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The 5α condensate state in ^{20}Ne

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The α clustering phenomenon, in which nucleons are arranged into α particles (^{4}He nuclei) within a nuclear system, is one of the most intriguing aspects of nuclear structure. It has been observed in various light nuclei, such as ^{8}Be, ^{12}C, ^{16}O, and ^{20}Ne, and is responsible for many exotic and fascinating phenomena, such as the Hoyle state in ^{12}C, which plays an essential role in stellar nucleosynthesis [1–6] as well as in heavy-ion collisions [7–9].

A particularly interesting question is whether the α clusters can form a Bose–Einstein condensate (BEC) state, in which the α particles occupy the same quantum state and behave as a coherent matter wave. Such a state has been observed in dilute atomic gases [10] and has been speculated to exist in nuclear systems [11, 12]. The BEC influences physical properties of nuclear matter. If the BEC is indeed an inherent nature of nucleon many-body systems, nuclear matter reduces its energy at low density by organizing α clusters and condensing them into the lowest-energy state. This softening of dilute nuclear matter is mitigated in asymmetric nuclear matter due to a decrease in the population of α clusters with increasing asymmetry. Thus, the BEC could increase the symmetry energy of nuclear matter and exert a significant impact on the equation of state (EOS) for nuclear matter. Construction of the EOS for nuclear matter is one of the ultimate goals in nuclear physics. It serves not only as a benchmark for our comprehension of strongly interacting fermions but also as a foundation for understanding astrophysical phenomena such as supernovae and neutron stars.

However, it remains unclear whether the BEC manifests in dilute nuclear matter. Establishing the BEC states in various nuclei is desired because the ubiquity of the BEC states in finite nuclei could be strong evidence that the BEC is an inherent nature of nuclear systems. Despite this significance, the presence of the BEC states has only been reported in a limited number of light nuclei.

In ^{8}Be, the ground state is regarded as a 2α BEC state. The 0_2^+ state in ^{12}C, known as the Hoyle state with a gas-like 3α structure, is also considered to be a manifestation of the 3α BEC [13–15]. Similarly, the 0_6^+ state in ^{16}O, which has a dominant 4α structure, is a strong candidate for the 4α BEC state [16–18], but the experimental confirmation is challenging. If the 0_6^+ state is really the 4α BEC state, its wave function has a large overlap with that of the Hoyle state as the 3α BEC state. However, the large overlap does not result in the large decay width of the 0_6^+ state to the Hoyle state due to the small decay energy which makes it difficult for α clusters to penetrate the Coulomb barrier [19].

Recently, we reported candidates of the 5α BEC states in ^{20}Ne by measuring α particles inelastically scattered from ^{20}Ne in coincidence with decay charged particles [20]. We found new states at $E_x = 21.2$, 21.8, and 23.6 MeV in ^{20}Ne, which dominantly decay to the 0_6^+ state in ^{16}O as the strong candidate for the 4α BEC state. Furthermore, we reported the candidates for the 6α BEC state and its excited states in ^{24}Mg from a measurement of the ^{12}C + ^{12}C scattering [21]. However, unfortunately, no fully microscopic calculation on the BEC states was available in ^{20}Ne and heavier nuclei at that time except for a phenomenological calculation assuming the α clusters as point-like bosons [22], although detailed comparison between experimental and theoretical results are essential to pin down the BEC states in atomic nuclei.

In a very recent work, Zhou et al. [3] extended their theoretical study of the α condensation phenomenon to the ^{20}Ne nucleus, which has a rich clustering structure and a 5α threshold at $E_x = 19.2$ MeV, using the Tohsaki–Horiuchi–Schuck-Röpke (THSR) wave function [23] for the first time. The THSR wave function is particularly suitable for describing the gas-like states and the authors found two 0^+ states above the 5α threshold, one of which has a clear 5α condensate character. Figure 1 illustrates their result that one of the predicted states is considered to be the 5α BEC state with a dilute gas-like nature where the 5α clusters occupy the lowest-energy $0S$ orbit, whereas the ground state has

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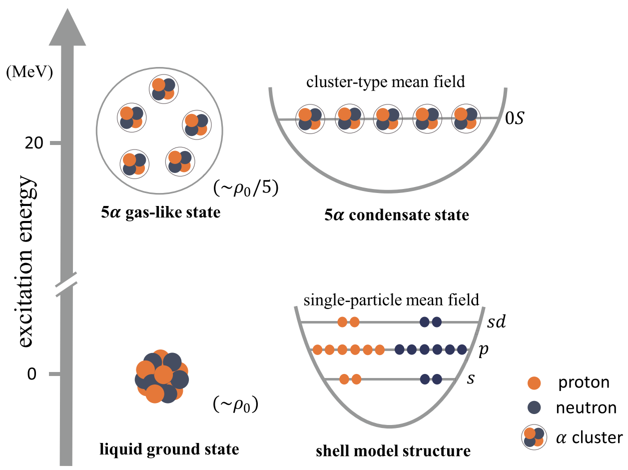

Fig. 1 (Color online) Diagrammatic representation for the shell-model-like ground state and the possible 5α condensate state in ^{20}Ne. See Ref. [3] for more details

a compact shell-model-like structure, where 20 nucleons occupy single-particle orbits in a mean-field potential and behaves like quantum liquid at normal density ρ_0. The structure and decay properties of this state show that it has a remarkable link with the 4α condensate state in ^{16}O. The results suggest that the α condensation phenomenon can occur in heavier nuclei under similar conditions as in ^{12}C and ^{16}O.

