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Strangeness to increase the density of finite nuclear systems in constraining the high-density nuclear equation of state

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Abstract As the high-density nuclear equation of state (EOS) is not very well constrained, we suggest that the structural properties from the finite systems can be used to extract a more accurate constraint. By including the strangeness degrees of freedom, the hyperon or anti-kaon, the finite systems can then have a rather high-density core which is relevant to the nuclear EOS at high densities directly. It is found that the density dependence of the symmetry energy is sensitive to the properties of multi-Λ hypernuclei, while the high-density EOS of symmetric matter correlates sensitively to the properties of kaonic nuclei.

Keywords Nuclear equation of state · Relativistic mean-field theory · Strangeness

1 Introduction

The nuclear equation of state (EOS) plays a crucial role in nuclear structures, reaction dynamics, and many issues in astrophysics. Yet, both the symmetric matter EOS and the density dependence of the symmetry energy at supra-normal densities have large uncertainties arising from the theoretical models [1–5] and the extractions from the heavy-ion reactions [5–8]. The extracted EOS of symmetric matter may suffer from a large relative error of 50% at high densities [6], whereas the extracted symmetry energy is even more diversified at high densities, see discussions in Ref. [9]. To reduce the large uncertainty, it is necessary to use the structural properties of the finite systems to constrain the EOS. The question is then how to create finite systems that have a high-density core to be relevant to the EOS in the density region of interest.

In virtue of heavy-ion collisions, it may possibly produce metastable exotic multihypernuclear objects (MEMOs), such as multi-Λ hypernuclei. Schaffner et al. showed that multi-Λ hypernuclei might be more strongly bound than normal nuclei because of the additional $\Lambda-\Lambda$ interaction, leading to a dense core of the density $(2.5\sim3\rho_0)$ [10]. More favorable finite systems with a high-density core would be the kaonic nuclei in the sense of the closeness to the available experiments together with the consistent theory. It is very striking that various heavy-ion collisions got almost the consistent K^- potential depth around $-$ 100 MeV, see Refs. [11–13]. Theoretically, one can observe in the Lagrangian of the relativistic mean-field (RMF) models that the anti-kaon actually couples coincidently to the scalar and vector mesons which contribute the same sign to the anti-kaon potential, giving rise to a deep potential of $-$ 100MeV. In other words, the RMF theory can support the currently available experiments for the anti-kaon potential depth, though it is yet to have direct experiments to produce kaonic nuclei. With the depth of $-$ 100MeV, the core density of light kaonic nuclei may reach as high as $2.5\rho_0$ [14] in the RMF theory.

In this work, we will study in the RMF model the finite systems with a high-density core produced by the inclusion

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of the strangeness, regardless of the effect of the possible hadron–quark transition in the dense core [15]. With the implantation of the multi-Λ in hypernuclei and the embedment of anti-kaon in light nuclei, the high-density core appears. Firstly, we will show that the neutron skin of multi-Λ hypernuclei is sensitive to the symmetry energy at supra-normal densities. Then, we demonstrate that a much higher density core in light kaonic nuclei appears with the relevant properties being sensitive to the stiffness of symmetric matter EOS at supra-normal densities. The paper is organized as follows. In Sect. 2, we present the necessary formalism. The numerical results and discussions follow in Sect. 3. The summary is given last.

2 Formalism

For the finite system including the hyperons, the effective Lagrangian density is given as follows [16]

$$\begin{aligned}\mathcal{L} = & \overline{\psi}_B[i\gamma_\mu\partial^\mu - M_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu\tau_3 b_0^\mu \\ & + \frac{f_{\omega B}}{2M_N}\sigma_{\mu\nu}\partial^\nu\omega_0^\mu - e\frac{1}{2}(1+\tau_c)\gamma_\mu A^\mu]\psi_B - U(\sigma,\omega^\mu,b_0^\mu) \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}m_\rho^2 b_{0\mu}b_0^\mu - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} + \mathcal{L}_Y + \mathcal{L}_K,\end{aligned} \tag{1}$$

and $\mathcal{L}_Y$ is for the strange meson–hyperon interactions and free fields of strange mesons

