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A novel encoding mechanism for particle physics

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Abstract
This study proposes a novel particle encoding mechanism that seamlessly incorporates the quantum properties of particles, with a specific emphasis on constituent quarks. The primary objective of this mechanism is to facilitate the digital registration and identification of a wide range of particle information. Its design ensures easy integration with different event generators and digital simulations commonly used in high-energy experiments. Moreover, this innovative framework can be easily expanded to encode complex multi-quark states comprising up to nine valence quarks and accommodating an angular momentum of up to 99/2. This versatility and scalability make it a valuable tool.

Keywords Multi-quark state · Encoding mechanism · Constituent quark · Particle physics

1 introduction

With the continuous development of high-energy heavy-ion collision experiments, an increasing number of new particles has been identified. The experimental characteristics of these particles, including mass, angular momentum, spin, and symmetry, are measured with increasing accuracy. Every year, the particle data group (PDG), operating at the Lawrence Berkeley Laboratory, in collaboration with the International Organization for Particle Physics Cooperation, releases updated electronic versions of the Review of Particle Physics. These comprehensive reviews have typically been published in prominent international journals during even-numbered years [1–4]. They provide detailed experimental data on various leptons, mesons, and baryons, along with their mass, angular momentum, symmetry, isospin, and other properties, such as full width and decay modes.

Initially, when the number of particles was limited, a simplistic classification using a few letters was often employed. Later, it was further subdivided based on quantum numbers such as isospin and angular momentum. However, with the continuous advancement of experimental technology, an increasing number of new peaks in the mass spectrum have been identified as new particles that share similar quantum properties. Consequently, it is necessary to add the mass of each particle to distinguish between them. For instance, π, $\pi(1300)$, $\pi(1800)$, $\pi(2070)$, $\cdots$, $a_1(1260)$, $a_1(1640)$, $a_1(2096)$, $a_1(2270)$, $\cdots$ have been newly identified [5, 6]. In computer simulation research on these experiments, it is necessary to assign a unique identification code to each particle. The concept of particle encoding was introduced by Yost et al. in 1988 [7], with significant revisions implemented in 1998 [8]. However, over the past two decades, the identification of newly discovered particles has been required. It has become evident that the approach of adding mass to identify particles is not sufficient for encoding, because different excited states with the same quark composition require different

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encoding. Experimental measurements that are considered a mixture of several possible particles also require separate encoding. However, in various simulation transport models, the excited states of a given particle with the same constituent quarks are often coded by program developers in a simple sequence based on their increasing mass trend, such as $\pi^0 - 111$, $\pi(1300)^0 - 100111$, $\pi(1800)^0 - 9010111$, $\pi(2070)^0 - ?$ [9], which lacks universality and sustainability.

In particular, the encoding of multi-quark states has become an urgent issue. In experiments, exotic states, in particular some four-quark and five-quark states ($X(3872)$ [10], $Z_c(3900)$ [11], $Z_c(4020)^{\pm}$ [12, 13], $X_1(2900)$ [14], T_{cc}^+ [15] etc.), have been discovered in the past 20 years. The fundamental theory of strong interaction, i.e., quantum chromodynamics (QCD), does not prohibit the existence of multi-quark states if they satisfy $q^m\bar{q}^n, m-n=3k(k=0,1,2,\cdots)$ [16]. Therefore, more exotic states are expected to be explored in the future. Efforts have been made to unify naming conventions for exotic hadrons [17]. The existing encoding mechanisms cannot encode these particles, which poses a challenge for current computer simulations when dealing with multi-quark states [18]. However, this cannot be solved by simply adding encoding bits. Therefore, a novel particle encoding mechanism is required.

This study proposes a durable and rational encoding mechanism to unify the treatment of normal and exotic hadrons. This mechanism may serve for an extensive period of time in the future. The remainder of this paper is organized as follows: In Sect. 2, we go over the naming rules in PDG data. Section 3 explains the proposed encoding mechanism. Section 4 explains the encoding of some common particles according to the proposed encoding mechanism. Finally, a summary is presented in Sect. 5.

2 Particle naming rules from PDG

The particle information disseminated annually by the PDG, which reflects up-to-date experimental confirmations, adheres to specific classification rules and nomenclature that can be summarized as follows:

1. Quarks and leptons are predominantly denoted by the initial letter of their names. A few exceptions to this rule are due to convention, e.g., the photon is denoted by the symbol γ.
2. Non-strange light mesons, characterized with zero net strangeness and heavy flavor quantum numbers (i.e., S = C = B = 0), are typically represented by lowercase letters, such as $\pi, \rho, \omega, \eta, \phi, a, f, b, h$. They are further divided into two subgroups based on their spin angular momentum S:

   $S=0: \pi, \eta, b, h$
   $S=1: \rho, \omega, \phi, a, f$

   In each subgroup, the total angular momentum of the particle is marked with a subscript, such as $\pi_2, \phi_3, \cdots$. However, it was observed that there were particles with the same spin and total angular momenta but different masses. To distinguish these particles, their masses, in units of MeV/c^2, are added in parentheses after the symbol, e.g., $\pi_1(1400)$, $\pi_1(1600)$, $f_1(1500)$, $f_1(1710)$. The electric charge is also specified as a superscript. It can be omitted for isospin singlet ($I=0$) states where only one charge value is possible for each particle.
3. Other mesons in the form of $q\bar{q}'$ are labeled with uppercase letters. Particles with spin 1 are represented by an additional asterisk in the upper right corner to distinguish them from those with spin 0, while the total angular momentum is added in the lower right corner. The charge property of these particles is derived from their internal quark composition.

   K Strange light mesons (S = ±1, C = B = 0) such as $K^+ = u\bar{s}, K^0 = d\bar{s}, \overline{K^0} = \bar{d}s, K^- = \bar{u}s$, others K_J^* similar.

   D Charm meson (C = ±1); if there is also S = ±1, it is represented by adding s to the lower right corner of the letter. For example, D_{s1}^{*+} represents a meson with a strange quark and angular momentum of 1 for the $c\bar{s}$ combination.

   B The bottom meson (B = ±1), similar to the charm meson, may also contain strange quarks. Therefore, the subscript may also be labeled as s. For example, B_{s2}^* represents a meson with a strange quark and angular momentum of 2 for the $s\bar{b}$ combination. The bottom meson may also contain charm; therefore, it is marked with the subscript c, e.g., B_c^+.
4. Heavy quarkonium refers to mesons in the form of $c\bar{c}$, $b\bar{b}$ combinations. They are represented by $\eta_c, \eta_b, h_c, h_b, \psi, \chi, Z_c, Z_b, \Upsilon$, etc.
5. Baryons are represented in uppercase letters, except for proton p and neutron n.

(a) Comprising only two types of light quarks, i.e., up and down, they can be divided into N and Δ series based on their isospin, $I = 1/2, 3/2$, respectively.
(b) Containing one strange quark, it can be divided into Λ and Σ series based on its isospin, $I = 0, 1$, respectively.
(c) The Ξ series contains two strange quarks and one up or down quark, while Ω consists of three strange quarks.
(d) Baryons containing a charm quark are marked with the subscript c, e.g., $\Lambda_c, \Sigma_c, \Xi_c, \Omega_c, \Xi_{cc}, \Omega_{cc}$.
(e) Baryons containing a bottom quark are marked with the subscript b, e.g., $\Lambda_b, \Xi_b, \Omega_b$.

6. The four-quark and five-quark states have not yet been listed as a separate category in the 2022 edition of the PDG (and possibly not in the 2024 edition either). In its section 'Monte Carlo Particle Numbering Scheme', only two codes of pentaquarks are listed. A naming method is proposed in Ref. [17] that uses T and P as the main names, supplemented by superscripts and subscripts, to indicate the composition and quantum properties of the corresponding particle. Mass, in units of MeV/c^2, must be added in parentheses, and the charge superscript must also be added where is appropriate. For particles with certain information, this naming method is applied in the following context.