The five-body calculations for the ^{20}Ne nucleus focused on the 0^+ states above the 5α threshold, which were candidates for the 5α cluster and condensate states. The two 0^+ states, denoted as $0^+_{\rm I}$ and $0^+_{\rm II}$, were found at about $E_x = 22$ MeV and 23 MeV, respectively. These energies are qualitatively consistent with the phenomenological calculation [22] and the experimental data reported from RCNP [20] and iThemba LABS [24] as presented in Fig. 2a. The structures and decay properties of these states were analyzed using various observables.

The authors first examined the reduced width amplitudes (RWA) of the $0^+_{\rm I}$ and $0^+_{\rm II}$ states in the channel of $^{16}{\rm O}(0^+_6) + \alpha$, which could show us the behavior of the relative wave function of $^{16}{\rm O}(0^+_6)$ and α in ^{20}Ne. As shown in Fig. 2b, the $0^+_{\rm I}$ and $0^+_{\rm II}$ states have significantly larger amplitudes in this channel than other states, indicating that they have a dominant $^{16}{\rm O}(0^+_6) + \alpha$ configuration. Moreover, the $0^+_{\rm I}$ state has a Gaussian-like RWA with a large amplitude and a long tail. This type of RWA behavior is an important feature of the BEC state, originating from the $(0S)$ motion between clusters. On the other hand, the $0^+_{\rm II}$ state has a relatively small amplitude in the inner region and a peak in the outer region with a strongly extended tail. It has one node in the RWA, suggesting that it is an excited state of the $0^+_{\rm I}$ state.

The overlap of the $0^+_{\rm I}$ and $0^+_{\rm II}$ states with the single-β THSR wave function were also examined. It can show the degree of similarity between the eigenstates and the single-β THSR wave function. The single-β THSR wave function is a simple and intuitive way to describe the gas-like states [25–27], in which the α particles are confined in a common Gaussian potential with a size parameter β. The larger β value, the more dilute and gas-like the state is. The authors found that the overlap of the $0^+_{\rm I}$ state with the single-β THSR wave function has a maximum value much larger than those of the neighboring states. This indicates that the $0^+_{\rm I}$ state has a clear 5α gas-like structure, in which the α particles can move relatively freely in a cluster-type mean field.

The α decay from the 5α condensate state into the 4α condensate state is another important aspect of the 5α condensate state. The calculated partial α decay width of the predicted 5α condensate state is as high as 0.7 MeV. Thus, this dominated decay channel can be measured directly in the experiment. The decay widths to the 0^+_2 state and the

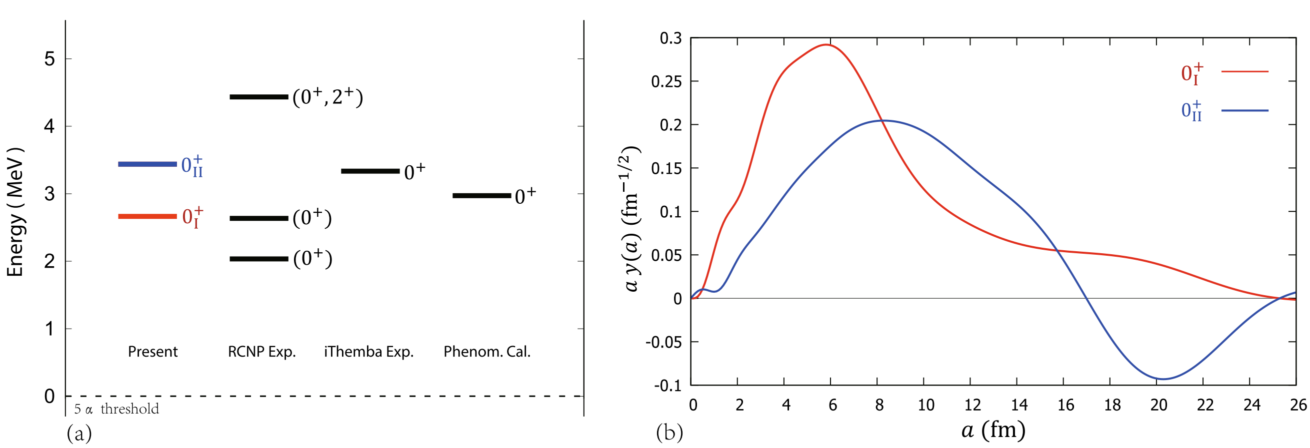

Fig. 2 (Color online) **a**) Comparison of theoretical predictions with the experimental results for the 5α cluster states above the threshold [20, 24] and the phenomenological calculation [22]. **b** The calculated reduced width amplitudes of the $0^+_{\rm I}$ state and $0^+_{\rm II}$ state in ^{20}Ne in the channel of $^{16}{\rm O}(0^+_6) + \alpha$. See Ref. [3] for more details

ground state of ^{16}O are also comparable and large enough. This information may help experiments to determine the 0_{I}^{+} state ($E_x \approx 22$ MeV) predicted in this work and offer a great opportunity for further exploration of the BEC in the nuclear system.

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