

Exploring the sensitivity of α -decay half-life to neutron skin thickness for nuclei around ^{208}Pb

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Abstract Based on the newest experimentally extracted nuclear density distributions for double-magic nucleus ^{208}Pb (Tarbert et al. in Phys Rev Lett 112:242502, 2014), the sensitivity of α -decay half-life to nuclear skin thickness is explored in the vicinity of the shell closure region around ^{208}Pb , i.e., isotopes of $Z = 82$ and isotones of $N = 126$. With the two-parameter Fermi (2PF) density distributions and an analytically derived formula, the α -decay half-life is found to be closely related to the magnitude of nuclear skin thickness. For α decays to the $Z = 82$ isotopes, the α -decay half-life is found to decrease with the increasing neutron skin thickness, while the opposite behavior is found for α decays to the $N = 126$ isotones. Therefore, it could be a possible way to extract the nuclear skin thickness from measured α -decay half-lives.

Keywords Density distribution · Neutron skin thickness · Density-dependent cluster model · α -decay half-life

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1 Introduction

With the efforts of the whole community, α -decay half-life $T_{1/2}$ of unstable nuclei is well known to be related to several quantities, such as the α -particle preformation factor [1, 2], the pre-exponential factor [3], and the penetration probability [4, 5]. The determining factor is the penetration probability which is extremely sensitive to α -decay energy, Q , as indicated by the Geiger–Nuttall empirical rule [6]. Besides the decay energy, Q , some other factors, such as the radius of daughter nucleus, R_d , are also important to calculate the penetration probability, but have been paid little attention to in α -decay half-life calculations. The sensitive correlation between R_d and the penetration probability, as well as $T_{1/2}$, has been pointed out in the nuclear textbook [6] that “the calculated half-lives are extremely sensitive to small changes in the assumed mean radius” of daughter nucleus.

The α -decay half-life $T_{1/2}$ is sensitive to R_d , which is closely related to the nuclear skin thickness, S , in the daughter nucleus; however, in many α -decay half-life calculations, $S = 0$ fm is often assumed for simplification [4–11]. This is possibly due to the fact that little experimental information is known about the neutron density distribution, $\rho_n(r)$, of most nuclei, as well as S . There are only few experiments [12–21] on measuring the neutron density distributions and most of them attempt to extract $\rho_n(r)$ and S_n in the double-magic nucleus ^{208}Pb , whose neutron skin thickness has been recently pointed out to show a correlation with the density slope of symmetry energy in nuclear matter [22–25]. Oppositely, the proton density distribution, $\rho_p(r)$, in many nuclei has been accurately investigated by electron scattering experiments and

the experimental data are analyzed by different parameterized approaches, such as the Fourier–Bessel (FB) expansion and the two-parameter Fermi (2PF) model [26–34]. For instance, the $\rho_p(r)$ of ^{208}Pb is already known from electron scattering experiments with the widely used 2PF model: $\rho_{p/n}(r) = \rho_{p/n}^0 / \{1 + \exp[(r - c_{p/n})/a_{p/n}]\}$, where the proton half-density radius, c_p , and diffuseness parameter, a_p , are extracted to be 6.680 fm and 0.447 fm [35]. From the very recent coherent pion photoproduction experiment [36], the $\rho_n(r)$ of ^{208}Pb is extracted with c_n and a_n to be $6.70 \pm 0.03(\text{stat.})$ fm and $0.55 \pm 0.01(\text{stat.})^{+0.02}_{-0.03}(\text{sys.})$ fm, respectively, and $S_n = 0.15 \pm 0.03(\text{stat.})^{+0.01}_{-0.03}(\text{sys.})$ fm. In this paper, based on these newest experimental data of ^{208}Pb , our main purpose is to explore the sensitivity of α -decay half-life, $T_{1/2}$, to nuclear skin thickness, S , for α emitters decaying to ^{208}Pb and other daughter nuclei nearby, namely isotopes of $Z_d = 82$ and isotones of $N_d = 126$.