In the transport simulation of high-energy elementary particle collisions and nuclear collisions, it is necessary to convert the aforementioned naming rules into numerical encoding. On the one hand, it is convenient to identify a large number of particles coexisting in the collision system; on the other hand, it is necessary to enable the program to read diverse information about the corresponding particles from their encoding.

3 Novel particle encoding scheme

In the proposed particle encoding mechanism, we focus on constituent quarks by expanding the information content. However, we preserve the original encoding ideas as much as possible to make our encoding scheme easy to implement. These ideas have been adopted in most event generators and transport simulation programs, such as PYTHIA [19] and PACIAE [20]. Numbers below 100 are reserved for special purposes, same as the existing encoding approaches [19, 20].

1. Numbers 1--10 are assigned to quarks whereas numbers 11--20 are assigned to leptons. There are also numbers reserved for possible fourth- and fifth-generation quarks and leptons beyond the standard model (see Table 1). Their corresponding anti-particles are identified with corresponding negative numbers.
2. Two-digit numbers in the range 21–30 are assigned to gauge bosons and Higgs in the standard model (see Table 1).
3. The boson content of a two-Higgs-doublet scenario and that of additional $SU(2) \times U(1)$ groups are assigned to the range 31–40.
4. 'One-of-a-kind' exotic particles are assigned numbers in the range 41–80.
5. The numbers 81-100 are reserved for generator-specific pseudoparticles and concepts.
6. In principle, di-quark states can also be fully encoded using the proposed encoding mechanism. Without considering their excited states, a simple four-digit code can represent their two different spin combinations. The first two digits are the flavor codes of the quarks with larger absolute value ranking first. The third digit is '0', whereas the last digit is '1' or '3', representing their corresponding spin as '0' or '1', i.e., a simple code for the di-quark state will be in the form of $qq'01$ or $qq'03$, such as "3201" (us quark combination with spin 0).

Table 1 Particle encoding in the standard model

Name	Code	Name	Code	Name	Code	Name	Code
d	1	e^-	11	g	21	h^0/H_1^0	31
u	2	ν_e	12	γ	22	Z'/Z_2^0	32
s	3	μ^-	13	Z^0	23	Z''/Z_3^0	33
c	4	ν_μ	14	W^+	24	W'/W_2^+	34
b	5	τ^-	15	G	25	H^0/H_2^0	35
t	6	ν_τ	16			A^0/H_3^0	36
b'	7	τ'^-	17			H^+	37
t'	8	$\nu_{\tau'}$	18				

The other composite particles are encoded based on the information of their component quarks, including the total number of quarks, number of positive quarks, quark composition, total spin, total angular momentum, isospin, radial quantum number, orbital quantum number, and three types of symmetry, namely G parity, parity, and charge conjugation (GPC), represented by P_3. The general form of the particle encoding is a string of numbers:

$N\ n\ q_N \cdots q_{N-n+1} \cdots q_1\ n_s\ n_J\ n_I\ n_r\ n_L\ P_3$.

Figure 1 represents this encoding.

The total number of digits in the particle code is $N+9$. Specific details are as follows:

1. The first number, 'N', represents the total number of constituent quarks in the particle (the proposed encoding mechanism is able to encode particles composed of up to nine quarks, and this can be easily extended to particles with more than nine quarks, which are not expected to be discovered for now). The second number, 'n', indicates the number of positive quarks in it. The following N-digit numbers are used to specify the component quarks; they are arranged as follows:

   (a) The n positive quarks are ranked first in descending order using the quark number code listed in Table 1, while the $N-n$ anti-quarks are ranked in descending order of their absolute values after the positive quarks.

   (b) The distinction between positive and negative particles is no longer defined according to the sign before the particle code. It depends on the summation of the charges of the constituent quarks. Thus, the PDG convention for positive or negative particles is maintained. The encoding of the anti-particle of a given particle is assigned by adding a negative sign before its code, i.e., a negative sign indicates an operation in which, if a negative sign is added before the code, all the constituent quarks require their anti-quark code according to the following expression:

$$\begin{aligned} &-Nnq_N \cdots q_{N-n+1} \cdots q_1 n_s \cdots \\ &= N(N-n) q_{N-n} \cdots q_1 q_N \cdots q_{N-n+1} n_s \cdots \end{aligned} \tag{1}$$

   Thus, it is straightforward to encode the particles in event generators and transport simulation programs once the constituent quarks are known. This does not require the knowledge whether it is a particle or an anti-particle.

2. The spin information of the particle is placed next to the quark composition and only requires one digit. Therefore, the total spin $2S+1$ should not exceed 9, which leads to $S \leq 4$. Thus, only particles with a total spin less than or equal to 4 can be encoded; still, it is sufficiently large for practical encoding.
3. The total angular momentum quantum number is in units of 1/2. Therefore, the total quantum number of the angular momentum of a particle with code J is $J \times \frac{1}{2}$. To consider high-angular-momentum particles, this information is represented by two digits, and the maximum total angular momentum can be encoded as 99/2.
4. The isospin quantum number, also measured in units of 1/2, only requires one digit.
5. n_r is used to label the radial quantum number for the excited states of particles. The radial quantum number is $1, 2, \cdots$.
6. The orbital quantum number n_L takes values of $0, 1, 2, 3, \cdots$, corresponding to $S, P, D, F, \cdots$, respectively. Both n_r and n_L require a single digit.
7. The last digit, P_3, represents a binary code composed of three symmetric combinations of GPC, which is converted back to decimal representation. Each symmetry is represented by '0' and '1' as '−' and '+', respectively. Items without symmetry are also represented by '0'. For example, the combination of GPC symmetry '+-+' is represented by the number '5'.

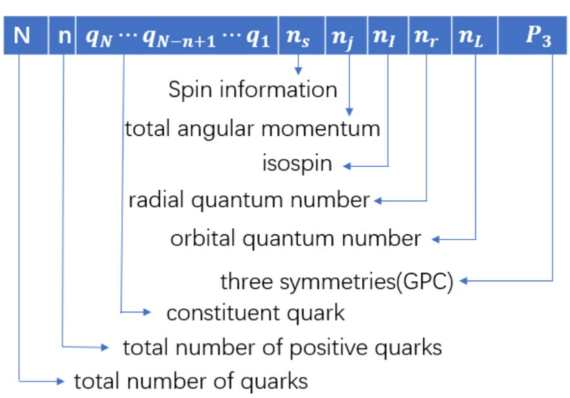

Fig. 1 encoding diagram

4 Particle encoding examples

According to the constituent quark model [21], the quantum numbers of the total angular momentum J, parity P, and charge-conjugation parity C (for charge-neutral states) of a meson ($q\bar{q}$ system) are given by

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}, \quad P = (-)^{L+1}, \quad C = (-)^{L+S}, \tag{2}$$

where L is the relative orbital angular momentum between q and $\bar{q}$ and S is the total spin. Thus, all possible JPCs

Table 2 J^{PC} that the $q\bar{q}$ system allows (up to D-wave)

L	S	J^{PC}	L	S	J^{PC}	L	S	J^{PC}
0	0	0^{-+}	1	0	1^{+-}	2	0	2^{-+}
0	1	1^{--}	1	1	0^{++}	2	1	1^{--}
			1	1	1^{++}	2	1	2^{--}
			1	1	2^{++}	2	1	3^{--}

can be expressed as listed in Table 2. Combinations $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ are not allowed in conventional $q\bar{q}$ systems [22]. In other words, they are exotic states with these quantum numbers. For example, $\pi_1(1400)$, $\pi_1(1600)$, $\pi_1(2015)$, and η_1 with $J^{PC} = 1^{-+}$ have been measured experimentally. Some studies suggested that these may be tetraquarks [23]. For convenience, we list the codes of some common particles in the proposed encoding mechanism, most of which have been discovered experimentally. The names of the particles (column 'notation' in the tables) and the corresponding information of the isospin, angular momentum, parity, etc. (columns $I^G(J^{PC})$ in the tables) were taken from PDG [4]. The information in the $n^{2S+1}L_j$ column was mostly obtained from the literature (Refs. [24, 25] for light mesons mainly), while the question mark '?' in column 'Ref.' indicates some uncertainty. If a question mark is immediately followed by a number or symbol, then this number or symbol is a possible speculation about the value represented by the question mark.

