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Effects of the phase variation and high-order momentum transfer components of NN amplitude on p-^{4}He elastic scattering

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Abstract Elastic scattering differential cross sections for a p-^{4}He system are calculated within the framework of optical limit approximation of the Glauber multiple scattering model. Three different ranges for proton energy (E_{lab}), $19 < E_{\mathrm{lab}} < 50$ MeV, $100 \leq E_{\mathrm{lab}} \leq 1730$ MeV, and $45 \leq E_{\mathrm{lab}} \leq 393$ GeV are considered. It is shown that the Pauli blocking fails to describe the data up to the proton energy, $E_{\mathrm{lab}} < 100$ MeV. For higher proton energies, a qualitative agreement is obtained. The observed elastic scattering differential cross section is nicely reproduced in the whole range of scattering angles in the center of mass system up to $\Theta_{\mathrm{c.m.}} < 200^\circ$ for $19 < E_{\mathrm{lab}} \leq 100$ MeV when the effect of both the nucleon–nucleon (NN) phase variation parameter γ_{NN} and higher-order momentum transfer components ($\lambda_n, n = 1$ and 2) of (NN) elastic scattering amplitude is included. In the range of $200 \leq E_{\mathrm{lab}} \leq 1730$ MeV, introducing λ_n plays a significant role in describing the data up to the momentum transfer, $q^2 \leq 1.2$ (GeV/c)^2. Moreover, it is found that considering only the effect of phase variation parameter, γ_{NN}, improved the agreement in the region of minima for elastic scattering differential cross section for $45 \leq E_{\mathrm{lab}} \leq 393$ GeV. The values of γ_{NN} and λ_n as a function of incident proton energies are presented.

Keywords Elastic scattering · Optical limit approximation · Glauber multiple scattering model

1 Introduction

The study of nuclear scattering is a challenging subject of nuclear physics in both theory and the laboratory. Scattering amplitude of light nuclei is more sensitive both to the interaction mechanism and to the parametrization of proton–nucleon (PN) scattering amplitude. Therefore, it can be used as a more critical test to theoretical models and their approximations. This also is useful to explain the nuclear structure of stable as well as exotic nuclei. Study p-^{4}He scattering is very interesting and important of the knowing for the properties of PN interaction and a few-body system. The ^{4}He nucleus (α particle) holds a special place among the lightest nuclei. This is a doubly magic nucleus and possesses a closed shell. (The mean nuclear density in ^{4}He is close to the nuclear density of the lead, i.e., it is essentially nuclear matter (in itself) and, consequently, it can be used for studying most of the effects occurring in complex nuclei [1]).

The importance of proton collision from relatively low energies ($E_{\mathrm{lab}} > 10$ MeV) to intermediate energies ($E_{\mathrm{lab}} > 100$ MeV) and then to high energies ($E_{\mathrm{lap}} > 1$ GeV) comes from two essential points. One of them is that the probing of the nucleus with intermediate and high-energy protons could be much more fruitful for nuclear structure research than analogous experiments at lower energies. One obvious reason is that a-500 MeV proton has a wavelength, $\lambda \simeq 0.2$ fm, which is much smaller than the internucleon spacing in the nucleus. The probe can

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therefore "see" the individual nucleons, which would not be the situation at $E_{\mathrm{lab}} \simeq 20$ MeV where $\lambda = 1$ fm. The second point is related to the sensitivity of the proton as a probe to the nuclear in-medium effects. Of greater importance though, when a low-energy proton is introduced into a nucleus, there is little to distinguish it from one of the target nucleons. It participates in the many body dynamics. On the other hand, a high-energy proton passing through a nucleus is on the average hardly deflected.

On theoretical front, many authors have investigated the p-^{4}He elastic scattering over 120 MeV by means of the phase shift analysis [2], the phenomenological analysis with optical potential [3], and the microscopic resonating group method (RGM) with a complex effective NN potential [4, 5]. In addition, analysis of proton-nucleus (PA) data has generally been investigated using the Glauber multiple scattering models (GMSM) [6, 7]. One of the attractive features of the formalism of this semiclassical model is that it connects the directly measurable (free) NN amplitude to the nucleon-nucleus one in a mathematically tractable way. This shows that the information about the microscopic details of the target nucleus does not only depend on the validity of the reaction mechanism used, but also depend on the accuracy of the NN amplitude. Several studies were established using Glauber multiple scattering expansion in conjunction with many nuclear in-medium effects and the deviation of the projectile due to both the coulomb and nuclear potential [coulomb-modified correlation expansion of the Glauber model (CMGM)], for proton-nucleus [8–12] and nucleus–nucleus [13–16]. On the other hand, for p-nucleus system for light nuclei, the double and higher-order scattering terms in the evaluation of scattering amplitude are important. The consideration of these higher-order terms is not only difficult, but also beset with poor knowledge of higher-order correlations in nuclei. Nevertheless, the region of energy and of momentum transfer where the formalism of the GMSM may be reliably applied is not well known for hadron scattering on nuclei. So, it is interesting to investigate the validity of this model at lower energies where one would not expect the assumptions, which are made to be correct [17].