2 The correlation between α -decay half-life, $T_{1/2}$, and neutron skin thickness S

The close correlation between α -decay half-life, $T_{1/2}$, and neutron skin thickness, S , in the daughter nucleus can be analytically derived with a simplified α -decay model [6, 7], which assumes that only the point-charge Coulomb potential $V_C(R) = Z_\alpha Z_d e^2 / R$ operates beyond mother nucleus surface, where Z_α and Z_d are the proton numbers of the α particle and daughter nucleus, respectively. Then α -decay half-life $T_{1/2}$ can be written as [4–7]

$$T_{1/2} = \frac{\hbar \ln 2}{P_\alpha F \frac{\hbar^2}{4\mu} \exp\left[-2 \int_{R_2}^{R_3} \sqrt{\frac{2\mu}{\hbar^2} |Q - V_C(R)|} dR\right]}, \quad (1)$$

where P_α is the α -particle preformation factor and F is a normalization factor well defined by the two-potential approach [3]. μ is the reduced mass of α -particle and daughter nucleus. R_2 and R_3 are the second and third classical turning points. Since we investigate favored α decays in this paper, the centrifugal potential is not involved. By inserting $R_3 = Z_\alpha Z_d e^2 / Q$ and $R_2 = R_d + R_\alpha$ into Eq. (1), where R_α is the radius of α -particle, one can obtain [4–9]

$$\log_{10} T_{1/2} = \log_{10} \frac{4\mu \ln 2}{\hbar P_\alpha F} + \frac{2\sqrt{2}\mu e^2}{\hbar \ln 10} Z_\alpha Z_d Q^{-1/2} \times [\arccos(x) - x\sqrt{1-x^2}], \quad (2)$$

where $x = \sqrt{(R_d + R_\alpha)/R_3} \ll 1$. With $\arccos(x)$ and $x\sqrt{1-x^2}$ approximated by $(\pi/2 - x)$ and x , respectively, the following formula can be derived

$$\log_{10} T_{1/2} = a\sqrt{\mu Z_\alpha Z_d} Q^{-1/2} + b\sqrt{\mu Z_\alpha Z_d (R_d + R_\alpha)} + c, \quad (3)$$

where the parameters are $a = \frac{\sqrt{2}\pi e^2}{\hbar \ln 10}$, $b = -\frac{4e\sqrt{2}}{\hbar \ln 10}$, and $c = \log_{10} \frac{4\mu \ln 2}{\hbar P_\alpha F}$. Since P_α and F are not always constants for different α decays and the point-charge Coulomb potential is assumed, the parameters a , b , and c can be determined by fitting experimental data. Besides, it is clearly shown that the α -decay half-life, $T_{1/2}$, is related to the radius of daughter nucleus, R_d , and $\log_{10} T_{1/2} \sim \sqrt{R_d + R_\alpha}$. The radius R_d can be obtained from the root-mean-square radius, R_s , by applying the simple relation: $R_s = \sqrt{\frac{3}{5}} R_d$ [37]. The radius R_s is calculated from the proton and neutron density distributions $\rho_p(r)$ and $\rho_n(r)$ of daughter nucleus

$$R_s = \sqrt{\frac{\int 4\pi [\rho_p(r) + \rho_n(r)] r^4 dr}{A_d}}, \quad (4)$$

where A_d is the mass number of daughter nucleus. The difference between $\rho_p(r)$ and $\rho_n(r)$ can be characterized by the so-called proton and neutron skin thicknesses

$$S_p = R_p - R_n, \quad S_n = R_n - R_p, \\ R_p = \sqrt{\frac{\int 4\pi \rho_p(r) r^4 dr}{Z_d}}, \quad R_n = \sqrt{\frac{\int 4\pi \rho_n(r) r^4 dr}{N_d}}, \quad (5)$$

where R_p and R_n are the proton and neutron root-mean-square (RMS) radii, respectively. $N_d = A_d - Z_d$ is the neutron number of the daughter nucleus. The correlation between $T_{1/2}$ and S can be directly shown by the α -decay half-life ratio derived from Eq. (3)

$$T_{1/2}^S / T_{1/2}^{S=0} = 10^{b\sqrt{\mu Z_\alpha Z_d} (\sqrt{R_d^S + R_\alpha} - \sqrt{R_d^{S=0} + R_\alpha})}. \quad (6)$$

It is obvious that this ratio is irrelevant to the α -decay energy, Q , but exponentially dependent on the variation of $\sqrt{R_d^S + R_\alpha}$, while the uncertainties coming from P_α and other factors are all canceled out.