Table 3 summarizes the particle codes for light mesons. Note that the quark combination $d\bar{d}$ represents a mixed state of $u\bar{u}$ and $d\bar{d}$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}).$$

Table 4 lists the codes for some strange, charmed bottom mesons, and heavy quarkonium. For two particle states with the same quantum numbers but different lifetimes, such as K^0_S and K^0_L, a strategy of subtracting and adding 1 at the spin bit to distinguish their encodings is applied. This strategy can also be applied to particles which are the two different mass eigenstates, such as B^0_L, B^0_H. Different models were used for the excited states of the D and D_s meson series in Refs. [28] and [30], yielding similar computational results. However, there are multiple structural models of heavy quarkonium. For example, $Z_c(3900)$ can be assumed to be a hadro-quarkonium, $c\bar{c}$ [49], $\bar{D}^*D$ molecule [50], or a tetraquark [51]. Because of the increase in charmonium-like states, it was not easy to experimentally determine whether they were hybrid, molecular, or multi-quark states, so they were all represented by X(xxxx), Y(xxxx), Z(xxxx), (where xxxx is the particle mass in units of MeV/c^2), collectively referred to as XYZ particles [52]. For instance, $X(3823)$, $X(3872)$, $Y(4260)$, $Z_c(3900)\cdots$, and $X(3872)$ are represented as $\chi_{c1}(3872)$; $X(3823)$ is similar to $\psi_2(1^3D_2)$ [17, 31]. However, in computer simulation programs, these particles can have a definite quark composition; therefore, there must be a definite encoding. Table 4 lists the representative particles in the PDG classified into heavy flavor quarkonium classes. The symbols '?' in the table must be verified. Some assignments after '?' are only tentative, that is, they are the best estimates reported so far.

Table 5 lists the particle codes for certain baryons. The name, isospin, total angular momentum, and symmetry information of these particles were obtained from Ref. [4], whereas the radial and orbital quantum numbers were mostly obtained from theoretical calculations of the hypercentral constituent quark model (hCQM) [53]. In hCQM, the three-body interaction of quarks inside a baryon is described in the form of the Jacobi coordinates ρ and λ, which are combinations of the inter-quark distance r_i,

$$\rho = \frac{1}{\sqrt{2}}(r_1 - r_2),$$
$$\lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3),$$

and ultimately reduces to the hyperradius x and hyperangle ξ,

$$x = \sqrt{\rho^2 + \lambda^2}, \quad \xi = \arctan\frac{\rho}{\lambda}.$$

The total wave function can be expressed as ρ-and λ-type harmonic oscillators, which can provide radial and orbital quantum numbers. The values of n and L in the tables were mostly derived from calculations reported in the literature, which are listed in column 'References'. Some entries marked with '?' indicate that the values are indeterminate. However, this does not affect the proposed encoding mechanism, because the code can be encoded with temporary values until the final confirmation. Different charge states

