



Quantifying the strength of magnetic fields using baryon electric charge correlation

Xu-Guang Huang^{1,2,3}

Received: 24 June 2024 / Revised: 24 June 2024 / Accepted: 26 June 2024 / Published online: 31 July 2024

© The Author(s), under exclusive licence to China Science Publishing & Media Ltd. (Science Press), Shanghai Institute of Applied Physics, the Chinese Academy of Sciences, Chinese Nuclear Society 2024

Although heavy-ion collisions generate strong magnetic fields, their direct measurement is a challenging task. A new observable, the baryon electric charge correlation, was recently found to be sensitive to the magnetic field strength and thus could be used as a magnetometer for heavy-ion collisions. Additionally, this observable may shed light on the equation of state and phase structure of quantum chromodynamics (QCD) under magnetic fields.

Determining and understanding the phase structure of quantum chromodynamics (QCD) is important in contemporary physics [1]. Owing to the confinement property of QCD, QCD matter at low temperatures and baryon densities is in the confined hadronic phase. To explore the deconfined phases, relativistic heavy-ion colliders have been constructed, such as the relativistic heavy-ion collider (RHIC) and Large Hadron Collider (LHC). These colliders can accelerate ions to relativistic energies and then make them collide to generate deconfined QCD matter in which quarks and gluons are the fundamental degrees of freedom. Such matter is usually referred to as quark–gluon plasma (QGP). Furthermore, relativistic heavy-ion collisions can generate

very strong magnetic fields [2–4] because the moving ions form two strong transient electric currents that induce a strong magnetic field along the reaction plane. Several interesting physical effects can be induced by a strong magnetic field including the well-known chiral magnetic effect (CME), which is the induction of an electric current along the direction of a magnetic field if the QGP contains net chirality [5, 6]. This intriguing effect is crucial for detecting the possible parity violations of QCD in hot environments. Over the past 15 years, experimental efforts from both RHIC and LHC have focused on searching for the CME [7–12]. Moreover, the strong magnetic field introduces new dimensions into the QCD phase diagram, prompting questions regarding the phase structure of QCD on the temperature and magnetic field plane or on the baryon density and magnetic field plane. In this aspect, the magnetic catalysis of chiral symmetry breaking at low temperatures and inverse magnetic catalysis at temperatures near the QCD crossover temperature are perhaps the most interesting phenomena. Thus, magnetic fields can help us enrich and deepen our understanding of QCD matter [13–15].

However, despite theoretical calculations elucidating the strength of magnetic fields at the moment of collision, the complicated temporal evolution of the magnetic fields in the QGP hinders the estimation of the magnetic field strength in heavy-ion collisions [16–20]. Hence, the availability of observables capable of detecting the strength of magnetic fields is invaluable. Recently, Ding et al. [21] proposed that the baryon electric charge correlation could serve as an observable. Employing lattice QCD, the authors calculated various correlations among conserved charges, revealing that the baryon electric charge correlation, denoted as χ_{11}^{BQ} , is the most sensitive to the magnetic field, and thus can serve as a magnetometer for QCD.

The absence of a sign problem is notable in lattice QCD simulations conducted under strong magnetic fields. However, the necessity of discretizing the magnetic field using integer values of the magnetic flux limits the magnetic

This work was supported by the Natural Science Foundation of Shanghai (No. 23JC1400200), National Natural Science Foundation of China (Nos. 12225502, 12075061, and 12147101), and National Key Research and Development Program of China (No. 2022YFA1604900).

✉ Xu-Guang Huang
huangxuguang@fudan.edu.cn

¹ Physics Department and Center for Particle Physics and Field Theory, Fudan University, Shanghai 200433, China

² Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Institute of Modern Physics, Fudan University, Shanghai 200433, China

³ Shanghai Research Center for Theoretical Nuclear Physics, National Natural Science Foundation of China and Fudan University, Shanghai 200438, China

field strength of the simulation. Specifically, the maximum achievable strength is constrained by the square of the inverse lattice spacing, whereas the minimum achievable strength is limited by the square of the inverse spatial lattice size. This study employed the largest magnetic flux of six, thereby making the discretization error associated with the magnetic field negligible. Furthermore, the analysis of the conserved charge fluctuations relies on continuum estimates derived from lattices with temporal sizes (N_τ) of 8 and 12. The consistency between these continuum estimates for both χ_{11}^{BQ} and μ_Q/μ_B (the ratio of the electric and baryon chemical potentials) was established through additional lattice QCD calculations performed on $N_\tau = 16$ lattices. Thus, the findings underscore the reliability of the continuum estimates based on $N_\tau = 8$ and 12 lattices when applied to lattice QCD simulations in strong magnetic fields.