3 Numerical results and discussion

Based on Eq. (3), the sensitivity of α -decay half-life, $T_{1/2}$, to nuclear skin thickness, S , for α emitters decaying to ^{208}Pb , isotopes of $Z_d = 82$ and isotones of $N_d = 126$ are explored. Here the $\rho_p(r)$ for isotopes of $Z_d = 82$ and $\rho_n(r)$ for isotones of $N_d = 126$ are calibrated by the measured proton and neutron density distributions of ^{208}Pb [36]. The detailed parameters are given as follows

$$\begin{cases}
 \text{For isotopes of } Z_d = 82 : \\
 a_p = 0.447 \text{ fm and } c_p = 6.680 \text{ fm;} \\
 a_n = 0.55 \text{ fm and } c_n = 1.1308A_d^{1/3} \text{ fm;} \\
 \text{For isotones of } N_d = 126 : \\
 a_p = 0.447 \text{ fm and } c_p = 1.1274A_d^{1/3} \text{ fm;} \\
 a_n = 0.55 \text{ fm and } c_n = 6.70 \text{ fm,}
 \end{cases} \quad (7)$$

where the values 1.1308 and 1.1274 of the half-density radii are obtained by fitting the experimental ones in ^{208}Pb [36].

By using Eqs. (5) and (7), the corresponding skin thicknesses in isotopes of $Z_d = 82$ and isotones of $N_d = 126$ are calculated, as shown in Fig. 1. It is clearly seen from Fig. 1a that the skin thickness in ^{190}Pb is approximately 0 fm. For isotopes $^{191-214}\text{Pb}$, the neutron skin thickness, S_n , increases with the increase of neutron number, N , while for isotopes $^{182-189}\text{Pb}$, the proton skin thickness, S_p , decreases with N . So the skin thickness varies smoothly with the neutron number for either the proton skin case or the neutron skin case. Similarly, as shown in Fig. 1b for isotones of $N_d = 126$, S_n decreases with the proton number, Z , as the neutron number is fixed for nuclei from ^{209}Bi to ^{215}Ac .

The radius, R_d , of each daughter nucleus is computed based on Eqs. (4) and (7). Then the corresponding α -decay half-lives, $T_{1/2}^S$, are calculated by using Eq. (3), where the parameters $a = 0.0136$, $b = -0.0165$, and $c = -14.0903$ are fitted by the experimental data of α -decay energy, Q , and half-life $T_{1/2}^{\text{exp}}$ obtained from Refs. [38, 39]. The α -

decay half-lives $T_{1/2}^{S=0}$ corresponding to the case of $S = 0$ fm are also calculated with the same values of a , b , and c . The ratio $T_{1/2}^S/T_{1/2}^{S=0}$ with the variation of S is shown in Fig. 2. It is found in Fig. 2a that the ratio for ^{190}Pb is approximately 1.0 as its skin thickness is $S \approx 0$ fm. For α decays to $^{191-214}\text{Pb}$, the ratio decreases with the increasing S_n , and they are all less than 1.0 because of the negative sign of b [see Eq. (6)]. For instance, the ratio is as small as 0.466 for ^{214}Pb . In contrast, for α decays to $^{182-189}\text{Pb}$ the ratio increases with S_p and the maximum one is 1.266 for ^{182}Pb . For α decays to isotones of $N_d = 126$ in Fig. 2b, the ratio increases with S_n and ranges from 1.279 to 1.415. Besides, it is noted that the nuclei ^{205}Pb and ^{207}Pb are not included in Fig. 2a since the involved α transitions are unfavored.

The detailed values of the skin thicknesses, S , and the ratios $T_{1/2}^S/T_{1/2}^{S=0}$ are given in Table 1. The first and second columns of Table 1 are the α -decay mother nucleus and its isospin asymmetry $\delta = (N - Z)/A$, respectively. The third column is the calculated radius, R_d , of the daughter nucleus, and the corresponding skin thickness S_p/S_n is given in column four. It is clearly shown that the value of S_p decreases with the increasing isospin asymmetry, δ , while S_n increases with δ . It is found from the ratio $T_{1/2}^S/T_{1/2}^{S=0}$ of the fifth column that there exists a strong correlation between the α -decay half-life and the skin thickness S_p/S_n . In the last column, we give the comparison of calculated α -decay half-lives $T_{1/2}^S$ using Eq. (3) with the experimental ones $T_{1/2}^{\text{exp}}$ from Refs. [38, 39]. We found that the calculated α -decay half-lives $T_{1/2}^S$ agree with $T_{1/2}^{\text{exp}}$ well for both the $Z = 82$ isotopes and the $N = 126$ isotones. For