Table 3 Summary of encoding of light unflavored mesons

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
π^+	$u\bar{d}$	$1^-(0^-)$	1^1S_0	[24]	21211002101	$\rho(770)$	$d\bar{d}$	$1^+(1^{--})$	1^3S_1	[24]	21113022104
π^-	$d\bar{u}$	$1^-(0^-)$	1^1S_0	[24]	21121002101	$\omega(782)$	$u\bar{u}$	$0^-(1^{--})$	1^3S_1	[24]	21223020100
π^0	$d\bar{d}$	$1^-(0^{-+})$	1^1S_0	[24]	21111002101	$\phi(1020)$	$s\bar{s}$	$0^-(1^{--})$	1^3S_1	[24]	21333020100
η	$u\bar{u}$	$0^+(0^{-+})$	$?^1?_0$	?1?S	21221000105	$\rho(1450)$	$d\bar{d}$	$1^+(1^{--})$	2^3S_1	[24]	21113020204
η'	$s\bar{s}$	$0^+(0^{-+})$	$?^1?_0$	?1?S	21331000105	$\omega(1420)$	$u\bar{u}$	$0^-(1^{--})$	2^3S_1	[24]	21223020200
$\pi(1300)$	$d\bar{d}$	$1^-(0^{-+})$	2^1S_0	[24]	21111002201	$\phi(1680)$	$s\bar{s}$	$0^-(1^{--})$	2^3S_1	[24]	21333020200
$\eta(1295)$	$u\bar{u}$	$0^+(0^{-+})$	2^1S_0	[24]	21221000205	$\rho(1900)$	$d\bar{d}$	$1^+(1^{--})$	3^3S_1	[24]	21113020304
$\eta(1475)$	$s\bar{s}$	$0^+(0^{-+})$	2^1S_0	[24]	21331000205	$\omega(1960)$	$u\bar{u}$	$0^-(1^{--})$	3^3S_1	[24]	21223020300
$\pi(1800)$	$d\bar{d}$	$1^-(0^{-+})$	3^1S_0	[24]	21111002301	$\phi(2170)$	$s\bar{s}$	$0^-(1^{--})$	3^3S_1	[24]	21333020300
$\eta(1760)$	$u\bar{u}$	$0^+(0^{-+})$	3^1S_0	[24]	21221000305	$\rho(2265)$	$d\bar{d}$	$1^+(1^{--})$	4^3S_1	[24]	21113020404
$\eta(2100)$	$s\bar{s}$	$0^+(0^{-+})$	3^1S_0	[24]	21331000305	$\omega(2205)$	$u\bar{u}$	$0^-(1^{--})$	4^3S_1	[24]	21223020400
$\pi(2070)$	$d\bar{d}$	$1^-(0^{-+})$	4^1S_0	[24]	21111002401	$\rho(2490?)$	$d\bar{d}$	$1^+(1^{--})$	5^3S_1		21113020504
$\eta(2010)$	$u\bar{u}$	$0^+(0^{-+})$	4^1S_0	[24]	21221000405	$\omega(?)$	$u\bar{u}$	$0^-(1^{--})$	5^3S_1		21223020500
$\eta(2225)$	$s\bar{s}$	$0^+(0^{-+})$	4^1S_0	?	21331000405	$a_0(1450)$	$d\bar{d}$	$1^-(0^{++})$	1^3P_0	[24]	21113002113
$\pi(2360)$	$d\bar{d}$	$1^-(0^{-+})$	5^1S_0	[24]	21111002501	$f_0(1370)$	$u\bar{u}$	$0^+(0^{++})$	1^3P_0	[24]	21223000117
$\eta(2320)$	$u\bar{u}$	$0^+(0^{-+})$	5^1S_0	[24]	21221000505	$f_0(1500)$	$s\bar{s}$	$0^+(0^{++})$	1^3P_0	[24]	21333000117
$\pi_2(1670)$	$d\bar{d}$	$1^-(2^{-+})$	1^1D_2	[24]	21111042121	$a_0(1710?)$	$d\bar{d}$	$1^-(0^{++})$	2^3P_0		21113002213
$\eta_2(1645)$	$u\bar{u}$	$0^+(2^{-+})$	1^1D_2	[24]	21221040125	$f_0(1724)$	$u\bar{u}$	$0^+(0^{++})$	2^3P_0	[24]	21223000217
$\eta_2(1870)$	$s\bar{s}$	$0^+(2^{-+})$	1^1D_2	[24]	21331040125	$a_0(2025)$	$d\bar{d}$	$1^-(0^{++})$	3^3P_0	[24]	21113002313
$\pi_2(1880)$	$d\bar{d}$	$1^-(2^{-+})$	2^1D_2	[24]	21111042221	$f_0(1992)$	$u\bar{u}$	$0^+(0^{++})$	3^3P_0	[24]	21223000317
$\eta_2(2030)$	$u\bar{u}$	$0^+(2^{-+})$	2^1D_2	[24]	21221040225	$f_0(2314)$	$s\bar{s}$	$0^+(0^{++})$	3^3P_0	[24]	21333000317
$\pi_2(2245)$	$d\bar{d}$	$1^-(2^{-+})$	3^1D_2	[24]	21111042321	$a_0(2265?)$	$d\bar{d}$	$1^-(0^{++})$	4^3P_0	[24]	21113002413
$\eta_2(2248)$	$u\bar{u}$	$0^+(2^{-+})$	3^1D_2	[24]	21221040325	$f_0(2189)$	$u\bar{u}$	$0^+(0^{++})$	4^3P_0	[24]	21223022417
$b_1(1235)$	$d\bar{d}$	$1^+(1^{+-})$	1^1P_1	[24]	21111022116	$\rho(1570)$	$d\bar{d}$	$1^-(1^{--})$	1^3D_1	[24]	21113022120
$h_1(1170)$	$u\bar{u}$	$0^-(1^{+-})$	1^1P_1	[24]	21221020112	$\omega(1670)$	$u\bar{u}$	$0^+(1^{--})$	1^3D_1	[24]	21223020124
$h_1(1380)$	$s\bar{s}$	$0^-(1^{+-})$	1^1P_1	[24]	21331020112	$\rho(1909)$	$d\bar{d}$	$1^-(1^{--})$	2^3D_1	[24]	21113022220
$b_1(?)$	$d\bar{d}$	$1^+(1^{+-})$	2^1P_1		21111022216	$\omega(2290)$	$s\bar{s}$	$0^+(1^{--})$	2^3D_1	[24]	21333020224
$h_1(1595)$	$u\bar{u}$	$0^-(1^{+-})$	2^1P_1		21221020212	$\rho(2149)$	$d\bar{d}$	$1^-(1^{--})$	3^3D_1	[24]	21113022320
$h_1(?)$	$s\bar{s}$	$0^-(1^{+-})$	2^1P_1		21331020212	$\rho_2(?)$	$d\bar{d}$	$1^+(2^{--})$	1^3D_2	[24]	21113042124
$b_1(1960)$	$d\bar{d}$	$1^+(1^{+-})$	3^1P_1	[24]	21111022316	$\omega_2(?)$	$s\bar{s}$	$0^-(2^{--})$	1^3D_2	[24]	21333040120
$h_1(1965)$	$u\bar{u}$	$0^-(1^{+-})$	3^1P_1	[24]	21221020312	$\rho_2(1940)$	$d\bar{d}$	$1^+(2^{--})$	2^3D_2	[24]	21113042224
$b_1(2240)$	$d\bar{d}$	$1^+(1^{+-})$	4^1P_1	[24]	21111022416	$\omega_2(1975)$	$u\bar{u}$	$0^-(2^{--})$	2^3D_2	[24]	21223040220
$h_1(2215)$	$u\bar{u}$	$0^-(1^{+-})$	4^1P_1	[24]	21221020412	$\rho_2(2225)$	$d\bar{d}$	$1^+(2^{--})$	3^3D_2	[24]	21113042324
$b_1(?)$	$d\bar{d}$	$1^+(1^{+-})$	5^1P_1		21111022516	$\omega_2(2195)$	$u\bar{u}$	$0^-(2^{--})$	3^3D_2	[24]	21223040320
$h_1(?)$	$u\bar{u}$	$0^-(1^{+-})$	5^1P_1		21221020512	$a_2(1320)$	$d\bar{d}$	$1^-(2^{++})$	1^3P_2	[24]	21113042113
$\rho_3(1690)$	$d\bar{d}$	$1^-(3^{--})$	1^3D_3	[24]	21113062120	$f_2(1270)$	$u\bar{u}$	$0^+(2^{++})$	1^3P_2	[24]	21223040117
$\omega_3(1667)$	$u\bar{u}$	$0^+(3^{--})$	1^3D_3	[24]	21223060124	$f_2'(1525)$	$s\bar{s}$	$0^+(2^{++})$	1^3P_2	[24]	21333040117
$\phi_3(1854)$	$s\bar{s}$	$0^+(3^{--})$	1^3D_3	[24]	21333060124	$a_2(1700)$	$d\bar{d}$	$1^-(2^{++})$	2^3P_2	[24]	21113042213
$\rho_3(2066?)$	$d\bar{d}$	$1^-(3^{--})$	2^3D_3	[24]	21113062220	$f_2(1755)$	$u\bar{u}$	$0^+(2^{++})$	2^3P_2	[24]	21223040217
$\omega_3(2338?)$	$s\bar{s}$	$0^+(3^{--})$	2^3D_3	[24]	21333060224	$f_2(2010)$	$s\bar{s}$	$0^+(2^{++})$	2^3P_2	[24]	21333040217
$\rho_3(2300)$	$d\bar{d}$	$1^-(3^{--})$	3^3D_3	[24]	21113062320	$a_2(2050)$	$d\bar{d}$	$1^-(2^{++})$	3^3P_2	[24]	21113042313
$\omega_3(2278)$	$u\bar{u}$	$0^+(3^{--})$	3^3D_3	[24]	21223060324	$f_2(2001)$	$u\bar{u}$	$0^+(2^{++})$	3^3P_2	[24]	21223040317
$a_1(1260)$	$d\bar{d}$	$1^-(1^{++})$	1^3P_1	[24]	21113022113	$f_2(2300)$	$s\bar{s}$	$0^+(2^{++})$	3^3P_2	[24]	21333040317
$f_1(1285)$	$u\bar{u}$	$0^+(1^{++})$	1^3P_1	[24]	21223020117	$a_2(2280)$	$d\bar{d}$	$1^-(2^{++})$	4^3P_2	[24]	21113042413
$f_1(1420)$	$s\bar{s}$	$0^+(1^{++})$	1^3P_1	[24]	21333020117	$f_2(2300)$	$u\bar{u}$	$0^+(2^{++})$	4^3P_2	[24]	21223040417
$a_1(1640)$	$d\bar{d}$	$1^-(1^{++})$	2^3P_1	[24]	21113022213	$a_2(1797?)$	$d\bar{d}$	$1^-(2^{++})$	1^3F_2	[24]	21113042133
$f_1(?)$	$u\bar{u}$	$0^+(1^{++})$	2^3P_1	[24]	21223020217	$f_2(1815)$	$u\bar{u}$	$0^+(2^{++})$	1^3F_2	[24]	21223040137

Table 3 (continued)

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
$f_1(1971)$	$s\bar{s}$	$0^+(1^{++})$	2^3P_1	[24]	21333020217	$f_2(2156)$	$s\bar{s}$	$0^+(2^{++})$	1^3F_2	[24]	21333040137
$a_1(2096)$	$d\bar{d}$	$1^-(1^{++})$	3^3P_1	[24]	21113022313	$a_2(2100)$	$d\bar{d}$	$1^-(2^{++})$	2^3F_2	[24]	21113042233
$f_1(?)$	$u\bar{u}$	$0^+(1^{++})$	3^3P_1		21223020317	$f_2(2141)$	$u\bar{u}$	$0^+(2^{++})$	2^3F_2	[24]	21223040237
$a_1(2270)$	$d\bar{d}$	$1^-(1^{++})$	4^3P_1	[24]	21113022413	$f_1(2310)$	$u\bar{u}$	$0^+(1^{++})$	4^3P_1	[24]	21223020417