The fluctuations and correlations considered in this study are defined by

$$\chi_2^{\text{B}} = \frac{\partial^2(P/T^4)}{\partial \hat{\mu}_B^2}, \chi_2^{\text{Q}} = \frac{\partial^2(P/T^4)}{\partial \hat{\mu}_Q^2}, \chi^{\text{BQ}} = \frac{\partial^2(P/T^4)}{\partial \hat{\mu}_B \partial \hat{\mu}_Q}, \quad (1)$$

evaluated at $\mu_B = \mu_Q = 0$. Here, P denotes the total pressure of the system; T is the temperature; and $\hat{\mu}_{B,Q} = \mu_{B,Q}/T$. These quantities are generally functions of eB . However, the lattice simulations showed that the correlation χ_{11}^{BQ} exhibited the highest sensitivity. The result of χ_{11}^{BQ} normalized by its value at zero magnetic field along the crossover transition line is shown in Fig. 1. The crossover temperature must be examined, because these fluctuations and correlations exhibit critical behavior, leading to a peak near the crossover temperature [22–24]. A strong dependence on eB is observed; as eB grows from zero to 0.15 GeV^2 , the ratio $\chi_{11}^{\text{BQ}}(eB)/\chi_{11}^{\text{BQ}}(0)$ approximately doubles. The crossover temperature also depends on eB ; however, this dependence

is very weak when $eB < 0.15 \text{ GeV}^2$, as shown in Fig. 2. The slowly decreasing behavior of T_{pc} versus eB is simply the inverse magnetic catalysis of the chiral phase transition which was discovered a decade ago in lattice simulations [25, 26].

Additionally, Fig. 1 presents the results from the hadron resonance gas (HRG) model, which explain the lattice data in the studied eB window. This demonstrates that the behavior of $\chi_{11}^{\text{BQ}}(eB)/\chi_{11}^{\text{BQ}}(0)$ has a thermodynamic basis.

Several interesting consequences can arise from these results, as discussed in the following section.

First, a STAR collaboration data analysis was recently performed primarily by a joint team of Fudan University, Brookhaven National Laboratory, University of California—Los Angeles, and Institute of Modern Physics of Chinese Academy of Sciences, led by Jinhui Chen, Diyu Shen, Yu-Gang Ma, Aihong Tang, Gang Wang, Aditya Prasad Dash, Subhash Singha, and Dhananjaya Thakur. This analysis revealed a nontrivial sign change in the directed-flow splitting of charged hadrons, such as $\Delta v_1^p = v_1^p - v_1^{\bar{p}}$ and $\Delta v_1^\pi = v_1^\pi - v_1^{\bar{\pi}}$, with increasing centrality [27, 28]. This strongly indicates the presence of magnetic fields. However, precisely extracting the strength of the magnetic field from v_1 splitting remains challenging, owing to the complex dynamic evolution of the hot medium and the magnetic field itself. This challenge stems largely from the absence of a robust model describing the coupled spacetime evolution of the hot medium and magnetic field. Now, the correlation χ_{11}^{BQ} emerges as a complementary observable, which provides a model-independent approach to quantify the magnetic field strength. This is because χ_{11}^{BQ} can be solely determined from the final-state hadron spectra (although, naturally, this approach remains subject to considerations, such as kinematic acceptance and detector corrections).

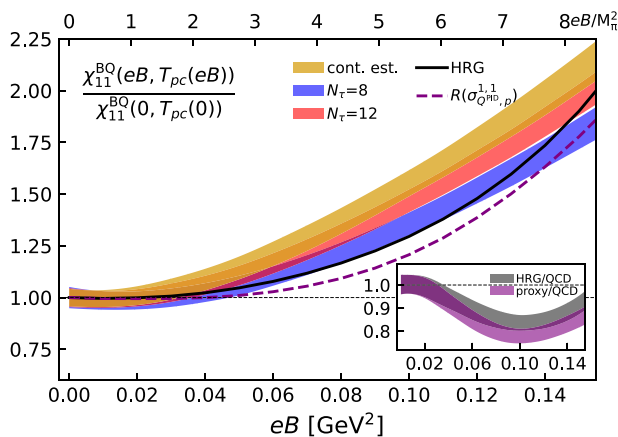


Fig. 1 (Color online) The ratio $\chi_{11}^{\text{BQ}}(eB)/\chi_{11}^{\text{BQ}}(0)$ at the crossover temperature. The result from the HRG model (dashed line) is also shown (Ref. [21])