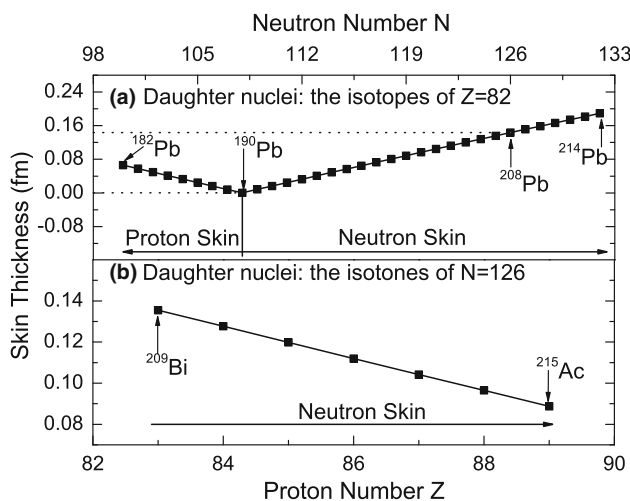


Fig. 1 The skin thickness, S , with the variation of neutron number, N , for isotopes of $Z_d = 82$ (a) and of proton number, Z , for isotones of $N_d = 126$ (b)

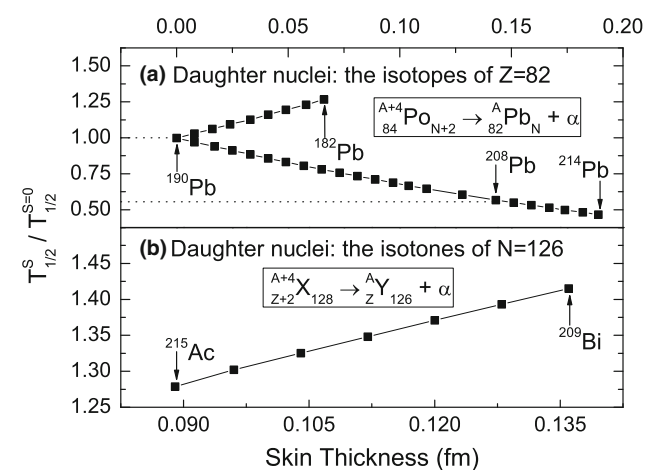


Fig. 2 The correlation between α -decay half-life ratio $T_{1/2}^S/T_{1/2}^{S=0}$ and nuclear skin thickness, S , for α decays to isotopes of $Z_d = 82$ (a) and to isotones of $N_d = 126$ (b)

Table 1 The values of the skin thicknesses, S , and the ratios $T_{1/2}^S/T_{1/2}^{S=0}$ for α decays to isotopes of $Z_d = 82$ and isotones of $N_d = 126$