$$\begin{aligned}\mathcal{L}_Y = & \overline{\psi}_Y[g_{\sigma^* Y}\sigma^* - g_{\phi Y}\gamma_\mu\phi^\mu]\psi_Y + \frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) \\ & - \frac{1}{4}(\partial^\mu\phi^\nu - \partial^\nu\phi^\mu)(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu) + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu,\end{aligned} \tag{2}$$

where ψ_B, σ, ω, and b_0 are the fields of the baryon, scalar, vector, and charge-neutral isovector–vector mesons, with their masses M_B, m_σ, m_ω, and m_ρ, respectively. A_μ is the field of the photon. $g_{iB}(i=\sigma,\omega,\rho)$ and $f_{\omega B}$ are the corresponding meson–baryon couplings. τ_3 is the third component of isospin Pauli matrix for nucleons, and $\tau_3 = 0$ for the Λ hyperon. τ_c is a constant relating to the baryon charge. $F_{\mu\nu}, B_{\mu\nu}$, and $A_{\mu\nu}$ are the strength tensors of the ω, ρ mesons, and the photon, respectively. The self-interacting terms of σ, ω-mesons and the isoscalar–isovector ones are given as

$$\begin{aligned}U(\sigma,\omega^\mu,b_0^\mu) = & \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 - \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 \\ & - 4g_{\rho N}^2\Lambda_v g_{\omega N}^2\omega_\mu\omega^\mu b_{0\mu}b_0^\mu\end{aligned}. \tag{3}$$

The Lagrangian of kaonic sector, $\mathcal{L}_K$, is written as

$$\mathcal{L}_K = (\mathcal{D}_\mu K)^\dagger(\mathcal{D}^\mu K) - (m_K^2 - g_{\sigma K}m_K\sigma)K^\dagger K, \tag{4}$$

where the covariant derivative is given by

$$\mathcal{D}_\mu \equiv \partial_\mu + ig_{\omega K}\omega_\mu + ig_{\rho K}b_{0\mu} + ie\frac{1+\tau_3}{2}A_\mu, \tag{5}$$

with the $g_{iK}(i=\sigma,\omega,\rho)$ being the corresponding K^--meson coupling constants. Here, K and $K^\dagger$ denote the kaon and anti-kaon doublet, i.e., $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ and $K^\dagger = (K^-, \bar{K}^0)$, respectively.

The equations of motion, derived from the Lagrangian, are the Dirac equations for baryons and Klein–Gordon equations for the K^-. The mesons such as σ, ω, and ρ produce the mean fields with which the baryons and K^- couple. The set of the coupled equations are solved in a self-consistently iterative method; for details, see Ref. [17].

3 Results and discussion

We perform calculations with the RMF parameter set as NL3 [18] whose Fermi momentum is $k_F = 1.30\,\mathrm{fm}^{-1}$ ($\rho_0 = 0.148\,\mathrm{fm}^{-3}$). The ω and ρ meson couplings with the hyperons are given by the SU(3) relations: $g_{\omega\Lambda} = 2/3 g_{\omega N}$ and $g_{\rho\Lambda} = 0$. The σ meson coupling with the hyperons is determined by the reasonable hyperon potential in nuclear matter at saturation density: $U_\Lambda^{(N)} = -30$ MeV. The coupling constant, $g_{\phi\Lambda}$, is taken to satisfy the SU(6) relation: $g_{\phi\Lambda}/g_{\omega N} = -\sqrt{2}/3$, and the $g_{\sigma^*\Lambda}$ is fitted to improve the $\Lambda\Lambda$ interaction matrix element, $\Delta B_{\Lambda\Lambda}$ [10]. We take the ratio of the scalar coupling constant to be $g_{\sigma^*\Lambda}/g_{\sigma N} = 0.76$, see Ref. [16]. The ωNN tensor coupling is vanishing, and the $\omega\Lambda\Lambda$ tensor coupling is small but adjusted to simulate the vanishing spin–orbit splitting for the Λ hyperon observed in $^{16}_{\Lambda}$O.