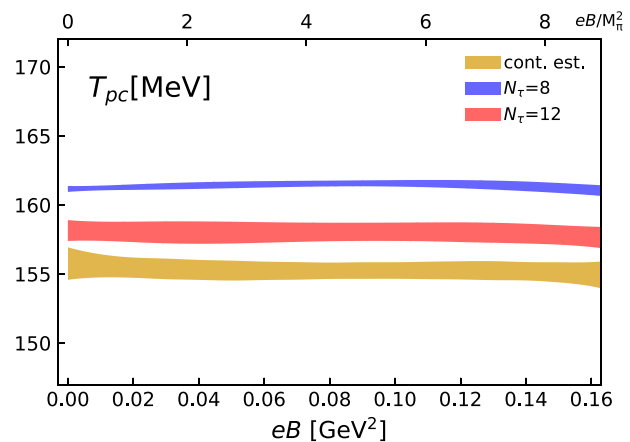


Fig. 2 (Color online) Crossover temperature as a function of eB for $eB \lesssim 0.16 \text{ GeV}^2$ (Ref. [21])

Second, the results show that the fluctuations χ_2^B and χ_2^Q exhibit a weaker dependence on the magnetic field within the studied parameter region. This is somewhat surprising, as the electric charge fluctuation is expected to be sensitive to electromagnetic fields. However, this behavior can be understood by considering the HRG model, which yields similar trends around the crossover temperature. Additionally, some insights can be gained from the high-temperature limit, where the system should predominantly consist of massless free quarks (and free gluons that do not interact with magnetic fields). In this scenario, as eB approaches zero, it can be shown that $\chi_2^B = \chi_2^Q/2 = 1/3$ and $\chi_{11}^{BQ} = 0$. Thus, when eB is turned on but remains weak (compared with the temperature), χ_{11}^{BQ} becomes sensitive to eB [29, 30].

Third, an intriguing insight is provided in Fig. 2 of Ref. [21], which illustrates the contributors to χ_2^B , χ_2^Q , and χ_{11}^{BQ} . According to the HRG model, within the considered range of eB , the primary contribution to χ_2^Q stems from charged pions, whereas the largest contribution to χ_2^B originates from protons, although other hadrons also make significant contributions. However, in the case of χ_{11}^{BQ} , protons dominate at $eB \lesssim 4m_\pi^2$, whereas doubly charged $\Delta(1232)$ baryons surpass protons at $eB \gtrsim 4m_\pi^2$. Because the proton contribution remains approximately constant with eB , the eB dependence of χ_{11}^{BQ} is primarily controlled by $\Delta(1232)$ baryons. This hinders the practical utilization of χ_{11}^{BQ} as a magnetometer in heavy-ion collisions, because $\Delta(1232)$ baryons are not directly measurable owing of their rapid decay into protons and pions. However, after accounting for such decays, the measurement of χ_{11}^{BQ} become quite reliable if a proxy for χ_{11}^{BQ} is constructed [21].

Finally, several interesting future directions were obtained. The lattice results for χ_2^B , χ_2^Q , and χ_{11}^{BQ} can be used to construct the equation of state of QCD matter under finite magnetic fields, small baryons, and electric chemical potentials. Higher-order fluctuations and correlations are necessary for determining a more precise equation of state or extending it to larger chemical potentials. Moreover, higher-order fluctuations and correlations are often considered as sensitive indicators of critical phenomena. Therefore, investigating the magnetic field dependence of these higher-order fluctuations and correlations near the crossover temperature is of considerable interest. Furthermore, the lattice results require the calculation of χ_{11}^{BQ} in other models that may complement the HRG model. Such studies can provide deeper insight into the magnetic field dependence of various fluctuations and correlations among conserved charges.

Acknowledgements The author thanks Heng-Tong Ding and Yu-Gang Ma for discussions.