Table 4 Summary of encoding of strange, charm, bottom mesons, and quarkoniums

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
Strange mesons						Bottom mesons					
$K^+(494)$	$u\bar{s}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21231001100	B^+	$u\bar{b}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21251001100
$K^-(494)$	$s\bar{u}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21321001100	B^-	$b\bar{u}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21521001100
$K^0(498)$	$d\bar{s}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21131001100	B^0	$d\bar{b}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21151001100
$\overline{K^0}(498)$	$s\bar{d}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21311001100	$\overline{B^0}$	$b\bar{d}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21511001100
$K^0_S(498)$	$d\bar{s}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21130001100	B^0_L	$d\bar{b}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21150001100
$K^0_L(498)$	$d\bar{s}$	$\frac{1}{2}(0^-)$	1^1S_0	[25]	21132001100	B^0_H	$d\bar{b}$	$\frac{1}{2}(0^-)$	1^1S_0	[26]	21152001100
$K^{*+}(892)$	$u\bar{s}$	$\frac{1}{2}(1^-)$	1^3S_1	[25]	21233021100	B^{*+}	$u\bar{b}$	$\frac{1}{2}(1^-)$	1^3S_1	[26]	21253021100
$K^{*0}(896)$	$d\bar{s}$	$\frac{1}{2}(1^-)$	1^3S_1	[25]	21133021100	B^{*0}	$d\bar{b}$	$\frac{1}{2}(1^-)$	1^3S_1	[26]	21153021100
$K^{*+}_0(1425)$	$u\bar{s}$	$\frac{1}{2}(0^+)$	1^3P_0	[25]	21233001112	$B_1(5721)^+$	$u\bar{b}$	$\frac{1}{2}(1^+)$	1^1P_1	[26]	21251021112
$K^{*0}_0(1425)$	$d\bar{s}$	$\frac{1}{2}(0^+)$	1^3P_0	[25]	21133001112	$B_1(5721)^0$	$d\bar{b}$	$\frac{1}{2}(1^+)$	1^1P_1	[26]	21121021112
$K^{*+}_2(1426)$	$u\bar{s}$	$\frac{1}{2}(2^+)$	1^3P_2	[25]	21233041112	$B^*_2(5747)^+$	$u\bar{b}$	$\frac{1}{2}(2^+)$	1^3P_2	[26]	21253041112
$K^{*0}_2(1432)$	$d\bar{s}$	$\frac{1}{2}(2^+)$	1^3P_2	[25]	21133041112	$B^*_2(5747)^0$	$d\bar{b}$	$\frac{1}{2}(2^+)$	1^3P_2	[26]	21153041112
$K^{*+}(1680)$	$u\bar{s}$	$\frac{1}{2}(1^-)$	1^3D_1	[25]	21233041112	B^0_s	$s\bar{b}$	$0(0^-)$	1^1S_0	[26]	21351000100
$K^{*0}(1718)$	$d\bar{s}$	$\frac{1}{2}(1^-)$	1^3D_1	[25]	21133041112	B^*_s	$s\bar{b}$	$0(1^-)$	1^3S_1	[26]	21353020100
$K^{*+}_3(1776)$	$u\bar{s}$	$\frac{1}{2}(3^-)$	1^3D_3	[25]	21233021120	$\overline{B^*_s}$	$b\bar{s}$	$0(1^-)$	1^3S_1	[26]	21533020100
$K^{*0}_3(1780)$	$d\bar{s}$	$\frac{1}{2}(3^-)$	1^3D_3	[25]	21133021120	$B_{s1}(5830)^0$	$s\bar{b}$	$0(1^+)$	1^1P_1	[26]	21351020112
$K^{*0}_4(2045)$	$d\bar{s}$	$\frac{1}{2}(4^+)$	1^3F_4	[25]	21233061120	$B^*_{s2}(5840)$	$s\bar{b}$	$0(2^+)$	1^3P_2	[26]	21353040112
$K^{*+}_5(2380)$	$u\bar{s}$	$\frac{1}{2}(5^-)$	1^3G_5	[25]	21233101140	B^+_c	$c\bar{b}$	$0(0^-)$	1^1S_0	[26]	21451000100
$K^{*0}_5(2380)$	$d\bar{s}$	$\frac{1}{2}(5^-)$	1^3G_5	[25]	21133101140	B^-_c	$b\bar{c}$	$0(0^-)$	1^1S_0	[26]	21541000100
$K^{*+}(1410)$	$u\bar{s}$	$\frac{1}{2}(1^-)$	2^3S_1	[25]	21233021200	Heavy quarkonium					
$K^{*0}(1410)$	$d\bar{s}$	$\frac{1}{2}(1^-)$	2^3S_1	[25]	21133021200	η_c	$c\bar{c}$	$0^+(0^{-+})$	1^1S_0	[27]	21441000105
$K^{*+}_0(1950)$	$u\bar{s}$	$\frac{1}{2}(0^+)$	2^3P_0	[25]	21233001212	$\eta_c(2S)$	$c\bar{c}$	$0^+(0^{-+})$	2^1S_0	[27]	21441000205
$K^{*0}_0(1950)$	$d\bar{s}$	$\frac{1}{2}(0^+)$	2^3P_0	[25]	21133001212	$h_c(1P)$	$c\bar{c}$	$0^-(1^{+-})$	1^1P_1	[27]	21441020114
Charmed mesons						J/ψ	$c\bar{c}$	$0^-(1^{--})$	1^1S_0	[27]	21441000100
D^+	$c\bar{d}$	$\frac{1}{2}(0^-)$	1^1S_0	[28]	21411001100	$\chi_{c0}(1P)$	$c\bar{c}$	$0^+(0^{++})$	1^1P_0	[29]	21441000117
D^-	$d\bar{c}$	$\frac{1}{2}(0^-)$	1^1S_0	[28]	21141001100	$\chi_{c1}(1P)$	$c\bar{c}$	$0^+(1^{++})$	1^1P_1	[29]	21441020117
D^0	$c\bar{u}$	$\frac{1}{2}(0^-)$	1^1S_0	[28]	21421001100	$\chi_{c2}(1P)$	$c\bar{c}$	$0^+(2^{++})$	1^1P_1	[29]	21441040117
$\overline{D^0}$	$u\bar{c}$	$\frac{1}{2}(0^-)$	1^1S_0	[28]	21241001100	$\chi_{c1}(3872)$	$c\bar{c}$	$0^+(1^{++})$	2^1P_1	[29]	21441020217
$D^*(2007)^0$	$c\bar{u}$	$\frac{1}{2}(1^-)$	1^3S_1	[30]	21423021100	$\chi_{c2}(3930)$	$c\bar{c}$	$0^+(2^{++})$	2^1P_2	[29]	21441040217
$D^*_0(2300)^0$	$c\bar{u}$	$\frac{1}{2}(0^+)$	1^3P_0	[30]	21423001112	$\psi(2S)$	$c\bar{c}$	$0^-(1^{--})$	2^1S_1	[29]	21441020200
$D_1(2420)^0$	$c\bar{u}$	$\frac{1}{2}(1^+)$	1^1S_1	?	21421021102	$\psi(3770)$	$c\bar{c}$	$0^-(1^{--})$	1^3D_1	[29]	21443020120
$D_1(2430)^0$	$c\bar{u}$	$\frac{1}{2}(1^+)$	1^1P_1	?	21421021112	$\psi_2(3823)$	$c\bar{c}$	$0^-(2^{--})$	1^3D_2	[31]	21443040120
$D^*_2(2460)^0$	$c\bar{u}$	$\frac{1}{2}(2^+)$	1^3P_2	[30]	21423041112	$Z_c(3900)$	$c\bar{c}$	$1^+(1^{+-})$	$?^??_1$	[32]	2144?020??6
$D_0(2550)^0$	$c\bar{u}$	$\frac{1}{2}(0^-)$	2^1S_0	[30]	21421001200	$\eta_b(1S)$	$b\bar{b}$	$0^+(0^{-+})$	1^1S_0	[32]	21551000105

Table 4 (continued)