The various isoscalar–isovector coupling Λ_v's are used to simulate different density dependencies of the symmetry energy. The symmetry energy is given as

$$E_{\mathrm{sym}} = \frac{k_F^2}{6E_F^*} + \frac{g_\rho^2}{3\pi^2}\frac{k_F^3}{m_\rho^{*2}}, \tag{6}$$

with $E_F^* = \sqrt{k_F^2 + M_N^{*2}}$ and $m_\rho^{*2} = m_\rho^2 + 8g_{\rho N}^2 g_{\omega N}^2\Lambda_v\omega_0^2$. For a given coupling Λ_v, we follow Ref. [19] to readjust the ρ_{NN} coupling constant, $g_{\rho N}$, so as to keep an average symmetry energy fixed as 25.7 MeV at $k_F = 1.15\ \mathrm{fm}^{-1}$. As a result, the binding energy of ^{208}Pb is nearly unchanged for various Λ_v's [19]. With various Λ_v's, we can obtain various symmetry energies at given densities.

An appealing result of the additional $\Lambda\Lambda$ interaction with the mediation of the strange mesons is the rise of the core density of hypernuclei with the reasonable increase in the Λ number before the maximum core density is reached. Here, we consider hypernuclei constructed upon ^{102}Ca. ^{102}Ca that is certainly out of the neutron drip line can be stabilized by stacking up the sufficient Λ hyperons. With the implantation of about 40 Λ's, the central core density is about $1.5\rho_0$, showing a moderate dependence on the symmetry energy, see Table 1. Though the Λ is an isoscalar, the isoscalar–isovector coupling results in the sensitivity of the baryon density to the symmetry energy. One may raise the question whether such a multi-Λ hypernuclei can be produced. This is possible, but seems not easily measured, as the analyses of data suggested that the limits on the existence of exotic objects such as strangelets and MEMOs were set as low as 10^{-7} for M/Z up to 120 [20].

Shown in Fig. 1 is the neutron skin of multi-Λ hypernuclei as a function of the square root of the symmetry energy at $1.5\rho_0$. It is interesting to see that the neutron skins of multi-Λ hypernuclei are sensitive to the symmetry energy at supra-normal densities. However, this sensitivity does not hold for ^{102}Ca, as shown in Fig. 1. Specifically, the neutron skins of multi-Λ hypernuclei are almost linear in the square root of the symmetry energy at $1.5\rho_0$. Here, we establish for the first time the direct correlation between the structural property of hypernuclei and the high-density symmetry energy.

For kaonic nuclei, the coupling constants $g_{\rho K}$ and $g_{\omega K}$ are determined by the SU(3) relation: $2g_{\omega K} = 2g_{\rho K} = g_{\rho\pi} = 6.04$, while the $g_{\sigma K}$ is fitted to the depth of K^- optical potential. Here, the K^- optical potential is defined as the difference between the K^- energy and its mass at zero momentum. Note that various heavy-ion collisions got the almost consistent K^- potential depth around − 100 MeV at saturation density [11–13], though various theoretical extrapolations give diversified values ranging from 40 to 200 MeV, see Refs. [14, 17, 21–23]. Thus, in the present study, the optical potential is taken as − 100 MeV at saturation density. As a result, the deep binding of K^- feeds back to the mean field by providing nucleons, more attractions, and yielding a high-density core. In this case, the properties of kaonic nuclei are not sensitive to the density dependence of the symmetry energy, because the isovector field that dominates the density profile of the symmetry energy is small compared to the deep nuclear binding. On the other hand, we find that the properties of kaonic nuclei are

