) that govern this transition depend much on the relative difference in low-resolution properties (e.g., deformations) between the two isobars [12, 13]. Therefore, if candidates for $0\nu\beta\beta$ decay were collided at RHIC or the CERN Large Hadron Collider (LHC), one could employ the methods developed by the STAR collaboration to provide new experimental results to benchmark calculations of the NME. All in all, a bright program of nuclear structure investigations at high-energy colliders lies ahead.

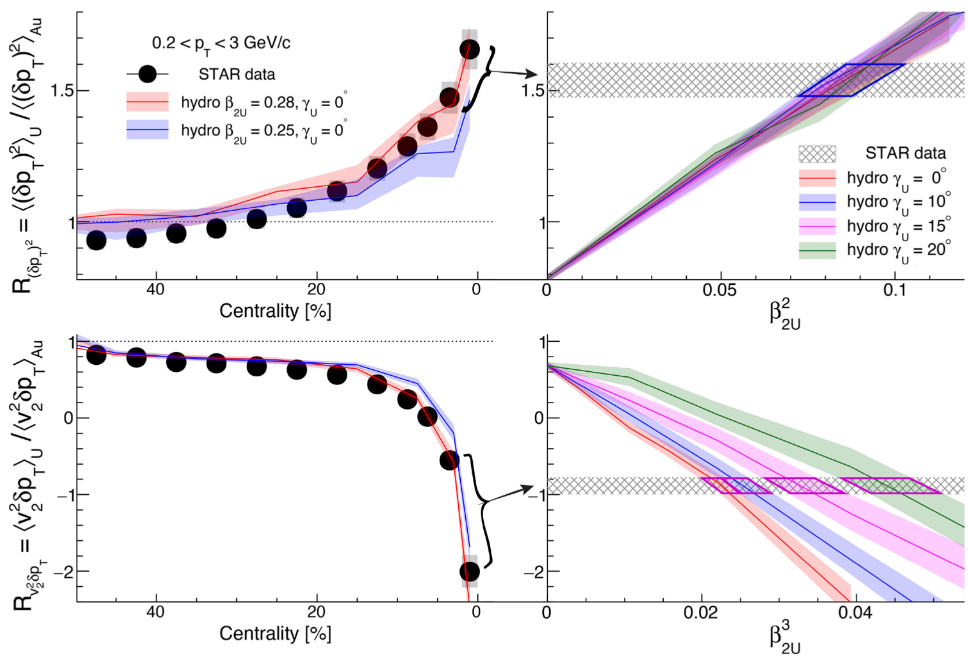

Fig. 3 (Color online) Measurements of the variance of the average transverse momentum, $\left\langle \left(\delta p_{\mathrm{T}}\right)^2 \right\rangle$, and of the statistical correlation, $\langle v_2^2 \delta p_{\mathrm{T}} \rangle$, between the squared elliptic flow, v_2, and the average transverse momentum, $[p_t]$, in the limit of fully-overlapping collisions (0% centrality) enable one to constrain the full quadrupole structure of the colliding ^{238}U ions. *Left:* Ratio between $\left\langle \left(\delta p_{\mathrm{T}}\right)^2 \right\rangle$ measured in ^{238}U+^{238}U collisions and the same quantity measured in ^{197}Au+^{197}Au collisions [top panel]. The same ratio is then constructed for $\langle v_2^2 \delta p_{\mathrm{T}} \rangle$ measured in the two systems [bottom panel]. The covariance in ^{238}U+^{238}U collisions becomes negative as we approach 0% centrality. This is driven by the nuclear quadrupole deformation parameter, β_2 as anticipated from the geometric arguments discussed in Fig. 2. Symbols are STAR collaboration results. Lines are hydrodynamic calculations implementing different values for β_2. *Right:* Sensitivity of the ratio of observables to the triaxiality parameter, γ, of ^{238}U. For both the top and the bottom panels, the hatched band represents the STAR result in the 0–5% centrality class. The lines (different colors) are hydrodynamic results for multiple input values of γ. As anticipated by Eq. (2), the ratio of $\left\langle \left(\delta p_{\mathrm{T}}\right)^2 \right\rangle$ constrains the value of β_2, while γ can be inferred from the ratio of $\langle v_2^2 \delta p_{\mathrm{T}} \rangle$. Figure adapted from Ref. [1]

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