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
$D_1^*(2600)^0$	$c\bar{u}$	$\frac{1}{2}(1^-)$	2^3S_1	[30]	21423021200	$\Upsilon(1S)$	$b\bar{b}$	$0^-(1^{--})$	1^3S_1	[32]	21553020100
$D_2(2740)^0$	$c\bar{u}$	$\frac{1}{2}(1^-)$	1^3D_2	[30]	21423041120	$\chi_{b0}(1P)$	$b\bar{b}$	$0^+(0^{++})$	1^3P_0	[32]	21553000117
$D_3^*(2750)^0$	$c\bar{u}$	$\frac{1}{2}(3^-)$	1^3D_3	[30]	21423061120	$\chi_{b1}(1P)$	$b\bar{b}$	$0^+(1^{++})$	1^3P_1	[32]	21553020117
D_s^+	$c\bar{s}$	$0(0^-)$	1^1S_0	[30]	21431000100	$\chi_{b2}(1P)$	$b\bar{b}$	$0^+(2^{++})$	1^3P_2	[32]	21553040117
D_s^-	$s\bar{c}$	$0(0^-)$	1^1S_0	[30]	21341000100	$\Upsilon(2S)$	$b\bar{b}$	$0^-(1^{--})$	2^3S_1	[32]	21553020200
D_s^{*+}	$c\bar{s}$	$0(1^-)$	1^3S_1	[30]	21433020100	$\Upsilon_2(1D)$	$b\bar{b}$	$0^-(2^{--})$	1^3D_2	[32]	21553040120
D_s^{*-}	$s\bar{c}$	$0(1^-)$	1^3S_1	[30]	21343020100	$\chi_{b0}(2P)$	$b\bar{b}$	$0^+(0^{++})$	2^3P_0	[32]	21553000217
$D_{s0}^*(2317)^+$	$c\bar{s}$	$0(0^+)$	1^3P_0	[30]	21433000112	$\chi_{b1}(2P)$	$b\bar{b}$	$0^+(1^{++})$	2^3P_1	[32]	21553020217
$D_{s0}^*(2317)^-$	$s\bar{c}$	$0(0^+)$	1^3P_0	[30]	21343000112	$\chi_{b2}(2P)$	$b\bar{b}$	$0^+(2^{++})$	2^3P_2	[32]	21553040217
$D_{s1}(2460)^+$	$c\bar{s}$	$0(1^+)$	1^1P_1	[30]	21431020112	$\Upsilon(3S)$	$b\bar{b}$	$0^-(1^{--})$	3^3S_1	[32]	21553020300
$D_{s1}(2460)^-$	$s\bar{c}$	$0(1^+)$	1^1P_1	[30]	21341020112	$\Upsilon(4S)$	$b\bar{b}$	$0^-(1^{--})$	4^3S_1	[32]	21553020400
$D_{s2}^*(2573)^+$	$c\bar{s}$	$0(2^+)$	1^3P_2	[30]	21433040112	$\chi_{b1}(3P)$	$b\bar{b}$	$0^+(1^{++})$	3^3P_1	[32]	21551020317
$D_{s2}^*(2573)^-$	$s\bar{c}$	$0(2^+)$	1^3P_2	[30]	21343040112	$Z_b(10610)$	$b\bar{b}$	$1^+(1^{+-})$	$?^??_1$	?	2155?020??6
$D_{s1}^*(2700)^+$	$c\bar{s}$	$0(1^-)$	2^3S_1	[30]	21433020200	$\Upsilon(10860)$	$b\bar{b}$	$0^-(1^{--})$	$?^??_1$	?	2155?020??0
$D_{s1}^*(2700)^-$	$s\bar{c}$	$0(1^-)$	2^3S_1	[30]	21343020200	$\Upsilon(11020)$	$b\bar{b}$	$0^-(1^{--})$	$?^??_1$	?	2155?020??0

Table 5 Summary of baryon encoding

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
p	uud	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[33]	332212011102	$\Delta(1232)^{++}$	uuu	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4S_{3/2}$	[34]	332224033102
n	udd	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[33]	332112011102	$\Delta(1232)^-$	ddd	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4S_{3/2}$	[34]	331114033102
$N(1440)^+$	uud	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$2^2S_{1/2}$	[33]	332212011202	$\Delta(1232)^+$	uud	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4S_{3/2}$	[34]	332214033102
$N(1440)^0$	udd	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$2^2S_{1/2}$	[33]	332112011202	$\Delta(1232)^0$	udd	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4S_{3/2}$	[34]	332114033102
$N(1520)^+$	uud	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[33]	332212031110	$\Delta(1600)^+$	uud	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$2^4S_{3/2}$	[34]	332214033202
$N(1520)^0$	udd	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[33]	332112031110	$\Delta(1600)^0$	udd	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$2^4S_{3/2}$	[34]	332114033202
$N(1535)^+$	uud	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[33]	332212011110	$\Delta(1620)^+$	uud	$\frac{3}{2}\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[33]	332212013110
$N(1535)^0$	udd	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[33]	332112011110	$\Delta(1620)^0$	udd	$\frac{3}{2}\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[33]	332112013110
$N(1650)^+$	uud	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[33]	332212011210	$\Delta(1700)^+$	uud	$\frac{3}{2}\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[33]	332212033110
$N(1650)^0$	udd	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[33]	332112011210	$\Delta(1700)^0$	udd	$\frac{3}{2}\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[33]	332112033110
$N(1675)^+$	uud	$\frac{1}{2}\left(\frac{5}{2}^-\right)$	$2^2P_{5/2}$	[33]	332212051210	$\Delta(1900)^+$	uud	$\frac{3}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[33]	332212013210
$N(1675)^0$	udd	$\frac{1}{2}\left(\frac{5}{2}^-\right)$	$2^2P_{5/2}$	[33]	332112051210	$\Delta(1900)^0$	udd	$\frac{3}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[33]	332112013210
$N(1680)^+$	uud	$\frac{1}{2}\left(\frac{5}{2}^+\right)$	$1^2D_{5/2}$	[35]	332212051122	$\Delta(1905)^+$	uud	$\frac{3}{2}\left(\frac{5}{2}^+\right)$	$1^4D_{5/2}$	[33]	332214053122
$N(1680)^0$	udd	$\frac{1}{2}\left(\frac{5}{2}^+\right)$	$1^2D_{5/2}$	[35]	332112051122	$\Delta(1905)^0$	udd	$\frac{3}{2}\left(\frac{5}{2}^+\right)$	$1^4D_{5/2}$	[33]	332114053122

Table 5 (continued)