Mother	δ	R_d (fm)	S (fm)	$T_{1/2}^S/T_{1/2}^{S=0}$	$T_{1/2}^S/T_{1/2}^{\text{exp}}$
^{186}Po	0.0968	6.9692	$S_p = 0.066$	1.266	0.284
^{187}Po	0.1016	6.9749	$S_p = 0.058$	1.230	0.230
^{188}Po	0.1064	6.9807	$S_p = 0.049$	1.195	0.564
^{189}Po	0.1111	6.9865	$S_p = 0.041$	1.160	0.599
^{190}Po	0.1158	6.9924	$S_p = 0.033$	1.127	0.906
^{191}Po	0.1204	6.9982	$S_p = 0.024$	1.094	0.433
^{192}Po	0.1250	7.0042	$S_p = 0.016$	1.061	1.081
^{193}Po	0.1295	7.0101	$S_p = 0.008$	1.030	0.534
^{194}Po	0.1340	7.0162	$S = 0.000$	0.999	1.115
^{195}Po	0.1385	7.0221	$S_n = 0.008$	0.969	0.746
^{196}Po	0.1429	7.0282	$S_n = 0.017$	0.940	1.339
^{197}Po	0.1472	7.0342	$S_n = 0.025$	0.912	0.618
^{198}Po	0.1515	7.0404	$S_n = 0.033$	0.884	1.093
^{199}Po	0.1558	7.0465	$S_n = 0.041$	0.857	0.762
^{200}Po	0.1600	7.0527	$S_n = 0.049$	0.831	0.863
^{201}Po	0.1642	7.0589	$S_n = 0.057$	0.805	0.657
^{202}Po	0.1683	7.0651	$S_n = 0.065$	0.781	0.821
^{203}Po	0.1724	7.0714	$S_n = 0.073$	0.756	0.589
^{204}Po	0.1765	7.0776	$S_n = 0.081$	0.733	0.693
^{205}Po	0.1805	7.0839	$S_n = 0.089$	0.710	0.616
^{206}Po	0.1845	7.0903	$S_n = 0.097$	0.688	0.624
^{207}Po	0.1884	7.0966	$S_n = 0.104$	0.666	0.351
^{208}Po	0.1923	7.1031	$S_n = 0.112$	0.645	0.372
^{210}Po	0.2000	7.1158	$S_n = 0.128$	0.605	0.235
^{212}Po	0.2075	7.1287	$S_n = 0.143$	0.567	1.090
^{213}Po	0.2113	7.1352	$S_n = 0.151$	0.549	0.917
^{214}Po	0.2150	7.1417	$S_n = 0.159$	0.531	2.411
^{215}Po	0.2186	7.1482	$S_n = 0.167$	0.514	2.023
^{216}Po	0.2222	7.1547	$S_n = 0.174$	0.498	3.449
^{217}Po	0.2258	7.1613	$S_n = 0.182$	0.482	2.789
^{218}Po	0.2294	7.1679	$S_n = 0.189$	0.466	4.174
^{213}At	0.2019	7.1321	$S_n = 0.136$	1.415	0.968
^{214}Rn	0.1963	7.1357	$S_n = 0.128$	1.393	1.178
^{215}Fr	0.1907	7.1392	$S_n = 0.120$	1.371	1.173
^{216}Ra	0.1852	7.1428	$S_n = 0.112$	1.348	1.197
^{217}Ac	0.1797	7.1464	$S_n = 0.104$	1.325	1.188
^{218}Th	0.1743	7.1502	$S_n = 0.096$	1.302	1.353
^{219}Pa	0.1689	7.1539	$S_n = 0.089$	1.279	1.596

The first and second columns are the α -decay mother nucleus and its asymmetry, $\delta = \frac{N-Z}{A}$. The third column is the calculated radius, R_d , of daughter nucleus, and the last column is the values of the ratio $T_{1/2}^S/T_{1/2}^{\text{exp}}$, where the experimental data, $T_{1/2}^{\text{exp}}$, are obtained from Refs. [38, 39]

the majority of cases, the experimental data are reproduced within a factor of two although a constant preformation probability of α cluster is used in Eq. (3). But for the ratio $T_{1/2}^S/T_{1/2}^{S=0}$, the shell effect and odd–even effect on the preformation probability are all canceled out and only the skin thickness effect is left. The skin thickness characterizes the difference between proton and neutron density distributions, especially the difference in the nuclear surface region. For the $Z = 82$ isotopes, the difference in neutron density distributions shall only affect the nuclear part of the α -core potential. However, for the $N = 126$ isotones, the change of proton density distribution will affect not only the nuclear potential but also the Coulomb potential. It is well known that the tunneling of the α particle in α decays takes place in the surface region; thus, the skin thickness effect should be included in α -decay half-life calculations.

4 Summary

In summary, with the analytically derived formula and newest experimental data of ^{208}Pb , the sensitivity of α -decay half-life to nuclear skin thickness is explored for daughter nucleus around ^{208}Pb , namely isotopes of $Z_d = 82$ and isotones of $N_d = 126$. From the numerical results, we find there is a close correlation between α -decay half-life and nuclear skin thickness, and the former one can be changed by a factor from 0.466 to 1.415. So it is necessary to consider the skin thickness for investigations on α -decay half-lives, and it could be a possible way to extract nuclear skin thickness from measured α -decay half-lives.

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