The absence of strong interaction theory and exact solution of few-body problems together with the "existence" of physical situations in which the interaction dynamics is simplified leads to the development of approximate methods for calculation of the measured quantities and extraction of data on the structure and properties of interacting nuclei [18]. Since evaluation of the complete multiple scattering terms of GMSM is a prohibitively difficult task in nucleon-nucleus scattering, several approximation methods have been developed for GMSM to evaluate the elastic S-matrix element, S(b), in the impact parameter space. One of them is the optical limit approximation (OLA) [6]. Most of the analysis for proton-nucleus PN scattering has been made by invoking the so-called OLA [9, 19–23]. The OLA depends upon the one-body density of the target nucleus and the NN elastic scattering amplitude, while the neglected terms depend upon the two-body and higher-order correlation functions. Therefore, it is indispensable to introduce nuclear in-medium corrections within OLA so that it becomes a more effective tool in the treatment of nucleon (nucleus)–nucleus scattering data.

There are two factors missing in the GMSM calculation with (one-term) Gaussian parameterizations (G_{NN}) for the input NN elastic scattering amplitude [24]. One is the large q behavior, which might be of some significance for collisions between lighter particles whose form factors fall rather smoothly. The other factor concerns the nuclear in-medium effects where one of the main problems is that the NN scattering seems to be quite drastically modified when the nucleon is embedded in a nuclear medium.

As discussed in Ref. [25], the G_{NN} may be well suited at relatively high energies where the NN scattering is mostly diffractive and peaked in the forward scattering. However, the same may not be very appropriate for describing the NN data at low energies, as the scattering in this case is non-diffractive. So, the parameterization of NN elastic scattering amplitude is needed for further investigations with nucleon and nuclear beams in a wide range of energies > 10 MeV. Earlier, it has been shown that the consideration of higher momentum transfer components, and hence the non-diffractive behavior of the NN amplitude [26], provides a more satisfactory account of the data than does the usually parameterized G_{NN} [27, 28]. On the other hand, Franco and Yin [29, 30] have suggested that the phase of the NN elastic scattering amplitude should vary with the momentum transfer where it has not been settled. The presence of the phase variation improves the results of p-A total reaction cross section [31] and elastic scattering differential cross section for p-A [32] and for A-A [33], especially at the minima regions.

Working within the framework of the coulomb-modified Glauber model [34, 35], it was shown by Chauhan and Khan [36] that inclusion of higher momentum transfer components and the NN phase variation has been applied successfully in describing the elastic scattering differential cross section for α-nucleus scattering at 25, 35, 40, 50, and 70 MeV/nucleon. This analysis suggested that the proper account of the higher momentum transfer components in G_{NN} may push down the GMSM to lower energies and may increase its validity in the region of relatively large momentum transfer. In addition, a satisfactory account of the total nuclear reaction cross section, σ_{R}, was obtained using the same model for the scattering of α-nucleus at

69.6, 117.2, 163.9, and 192.4 MeV [37]. The effects of both higher-order momentum transfer components and the NN phase parameter are also studied on the calculations of σ_R for p-α scattering in the energy range from 25 to 1000 MeV within OLA [31]. From this analysis, G_{NN} with the higher-order momentum components and phase variation could be taken as fairly stable to describe simultaneously the elastic angular distribution and the σ_R for a wide range of target nuclei and energies.

The main objective of this manuscript is to investigate the ability of OLA in conjunction with the modification of the NN elastic scattering amplitude in describing the p-^{4}He elastic scattering differential cross section over a wide range of proton energies from 19 MeV to 393 GeV. The modification included the effects of Pauli blocking through the different values of both proton–proton and proton–neutron total cross sections, phase variation parameter, γ_{NN}, and higher-order momentum transfer components λ_n with $n = 1$ and 2.