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
$N(1700)^+$	*uud*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$2^2P_{3/2}$	[33]	332212031210	$\Delta(1910)^+$	*uud*	$\frac{3}{2}\left(\frac{1}{2}^+\right)$	$1^4D_{1/2}$	[33]	332214013122
$N(1700)^0$	*udd*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$2^2P_{3/2}$	[33]	332112031210	$\Delta(1910)^0$	*udd*	$\frac{3}{2}\left(\frac{1}{2}^+\right)$	$1^4D_{1/2}$	[33]	332114013122
$N(1710)^+$	*uud*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$3^2S_{1/2}$	[33]	332212011302	$\Delta(1920)^+$	*uud*	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4D_{3/2}$	[33]	332212033122
$N(1710)^0$	*udd*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$3^2S_{1/2}$	[33]	332112011302	$\Delta(1920)^0$	*udd*	$\frac{3}{2}\left(\frac{3}{2}^+\right)$	$1^4D_{3/2}$	[33]	332112033122
$N(1720)^+$	*uud*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$3^2P_{3/2}$	[33]	332212031312	$\Delta(1930)^+$	*uud*	$\frac{3}{2}\left(\frac{5}{2}^-\right)$	$2^4P_{5/2}$	[33]	332214053210
$N(1720)^0$	*udd*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$3^2P_{3/2}$	[33]	332112031312	$\Delta(1930)^0$	*udd*	$\frac{3}{2}\left(\frac{5}{2}^-\right)$	$2^4P_{5/2}$	[33]	332114053210
$N(1875)^+$	*uud*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$2^2P_{3/2}$	?	332212031210	$\Delta(1950)^+$	*uud*	$\frac{3}{2}\left(\frac{7}{2}^+\right)$	$1^4D_{7/2}$	[33]	332214073122
$N(1875)^0$	*udd*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$2^2P_{3/2}$	[36]	332112031210	$\Delta(1950)^0$	*udd*	$\frac{3}{2}\left(\frac{7}{2}^+\right)$	$1^4D_{7/2}$	[33]	332114073122
$N(1880)^+$	*uud*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2D_{1/2}$	[36]	332212011122	$\Delta(2200)^+$	*uud*	$\frac{3}{2}\left(\frac{7}{2}^-\right)$	$1^2F_{7/2}$	[33]	332214073130
$N(1880)^0$	*udd*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2D_{1/2}$	[36]	332112011122	$\Delta(2200)^0$	*udd*	$\frac{3}{2}\left(\frac{7}{2}^-\right)$	$1^2F_{7/2}$	[33]	332114113130
$N(1895)^+$	*uud*	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[36]	332212011210	$\Delta(2420)^+$	*uud*	$\frac{3}{2}\left(\frac{11}{2}^+\right)$	$1^4G_{11/2}$	?	?
$N(1895)^0$	*udd*	$\frac{1}{2}\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[36]	332112011210	$\Delta(2420)^0$	*udd*	$\frac{3}{2}\left(\frac{11}{2}^+\right)$	$1^4G_{11/2}$	?	?
$N(1900)^+$	*uud*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$1^2P_{3/2}$	[36]	332212031112	$\Lambda(1116)$	*sud*	$0\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333212010102
$N(1900)^0$	*udd*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$1^2P_{3/2}$	[36]	332112031112	$\Lambda(1405)$	*sud*	$0\left(\frac{1}{2}^-\right)$	$?^2?_{1/2}$	?	?
$N(2060)^+$	*uud*	$\frac{1}{2}\left(\frac{5}{2}^-\right)$	$2^2P_{5/2}$	[36]	332212051220	$\Lambda(1520)$	*sud*	$0\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[37]	333212030110
$N(2060)^0$	*udd*	$\frac{1}{2}\left(\frac{5}{2}^-\right)$	$2^2P_{5/2}$	[36]	332112051220	$\Lambda(1600)$	*sud*	$0\left(\frac{1}{2}^+\right)$	$2^2S_{1/2}$	[37]	333212010202
$N(2100)^+$	*uud*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$4^2S_{1/2}$	[36]	332212011402	$\Lambda(1670)$	*sud*	$0\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[37]	333212010110
$N(2100)^0$	*udd*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$4^2S_{1/2}$	[36]	332112011402	$\Lambda(1690)$	*sud*	$0\left(\frac{3}{2}^-\right)$	$2^2P_{3/2}$	[37]	333212030210
$N(2120)^+$	*sud*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$?^??_{3/2}$	?	?	$\Lambda(1800)$	*sud*	$0\left(\frac{1}{2}^-\right)$	$2^2P_{1/2}$	[37]	333212010210
$N(2120)^0$	*sud*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$?^??_{3/2}$	?	?	$\Lambda(1820)$	*sud*	$0\left(\frac{5}{2}^+\right)$	$1^2D_{5/2}$	[37]	333212050122
$N(2190)^+$	*sud*	$\frac{1}{2}\left(\frac{7}{2}^-\right)$	$?^??_{7/2}$	?	?	$\Lambda(1830)$	*sud*	$0\left(\frac{5}{2}^-\right)$	$2^4P_{5/2}$	[37]	333214050210
$N(2190)^0$	*sud*	$\frac{1}{2}\left(\frac{7}{2}^-\right)$	$?^??_{7/2}$	?	?	…	…	…	…	…	…
…	…	…	…		…	Σ^+	*suu*	$1\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333222012102
Ξ^0	*ssu*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333322011102	Σ^0	*sud*	$1\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333212012102
Ξ^-	*ssd*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333312011102	Σ^-	*sdd*	$1\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[37]	333112012102
$\Xi(1530)^0$	*ssu*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$1^2S_{3/2}$	[37]	333322031102	$\Sigma(1385)^+$	*suu*	$1\left(\frac{3}{2}^+\right)$	$1^2S_{3/2}$	[37]	333222032102
$\Xi(1690)^0$	*ssu*	$\frac{1}{2}\left(\frac{?}{2}^?\right)$	?		?	$\Sigma(1660)^0$	*sud*	$1\left(\frac{1}{2}^+\right)$	$2^2S_{1/2}$	[37]	333212012202

Table 5 (continued)

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
$\Xi(1820)^0$	*ssu*	$\frac{1}{2}\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[37]	333322031110	$\Sigma(1670)^0$	*sud*	$1\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[37]	333212032110
$\Xi(1950)^0$	*ssu*	$\frac{1}{2}\left(\frac{?}{2}^?\right)$	?		?	$\Sigma(1750)^0$	*sud*	$1\left(\frac{1}{2}^-\right)$	$1^4P_{1/2}$	[37]	333214012110
$\Xi(2030)^0$	*ssu*	$\frac{1}{2}\left(\frac{?}{2}^?\right)$	?		?	$\Sigma(1775)^0$	*sud*	$1\left(\frac{5}{2}^-\right)$	$1^4P_{5/2}$	[37]	333214052110
…	…	…	…		…	…	…	…	…	…	…
Λ_c^+	*cud*	$0\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	334212010102	Ω^-	*sss*	$0\left(\frac{3}{2}^+\right)$	$1^2S_{3/2}$	[39]	333332030102
$\Lambda_c(2595)^+$	*cud*	$0\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[38]	334212010110	$\Omega(2250)^-$	*sss*	$0\left(\frac{5}{2}^+\right)$	$1^2D_{5/2}$	[39]	333332050122
$\Sigma_c(2455)^+$	*cud*	$1\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	334212012102	Ξ_b^0	*bsu*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	335322011102
Ξ_c^+	*csu*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	334324011112	$\Xi_b'(5935)^-$	*bsd*	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	335312011102
$\Xi_c(2645)^0$	*csd*	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	$1^4S_{3/2}$	[38]	334314031102	Ω_b^-	*bss*	$0\left(\frac{1}{2}^+\right)$	$1^2S_{1/2}$	[38]	335332010102
…	…	…	…		…	…	…	…	…	…	…

caused by different u and d components are also omitted from the table.

In the early days of this scientific field, researchers mainly started with the MIT bag model to study multiquark states composed of light quarks [54, 55]. In recent years, owing to the experimental discovery of multiple quark states containing heavy quarks, researchers have focused on studying the problems of tetraquarks and pentaquarks containing heavy quarks. Their mass spectra are mainly measured from particle reactions and decay products. In theory, the main approach is to analyze their quark composition through various model calculations. However, there is still extensive research to be conducted on their excited states, including radial and orbital excitations. Correspondingly, the standardization of identification and coding has become urgent. Table 6 lists some codes for possible tetraquarks and pentaquarks based on the proposed particle encoding mechanism. Many questions are still open to be resolved or confirmed based on new experimental data or research results. Moreover, even if the data listed in the table were calculated based on certain models, they may not necessarily be strictly accurate. For example, there are currently many models for particle structures, such as $a_0(980)$, $f_0(980)$, $\pi_1(1400)$, $\pi_1(1600)$, and η_1, which must be finished. The particle codes presented in the table were obtained by assuming them to be tetraquarks.

5 Summary

This study proposes a novel encoding mechanism for particle physics that is expected to meet the requirements of digital registration and computational simulation over a long period of time. Although the particle codes reported in this paper have not yet been fully determined and may even contain errors, users can make corresponding corrections based on experimental data or their own choices. New particles that are not included in the text or that will be discovered in the future can also be easily encoded by the proposed mechanism. This study was inspired by discussions held within an academic conference. Current computer simulation models require more distinguishable particle codes, and more information must be included in the particle codes.