References

1. A. Bzdak, S. Esumi, V. Koch et al., Mapping the phases of quantum chromodynamics with beam energy scan. *Phys. Rep.* **853**, 1–87 (2020). <https://doi.org/10.1016/j.physrep.2020.01.005>. [arXiv:1906.00936](https://arxiv.org/abs/1906.00936)
2. V. Skokov, A.Y. Illarionov, V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions. *Int. J. Mod. Phys. A* **24**, 5925–5932 (2009). <https://doi.org/10.1142/S0217751X09047570>. [arXiv:0907.1396](https://arxiv.org/abs/0907.1396)
3. W.T. Deng, X.G. Huang, Event-by-event generation of electromagnetic fields in heavy-ion collisions. *Phys. Rev. C* **85**, 044907 (2012). <https://doi.org/10.1103/PhysRevC.85.044907>. [arXiv:1201.5108](https://arxiv.org/abs/1201.5108)
4. J. Błoczynski, X.G. Huang, X. Zhang et al., Azimuthally fluctuating magnetic field and its impacts on observables in heavy-ion collisions. *Phys. Lett. B* **718**, 1529–1535 (2013). <https://doi.org/10.1016/j.physletb.2012.12.030>. [arXiv:1209.6594](https://arxiv.org/abs/1209.6594)
5. D.E. Kharzeev, L.D. McLerran, H.J. Warringa, The Effects of topological charge change in heavy ion collisions: “Event by event P and CP violation”. *Nucl. Phys. A* **803**, 227–253 (2008). <https://doi.org/10.1016/j.nuclphysa.2008.02.298>. [arXiv:0711.0950](https://arxiv.org/abs/0711.0950)
6. K. Fukushima, D.E. Kharzeev, H.J. Warringa, The chiral magnetic effect. *Phys. Rev. D* **78**, 074033 (2008). <https://doi.org/10.1103/PhysRevD.78.074033>. [arXiv:0808.3382](https://arxiv.org/abs/0808.3382)
7. K. Hattori, X.G. Huang, Novel quantum phenomena induced by strong magnetic fields in heavy-ion collisions. *Nucl. Sci. Tech.* **28**, 26 (2017). <https://doi.org/10.1007/s41365-016-0178-3>. [arXiv:1609.00747](https://arxiv.org/abs/1609.00747)
8. Y.C. Liu, X.G. Huang, Anomalous chiral transports and spin polarization in heavy-ion collisions. *Nucl. Sci. Tech.* **31**, 56 (2020). <https://doi.org/10.1007/s41365-020-00764-z>. [arXiv:2003.12482](https://arxiv.org/abs/2003.12482)
9. D.E. Kharzeev, J. Liao, P. Tribedy, Chiral magnetic effect in heavy ion collisions: the present and future. [arXiv:2405.05427](https://arxiv.org/abs/2405.05427)
10. D.E. Kharzeev, J. Liao, Chiral magnetic effect reveals the topology of gauge fields in heavy-ion collisions. *Nat. Rev. Phys.* **3**, 55–63 (2021). <https://doi.org/10.1038/s42254-020-00254-6>. [arXiv:2102.06623](https://arxiv.org/abs/2102.06623)
11. X.L. Zhao, G.L. Ma, Y.G. Ma, Electromagnetic field effects and anomalous chiral phenomena in heavy-ion collisions at intermediate and high energy. *Acta Phys. Sin.* **72**, 112502 (2023). <https://doi.org/10.7498/aps.72.20230245>
12. Q.Y. Shou, J. Zhao, H.J. Xu et al., Progress on the experimental search for the chiral magnetic effect, the chiral vortical effect, and the chiral magnetic wave. *Acta Phys. Sin.* **72**, 112504 (2023). <https://doi.org/10.7498/aps.72.20230109>
13. V.A. Miransky, I.A. Shovkovy, Quantum field theory in a magnetic field: from quantum chromodynamics to graphene and Dirac semimetals. *Phys. Rept.* **576**, 1–209 (2015). <https://doi.org/10.1016/j.physrep.2015.02.003>. [arXiv:1503.00732](https://arxiv.org/abs/1503.00732)
14. G. Cao, Extremely strong magnetic field and QCD phase diagram. *Nucl. Tech.* (in Chinese) **46**, 040003 (2023). <https://doi.org/10.11889/j.0253-3219.2023.hjs.46.040003>
15. H. Ding, S. Li, J. Liu, Progress on QCD properties in strong magnetic fields from lattice QCD. *Nucl. Tech.* (in Chinese) **46**, 040008 (2023). <https://doi.org/10.11889/j.0253-3219.2023.hjs.46.040008>
16. X.G. Huang, Electromagnetic fields and anomalous transports in heavy-ion collisions—A pedagogical review. *Rept. Prog. Phys.* **79**, 076302 (2016). <https://doi.org/10.1088/0034-4885/79/7/076302>. [arXiv:1509.04073](https://arxiv.org/abs/1509.04073)