Mathematical formulation and NN elastic scattering amplitude with its parameters are presented in Sect. 2. Section 3 is devoted to results and discussion. Conclusions are given in Sect. 4.

2 Mathematical formulation and OLA

According to GMSM, the scattering amplitude describing the elastic scattering of a projectile particle on a target nucleus with ground state wave function, Ψ_i, may be written as,

$$F_{ii}(\bar{q}) = \frac{i\mathbf{K}_{c.m.}}{2\pi}\int d^2b e^{i\bar{q}\cdot\bar{b}}\langle\Psi_i|\Gamma(\vec{b},\vec{s}_1,\vec{s}_2,\ldots,\vec{s}_A)|\Psi_i\rangle = \frac{i\mathbf{K}_{c.m.}}{2\pi}\int d^2b e^{i\bar{q}\cdot\bar{b}}(1-e^{i\chi(b)}), \tag{1}$$

where $\bar{q} = \bar{k_f} - \bar{k_i}$ is the momentum transferred by the hadron to the nucleus within scattering mechanism. ($\bar{k_f}$ and $\bar{k_i}$ are the momenta of hadron after and before collision in the nucleus rest frame.) $\vec{s}_j$ is the transverse component of the radius vector of the jth nucleon of the nucleus. $\vec{b}$ is the impact parameter vector, Ψ_i is the ground state wave function of the target nucleus, and $\tilde{\mathbf{K}}_{c.m.}$ represents the wave number of incident hadron in center of mass system. $\Gamma(\vec{b},\vec{s}_1,\vec{s}_2,\ldots,\vec{s}_A)$ represents hadron–nucleus nuclear profile function, which is expressed through the nuclear profile functions of scattering on a single nucleons of the target nucleus Γ_j as follows:

$$\Gamma(\vec{b},\vec{s}_1,\vec{s}_2,\ldots,\vec{s}_A) = 1 - e^{i\chi(\vec{b},\vec{s}_1,\vec{s}_2,\ldots,\vec{s}_A)} = 1 - e^{i\sum_{j=1}^{A}\chi(\vec{b}_j,\vec{s}_j)} = 1 - \prod_{j=1}^{A}(1-\Gamma_j(\bar{b}-\bar{s_j})), \tag{2}$$

where

$$\Gamma_j(\bar{b}-\bar{s_j}) = \frac{1}{2\pi ik}\int e^{i\bar{q}\cdot(\bar{b}-\bar{s_j})}f_j(\bar{q})d^2q \tag{3}$$

and $f_j(\bar{q})$ is the elastic scattering amplitude of a hadron from the jth nucleons of the target nucleus. The nuclear phase shift function corresponding to the first term in the expansion of the nuclear profile function, Eq. (2) can be written as [6, 7]

$$\chi^{(1)}(\bar{b}) = \chi^{(OLA)}(\bar{b}) = iA\int\rho(\bar{r})\Gamma_j(\bar{b}-\bar{s_j})d\bar{r}. \tag{4}$$

This leading term depends upon the one-body ground state density of the target nucleus, $\rho(\bar{r})$, and the hadron–nucleon (hN) nuclear profile function, $\Gamma_j(\bar{b}-\bar{s_j})$, while the neglected terms depend upon two-, three-, and higher-order correlation functions. This term is known as the so-called optical limit approximation (OLA) [6, 7].

The ^{4}He nucleus is well described by considering four nucleons in relative s-state. Higher-symmetry components are negligible. So, in this study, the ground state density for the target nucleus is taken in the form of Gaussian distribution [31].

$$\rho(\bar{r}) = \left(\frac{d}{\pi}\right)^{3/2}\exp[-dr^2] \tag{5}$$

and d is related to the root-mean-square radius by the relation

$$\langle r^2\rangle = \sqrt{\frac{1.5}{d}}, \tag{6}$$

d is adjusted to 0.6942 fm^{-2} according to $\langle r^2\rangle^{1/2} = 1.47 \pm 0.02$ fm which has been reproduced from electron scattering data [38]. This form is used successfully in the treatment of σ_R for p-^{4}He using OLA [31].

In the ordinary GMSM, the simplified model assumptions about the hadron–nucleon (hN) elastic scattering amplitude, $f_j(q)$, are considered as follows:

(a) The elastic scattering amplitude of hadron–proton (hp) and hadron–neutron (hn) scattering was supposed to be identical, i.e., the isospin dependence of amplitude was neglected;
(b) The spin dependence was neglected as well.