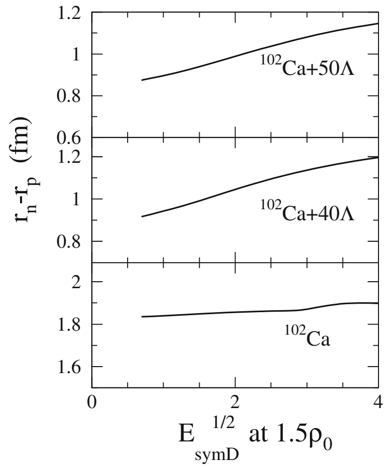

Fig. 1 Neutron skins of different finite objects as a function of the square root of the symmetry energy at $1.5\rho_0$. $E_{\rm symD}^{1/2}$ is given as $(E_{\rm sym} - 41\,{\rm MeV})^{1/2}$

sensitive to the EOS of symmetric matter at supra-normal densities. The stiffness of the high-density EOS of the symmetric matter can be characterized by the sound velocity square that is the partial derivative of the pressure over the energy density. Based on the parameter set NL3, we introduce the self-interaction of the ω meson, the so-called c_3 term, to simulate various stiffnesses of the high-density EOS, while other parameters are just moderately adjusted to keep the saturation properties unchanged and simultaneously the variation of the total binding energy of finite nuclei within 1 MeV [14].

As an example, we perform the calculation of finite systems constructed upon ^{16}O. The main results are tabulated in Table 2. We can see that the high-density EOS can affect sensitively the maximum density of the core and single-particle binding energies. The root-mean-square (rms) radii undergo a sufficient drop as the self-interaction of the ω meson is introduced, while they do not rely much on the specific c_3 with the rising softening of the high-density EOS. This result is relevant to the doubly magic structure of ^{16}O, since the change can be different in the light non-magic nuclei. In Ref. [14], the sensitivity of rms radii to the high-density EOS is very appealing in the candidate of halo nuclei arising from the embedment of K^-. Anyway, the light kaonic nuclei may serve as a good theoretical laboratory to extract the constraint of the high-density EOS of symmetric matter.

Table 1 Central core density ρ_c of $^{102}\mathrm{Ca} + 40\Lambda$ with respect to the slope of the symmetry energy L at ρ_0 and $1.5\rho_0$

$L(\rho_0)$	118.7	101.5	87.9	76.9	68.1	61.0	55.1	46.4
$L(1.5\rho_0)$	58.6	53.1	49.3	46.5	44.4	42.8	41.5	39.5
ρ_c in ρ_0	1.551	1.525	1.535	1.538	1.538	1.536	1.529	1.515

The L is defined as $L = 3\rho\partial E_{\rm sym}/\partial\rho$ in unit of MeV

Table 2 Proton and neutron radii R_p and R_n, the maximum nuclear density, and single-neutron binding energies in ${}^{16}_{K^-}$O with various sound velocity squares at $2\rho_0$

	w/o K^-	With K^-				
c_3	0.0	0.0	20.0	40.0	60.0	90.0
v_s^2	0.443	0.443	0.359	0.310	0.277	0.244
R_p	2.608	2.473	2.382	2.373	2.372	2.372
R_n	2.581	2.504	2.428	2.419	2.417	2.417
ρ_{Max}/ρ_0	1.14	2.46	2.01	1.76	1.67	1.62
$1s_{1/2}$	41.3	78.6	60.2	56.4	54.9	53.7
$1p_{3/2}$	21.7	26.8	26.4	26.2	26.0	25.9
$1p_{1/2}$	15.3	6.6	11.3	12.9	13.6	14.1

The column without K^- is obtained with the NL3 ($c_3 = 0$). The binding energies and radii are in units of MeV and fm, respectively

4 Summary

In this work, we demonstrate two cases of finite systems that are in turn sensitive to the differences in the symmetry energy and EOS of symmetric matter at supra-normal densities. In the first case, we include sufficient multi-Λ in the Ca isotope beyond the neutron drip line so as to stabilize the finite system against the β decay. It is found that the neutron skin in such exotic nuclei is sensitive to the symmetry energy at supra-normal densities, e.g., $1.5\rho_0$. In the second case, we explore the properties of light kaonic nuclei. It is found that the properties of light kaonic nuclei are sensitive to the EOS of symmetric matter at supra-normal densities. In these investigations, we have indeed proposed to extract the constraints of the high-density EOS of asymmetric matter through the structural properties of exotic finite nuclei with respectable accuracy. Finally, we would anticipate that these exotic nuclei may be created in laboratories in the near future.

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