17. L. Yan, X.G. Huang, Dynamical evolution of a magnetic field in the preequilibrium quark-gluon plasma. *Phys. Rev. D* **107**, 094028 (2023). <https://doi.org/10.1103/PhysRevD.107.094028>. arXiv:2104.00831
18. J.J. Zhang, X.L. Sheng, S. Pu et al., Charge-dependent directed flows in heavy-ion collisions by Boltzmann-Maxwell equations. *Phys. Rev. Res.* **4**, 033138 (2022). <https://doi.org/10.1103/PhysRevResearch.4.033138>. arXiv:2201.06171
19. Z. Wang, J. Zhao, C. Greiner et al., Incomplete electromagnetic response of hot QCD matter. *Phys. Rev. C* **105**, L041901 (2022). <https://doi.org/10.1103/PhysRevC.105.L041901>. arXiv:2110.14302
20. H. Li, X.L. Xia, X.G. Huang et al., Dynamic calculations of magnetic field and implications on spin polarization and spin alignment in heavy ion collisions. *Phys. Rev. C* **108**, 044902 (2023). <https://doi.org/10.1103/PhysRevC.108.044902>. arXiv:2306.02829
21. H.T. Ding, J.B. Gu, A. Kumar et al., Baryon electric charge correlation as a magnetometer of QCD. *Phys. Rev. Lett.* **132**, 201903 (2024). <https://doi.org/10.1103/PhysRevLett.132.201903>. arXiv:2312.08860
22. Y. Zhang, D. Zhang, X. Luo, Experimental study of the QCD phase diagram in relativistic heavy-ion collisions. *Nucl. Tech. (in Chinese)* **46**, 040001 (2023). <https://doi.org/10.11889/j.0253-3219.2023.hjs.46.040001>
23. K. Xu, M. Huang, QCD critical end point and baryon number fluctuation. *Nucl. Tech. (in Chinese)* **46**, 040005 (2023). <https://doi.org/10.11889/j.0253-3219.2023.hjs.46.040005>
24. Q. Chen, G. Ma, J. Chen, Transport model study of conserved charge fluctuations and QCD phase transition in heavy-ion collisions. *Nucl. Tech. (in Chinese)* **46**, 040013 (2023). <https://doi.org/10.11889/j.0253-3219.2023.hjs.46.040013>
25. G.S. Bali, F. Bruckmann, G. Endrodi et al., QCD quark condensate in external magnetic fields. *Phys. Rev. D* **86**, 071502 (2012). <https://doi.org/10.1103/PhysRevD.86.071502>. arXiv:1206.4205
26. G.S. Bali, F. Bruckmann, G. Endrodi et al., The QCD phase diagram for external magnetic fields. *J. High Energy Phys.* **02**, 044 (2012). [https://doi.org/10.1007/JHEP02\(2012\)044](https://doi.org/10.1007/JHEP02(2012)044). arXiv:1111.4956
27. M.I. Abdulhamid, B.E. Aboona, J. Adam et al., Observation of the electromagnetic field effect via charge-dependent directed flow in heavy-ion collisions at the Relativistic Heavy Ion Collider. *Phys. Rev. X* **14**, 011028 (2024). <https://doi.org/10.1103/PhysRevX.14.011028>. arXiv:2304.03430
28. S. Chen, Colossal magnetic field detected in nuclear matter. *APS Phys.* **17**, 31 (2024). <https://doi.org/10.1103/Physics.17.31>
29. A. Bazavov, T. Bhattacharya, C.E. DeTar et al., Fluctuations and correlations of net baryon number, electric charge, and strangeness: a comparison of lattice QCD results with the hadron resonance gas model. *Phys. Rev. D* **86**, 034509 (2012). <https://doi.org/10.1103/PhysRevD.86.034509>. arXiv:1203.0784
30. H.T. Ding, S.T. Li, Q. Shi et al., Fluctuations and correlations of net baryon number, electric charge and strangeness in a background magnetic field. *Eur. Phys. J. A* **57**, 202 (2021). <https://doi.org/10.1140/epja/s10050-021-00519-3>. arXiv:2104.06843