Therefore, in the present study, the nucleon–nucleon (NN) elastic scattering amplitude, [$f_j(\bar{q})$, in Eq. (3)] in conjunction with NN phase variation [29–31, 36] and higher momentum transfer components [31, 36, 37] can be written as:

$$f_{\rm NN}(\bar{q}) = \frac{k_{\rm c.m.}\sigma_{\rm t}^{\rm NN}}{4\pi}(i+\epsilon_{\rm NN})e^{-\frac{1}{2}(\beta_{\rm NN}+i\gamma_{\rm NN})q^2}(1+T(q)) \quad (7)$$

with

$$T(q) = \sum_{n=1,2,\ldots}\lambda_n q^{2(n+1)}. \quad (8)$$

$\sigma_{\rm t}^{\rm NN}$, $\epsilon_{\rm NN}$, and $\beta_{\rm NN}$ denote the nucleon-nucleon total cross section, ratio of real-to-imaginary forward scattering amplitude ($q=0$), and the slope parameter, respectively. $\gamma_{\rm NN}$ stands for the phase variation parameter. $T(q)$ represents the higher-order momentum transfer components.

$k_{\rm c.m.}$ represents the incident nucleon momentum ($p=\hbar k, \hbar=1$) in the NN center of mass system and q is the momentum transfer from the incident nucleon to the target nucleon. Both $\gamma_{\rm NN}$ and λ_n are taken as free adjustable parameters in the whole energy range.

The parameters $\sigma_t^{\rm NN}$, $\epsilon_{\rm NN}$, and $\beta_{\rm NN}$ in Eq. (7) are considered as follows:

1. In the energy range from 19.49 to 100 MeV, $\sigma_{\rm t}^{\rm NN}$ is calculated using formula [39]

$$\sigma_{\rm t}^{\rm NN} = \frac{Z\sigma_{\rm pp}+N\sigma_{\rm pn}}{A}. \quad (9)$$

Z, and N stand for protons and neutrons numbers in the target nucleus, respectively. The total cross section for proton–proton ($\sigma_{\rm pp}$) and proton–neutron ($\sigma_{\rm pn}$) is determined from [40, 41], where

$$\sigma_{\rm pp} = [13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^4] \times \frac{1+7.772E_{\rm lab}^{0.06}\varrho^{1.48}}{1+18.01\rho^{1.46}} \quad \text{and} \quad (10)$$

$$\sigma_{\rm pn} = [-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.58\beta] \times \frac{1+20.88E_{\rm lab}^{0.04}\varrho^{2.02}}{1+35.86\rho^{1.90}}, \quad (11)$$

where $\beta=\sqrt{1-\frac{1}{\eta^2}}$, $\eta=\frac{E_{\rm lab}}{931.5}+1$, $E_{\rm lab}$ is the incident proton energy in laboratory frame, and ϱ represents the nuclear density in units of fm^{-3}.
The ratio of real-to-imaginary forward ($q=0$) elastic scattering NN amplitude, $\epsilon_{\rm NN}$, is taken as [39],

$$\epsilon_{\rm NN} = \frac{Z\sigma_{\rm pp}\epsilon_{\rm pp}+N\sigma_{\rm pn}\epsilon_{\rm pn}}{Z\sigma_{\rm pp}+N\sigma_{\rm pn}} \quad (12)$$

where both $\epsilon_{\rm pp}$ and $\epsilon_{\rm pn}$ are parameterized from the phase shift and coulomb interference measurements [19, 31, 36, 37] as,

$$\epsilon_{\rm pp} = -0.386 + 1.224e^{-\frac{1}{2}\left(\frac{k_{\rm lab}-0.427}{0.178}\right)^2} + 1.01e^{-\frac{1}{2}\left(\frac{k_{\rm lab}-0.592}{0.638}\right)^2} \quad \text{and} \quad (13)$$

$$\epsilon_{\rm pn} = -0.666 + 1.437e^{-\frac{1}{2}\left(\frac{k_{\rm lab}-0.412}{0.196}\right)^2} + 0.617e^{-\frac{1}{2}\left(\frac{k_{\rm lab}-0.797}{0.291}\right)^2}. \quad (14)$$

$k_{\rm lab}$ is the incident proton laboratory momentum in units of GeV/c. The values of $k_{\rm lab}$ can be calculated using the relation [31, 42]

$$k_{\rm lab} = c\sqrt{E_{\rm lab}(E_{\rm lab}+2m_{\rm p})}. \quad (15)$$

Since for $E_{\rm lab} < 300$ MeV, only the elastic scattering is energetically possible as the pion production threshold is closed, the slope parameter, $\beta_{\rm NN}$, can be determined from formulas [20, 31, 43, 44]

$$\sigma_{\rm el}^{\rm NN} = \frac{1+\epsilon_{\rm NN}}{16\pi\beta_{\rm NN}}(\sigma_{\rm t}^{\rm NN})^2, \quad (16)$$

where $\sigma_{\rm el}^{\rm NN}=\sigma_{\rm t}^{\rm NN}$, and $\sigma_{\rm el}^{\rm NN}$ represents the total elastic NN cross section.