Table 6 Summary of encoding of multi-quark state candidates

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	Refer-ences	Code
Tetraquarks						Pentaquarks					
$a_0(980)$	$sd\bar{s}\bar{d}$	$1^-(0^{++})$	$1^??_1$	?1?P	4231311022111	$P_\psi^N(4312)^+$	$cuud\bar{c}$	$\frac{1}{2}^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[16]	54422142011100
$f_0(980)$	$su\bar{s}\bar{u}$	$0^+(0^{++})$	$1^??_1$	?1?P	4232321020115	$P_\psi^N(4380)^+$	$cuud\bar{c}$	$\frac{1}{2}^?\left(\frac{3}{2}^-\right)$	$1^2P_{3/2}$	[16]	54422142031110
$\pi_1(1400)$	$ud\bar{u}\bar{d}$	$1^-(1^{-+})$	$1^??_1$	?1?P	4221211022111	$P_\psi^N(4450)^+$	$cuud\bar{c}$	$\frac{1}{2}^?\left(\frac{5}{2}^-\right)$	$1^4P_{5/2}$	[16]	54422144051110
$\eta_1(?)$	$ud\bar{u}\bar{d}$	$0^+(1^{-+})$	$1^??_1$	?1?P	4221211020115	$P_\psi^N(4457)^+$	$cuud\bar{c}$	$\frac{1}{2}^?\left(\frac{1}{2}^-\right)$	$1^2P_{1/2}$	[16]	54422142011110
$f_4(2300)$	$sd\bar{s}\bar{d}$	$0^+(4^{++})$	$1^??_4$	?5?D	4231315080127	$P_\psi^N(4440)^+$	$cuud\bar{c}$	$\frac{1}{2}^?\left(\frac{3}{2}^-\right)$	$1^4P_{3/2}$	[16]	54422144031110
$f_2(1640)$	$su\bar{s}\bar{u}$	$0^+(2^{++})$	$1^??_2$	?5?D	4232325040127	$P_{\psi s}^\Lambda(4338)^0$	$cuud\bar{c}$	$0^+\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[40]	54422142030104
$f_3(2300)$	$su\bar{s}\bar{u}$	$0^+(3^{++})$	$1^??_3$	?5?D	4232325060127	$P_{\pi_u^0}^{\Lambda_c^+}$	$cuud\bar{u}$	$0^+\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[41]	54422122010104
$\omega(2290)$	$su\bar{s}\bar{u}$	$0^-(1^{--})$	$1^??_3$	[42]	423232?020??0	$P_{\pi_d^0}^{\Lambda_c^+}$	$cudd\bar{d}$	$0^+\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[41]	54421112010104
$X(3250)$	$su\bar{s}\bar{u}$	$?^?(?^{??})$	$?^??_?$	[42]	423232???????	$P_{\pi^-}^{\Lambda_c^+}$	$cudd\bar{u}$	$0^+\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[41]	54421122010104
$K(3100)^0$	$ud\bar{s}\bar{u}$	$?^?(?^{??})$	$?^??_?$	[42]	422132???????	…	…	…	…	…	…
$T_{cs0}(2900)^0$	$ud\bar{c}\bar{s}$	$?^?(0^{+0})$	$?^??_0$	[42]	422143?00????	$P_{\eta_c}^\Lambda$	$csud\bar{c}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5443214201010?
$T_{cs1}(2900)^0$	$sd\bar{c}\bar{u}$	$?^?(1^{-0})$	$?^??_1$	[42]	423142?02????	$P_{J\psi}^\Lambda$	$csud\bar{c}$	$0^?\left(\frac{3}{2}^-\right)$	$1^4S_{3/2}$	[43]	54432144030010?
$T_{c\bar{s}0}(2900)^0$	$cd\bar{s}\bar{u}$	$?^?(0^?)$	$?^??_0$	[42]	424132?00????	$P_{\eta_c}^\Sigma$	$csud\bar{c}$	$1^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5443214201110?
$T_{c\bar{s}0}(2900)^{++}$	$cu\bar{s}\bar{d}$	$?^?(0^?)$	$?^??_0$	[42]	424231?00????	$P_{J\psi}^\Sigma$	$csud\bar{c}$	$1^?\left(\frac{3}{2}^-\right)$	$1^4S_{3/2}$	[43]	5443214403110?
$X(3872)$	$su\bar{s}\bar{u}$	$0^+(1^{++})$	1^1S_1	[44]	4232321020107	$P_{D_s}^\Lambda$	$ssud\bar{c}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5433214201010?
$T_{cc}(3875)^+$	$cc\bar{u}\bar{d}$	$?^?(?^{??})$	$?^??_?$	[42]	424421???????	$P_{D_s^*}^\Lambda$	$ssud\bar{c}$	$0^?\left(\frac{3}{2}^-\right)$	$1^4S_{1/2}$	[43]	5433214403010?
$T_{\psi 1}^b(3900)^+$	$cu\bar{c}\bar{d}$	$1^+(1^{+-})$	$?^??_1$	[45]	424242?020???	$P_{D_s}^\Sigma$	$ssud\bar{c}$	$1^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5433214201110?
$T_{c\bar{c}1}(4200)^+$	$cu\bar{c}\bar{d}$	$0^+(1^{+-})$	$?^??_1$	[46]	424241?020???	$P_{D_s^*}^\Sigma$	$ssud\bar{c}$	$1^?\left(\frac{1}{2}^-\right)$	$1^4S_{1/2}$	[43]	5433214403110?
$R_{c0}(4240)^+$	$cu\bar{c}\bar{d}$	$?^?(0^{--})$	$?^??_0$	[42]	424241?00????	$P_K^{\Xi_{cc}}$	$ccud\bar{s}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5433213201010?
$Z_{cs}(4220)^+$	$cu\bar{c}\bar{s}$	$?^?(1^{+0})$	$?^??_1$	[42]	424243?02????	$P_{\eta_c}^{\Lambda_c}$	$ccud\bar{c}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5433214201010?

Table 6 (continued)

Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code	Notation	$q_N \cdots q_1$	$I^G(J^{PC})$	$n^{2S+1}L_j$	References	Code
$T_{\psi s1}(4000)^0$	$cd\bar{c}\bar{s}$	$?^?(1^{+0})$	$?^??_1$	[42]	424143?02????	$P^{\Sigma}_{D_s^*}$	$ssud\bar{b}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5433215201010?
$\chi_{c0}(4500)$	$cs\bar{c}\bar{s}$	$?^?(0^{++})$	$?^??_0$	[42]	424343?00????	P^p_K	$uudd\bar{s}$	$0^?\left(\frac{1}{2}^-\right)$	$1^2S_{1/2}$	[43]	5422114201010?
$\chi_{c1}(4685)$	$cs\bar{c}\bar{s}$	$?^?(1^{++})$	$?^??_1$	[42]	424343?02????	$P^p_{K^*}$	$uudd\bar{s}$	$0^?\left(\frac{5}{2}^-\right)$	$1^4P_{5/2}$	[43]	5422114405011?
$X(4630)$	$cs\bar{c}\bar{s}$	$1^-(1^{-+})$	$?^??_1$	[46]	424343?022???	P^{Δ}_K	$uuud\bar{s}$	$1^?\left(\frac{5}{2}^-\right)$	$1^4P_{5/2}$	[43]	5422214405111?
$Y(4008)$	$cs\bar{c}\bar{s}$	$1^-(1^{--})$	1^1P_1	[46]	4243431022110	…	…	…	…	…	…
$Y(4260)$	$cs\bar{c}\bar{s}$	$1^-(1^{--})$	1^3P_1	[46]	4243433022110						
$Y(4360)$	$cs\bar{c}\bar{s}$	$1^-(1^{--})$	2^1P_1	[47]	4243431022210						
$Y(4660)$	$cs\bar{c}\bar{s}$	$1^-(1^{--})$	2^3P_1	[47]	4243433022210						
$T_{4c}(6020)$	$cc\bar{c}\bar{c}$	$0^+(1^{+-})$	1^3S_1	[48]	4244443020106						
$T_{4c}(6600)$	$cc\bar{c}\bar{c}$	$0^+(2^{--})$	1^5P_2	[48]	4244445020114						
…	$cc\bar{c}\bar{c}$	…	…	…	…						

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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