2. In the energy range from 200 to 800 MeV, $\sigma_{\rm t}^{\rm NN}$ is determined from Eqs. (9, 10, 11), while both $\beta_{\rm NN}$ and $\epsilon_{\rm NN}$ are taken from Ref. [45], while for $E_{\rm lab} = 1030, 1240$ and 1730 MeV, $\sigma_{\rm t}^{\rm NN}, \beta_{\rm NN}$, and $\epsilon_{\rm NN}$ are considered from Ref. [46].
3. In the range of energy from 45 to 393 GeV, the parameters of NN elastic scattering amplitude are taken from Table (IX) in the work of Bujak et al. [47].

The angular distribution for elastic scattering observable can be determined from Ref. [6, 7],

$$\frac{d\sigma}{dq^2} = \frac{\pi}{\mathbf{K}_{\rm c.m.}^2}|F(q)|^2 = \frac{\pi}{\mathbf{K}_{\rm c.m.}^2}\frac{d\sigma}{d\Omega} \quad (17)$$

from Eq. (1) $F(q)$ has the following expression [6, 7]

$$F(q) = i\mathbf{K}_{\rm c.m.}R(q)\int b db J_0(qb)(1-e^{i\chi_{\rm opt}(\bar{b})}), \quad (18)$$

$R(q)$ is a center of mass correction function [6, 7]

$$R(q) = e^{\langle r^2\rangle q^2/6A}, \quad (19)$$

and $\mathbf{K}_{\rm c.m.}$ can be formulated as [31, 42]

$$\mathbf{K}_{\rm c.m.} = \frac{m_{\rm t}k_{\rm lab}}{\sqrt{(m_{\rm p})^2+(m_{\rm t})^2+2m_{\rm t}\sqrt{(k_{\rm lab})^2+(m_{\rm p})^2}}}, \quad (20)$$

where $m_{\rm p}$ and $m_{\rm t}$ represent projectile and target masses, respectively. $k_{\rm lab}$ is presented in Eq. (15).

3 Results and discussion

3.1 Effect of Pauli blocking

Figure 1 represents the results of the calculations of $\frac{d\sigma}{dq^2}$ for p-^{4}He in the range of proton energy from 19.94 to 1030 MeV, considering σ_t^{NN} (Eq. 9) by aiding (Eqs. 10, 11) in two cases, namely σ_t^{NN} (free), where $\varrho = \varrho_o = 0$ (solid curves), and σ_t^{NN} (bound) due to Pauli blocking, where $\varrho = \varrho_o = 0.17$ fm^{-3} (dashed curves) in the absence of γ_{NN} and λ_n. It is apparent that the two cases fail seriously to account for the available experimental data for $E_{lab} = 19.94, 30.43, 39.8$, and 45 MeV. At a higher proton energy, beginning from 100 MeV, a noticeable agreement is obtained, especially in the forward direction up to the first minima, with both $\varrho = \varrho_o = 0$ and $\varrho = \varrho_o = 0.17$ fm^{-3}. However, a qualitative agreement after the forward regions is obtained. For higher proton energies, i.e., $E_{lab} > 1$ GeV, the two cases almost produced the same results, where σ_t^{NN} (free) $\simeq \sigma_t^{NN}$ (bound). Let us now proceed to consider the effects of both γ_{NN} and λ_n ($n = 1$ and 2) in the case of $\varrho = \varrho_o = 0$ on the calculation of elastic scattering differential cross section.

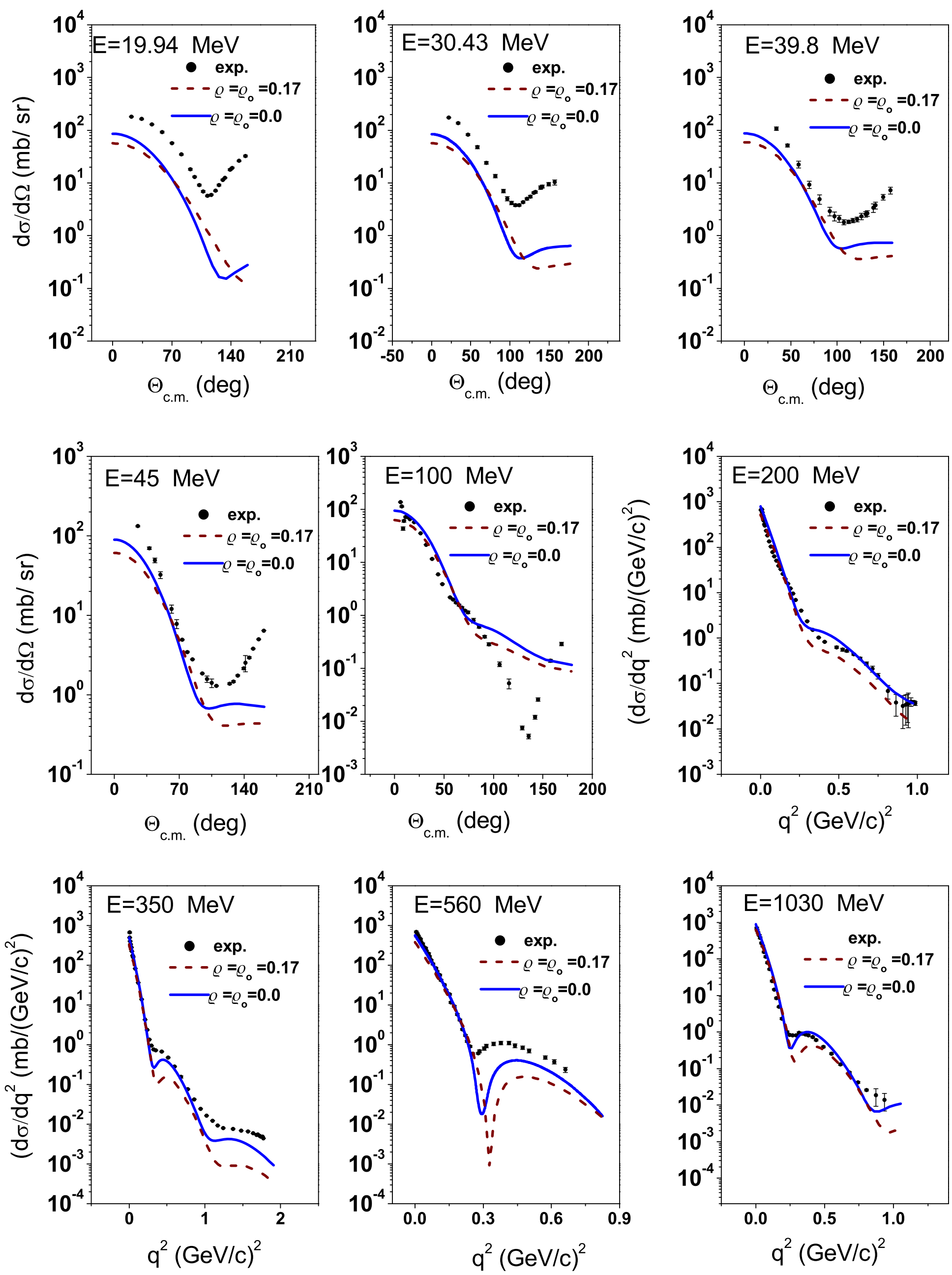

Fig. 1 Elastic scattering differential cross section for p-^{4}He from 19.94 to 1030 MeV. The experimental data from 19.49 to 100 MeV are taken from Ref. [48–50], while from 200 to 1030 MeV are considered from Ref. [52, 53] $\gamma_{NN} = \lambda_n = 0$

3.2 $\frac{d\sigma}{d\Omega}$ for 19.9 ≤ E_{lab} ≤ 100 MeV

The elastic scattering differential cross section for proton-^{4}He is calculated in the laboratory proton energy from 19.9 to 100 MeV by introducing the effects of both the NN phase variation and high-order momentum transfer components, as referred in Eq. (8).

Figures 2 and 3 represent our results in the above-mentioned range of proton energies. It is obvious that the usual NN elastic scattering amplitude without the effects of both γ_{NN} and λ_n ($n = 1, 2$) cannot reproduce the experimental data [48–50] in the whole range of scattering angles $\Theta_{c.m.}$. Taking into consideration the effect of γ_{NN} only ($\lambda_n = 0$), as represented by dashed curves, slightly improves the situation in the region $\Theta_{c.m.} > \sim 70°$ for $19.9 \le E_{lab} < 30$ MeV and in the region $\Theta_{c.m.} > \sim 50°$ for $30 < E_{lab} < 100$ MeV. However, unsatisfactory agreement still remains over the whole range of scattering angles.

Moreover, the theoretical results in the case of $\gamma_{NN} = 0$ (dotted curves) and $\gamma_{NN} \neq 0$ (dashed curves) are nearly equivalent in the forward scattering angles $\Theta_{c.m.} < \sim 70°$. This attitude clarifies that the NN phase variation plays a minute role in the forward scattering angles (very small momentum transfer). Considering only λ_n (n = 1 and 2) where $\gamma_{NN} = 0$ is shown by dashed-dotted curves led to unrealistic results in the proton energy range, $E_{lab} < 45$

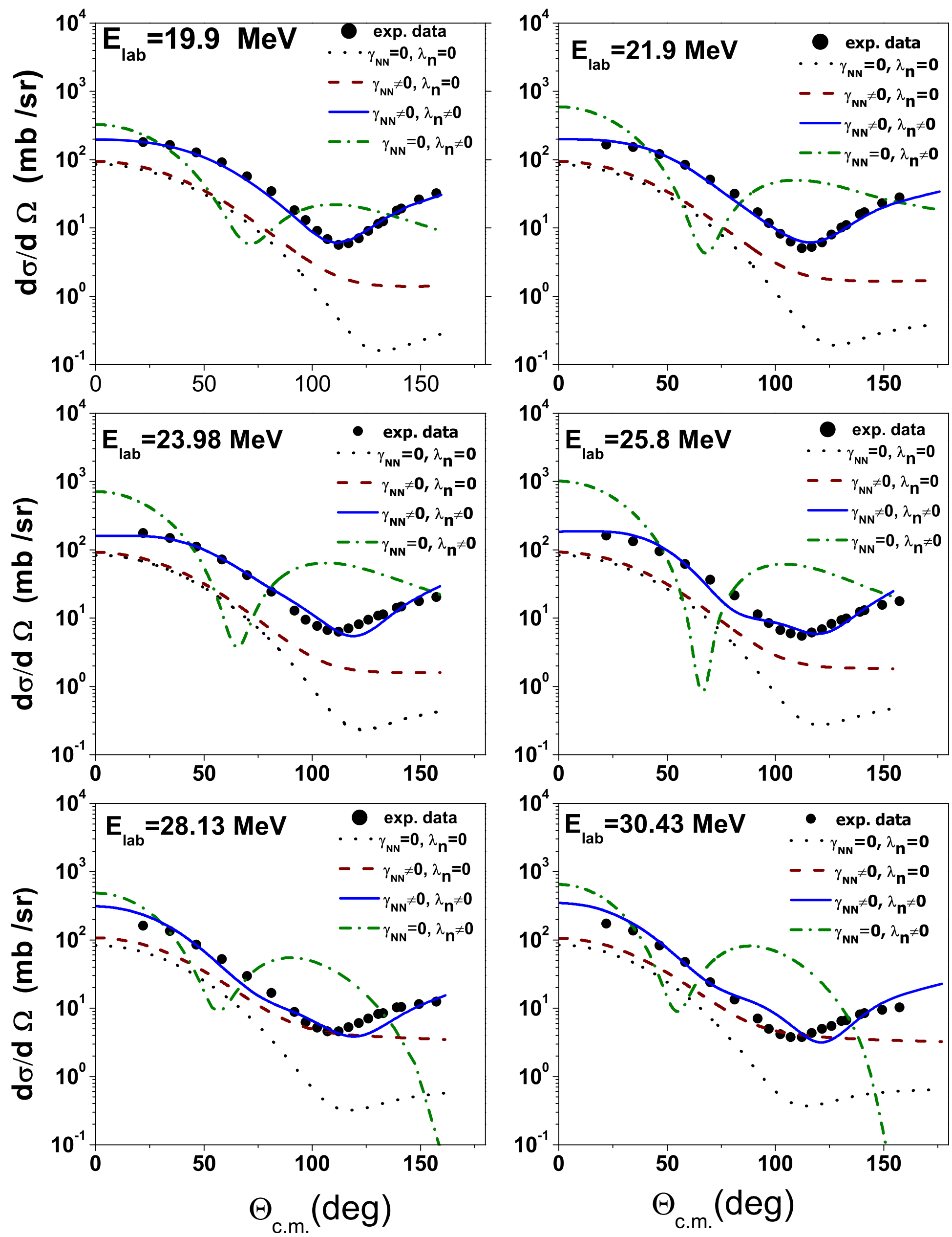

Fig. 2 Elastic scattering differential cross section for p-^{4}He in the laboratory proton energy, $19.9 \le E_{lab} \le 30.43$ MeV. The experimental data are taken from [48–50]

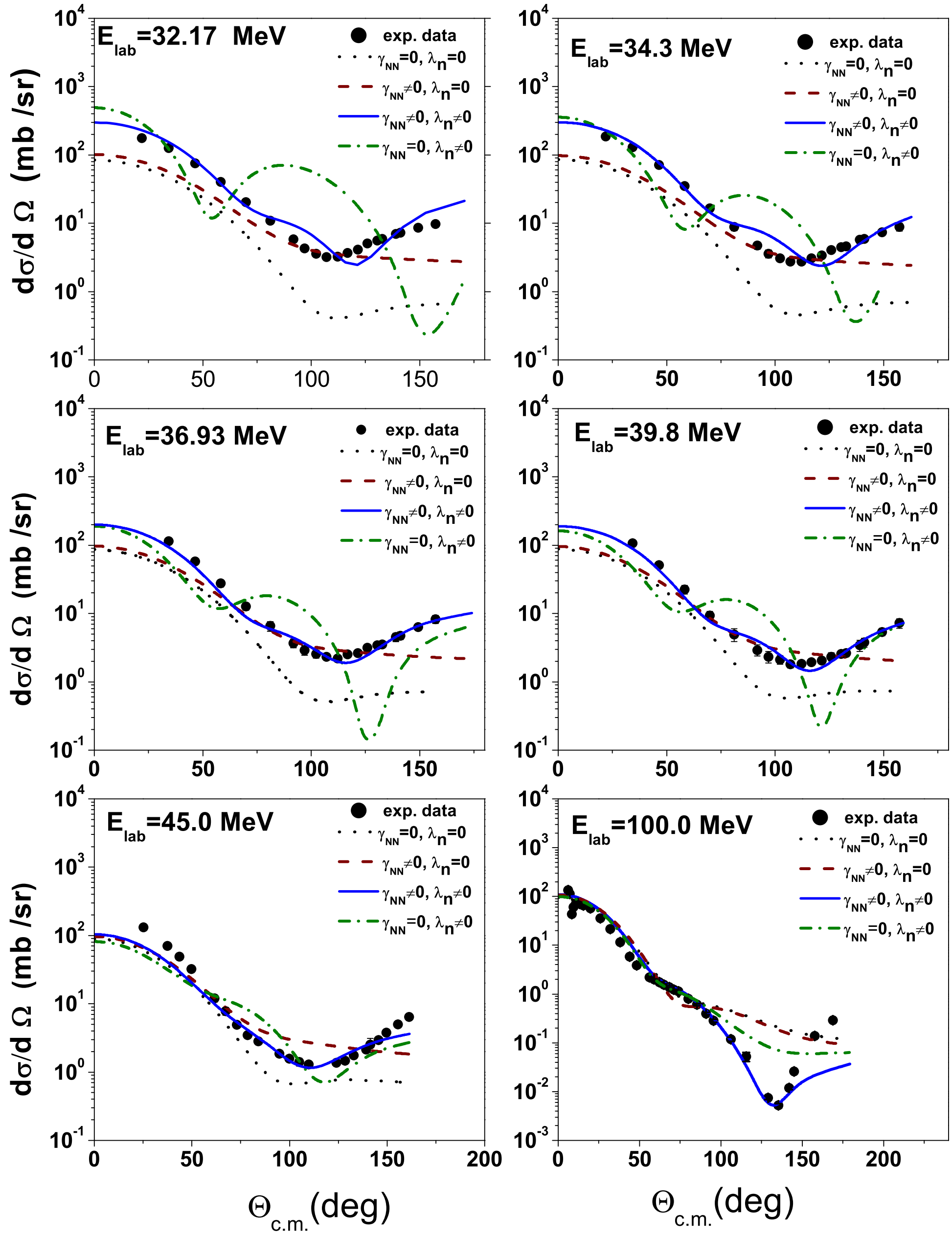

Fig. 3 Same as Fig. 2, but for $32.17 \leq E_{\mathrm{lab}} \leq 100$ MeV

MeV. The values of λ_n which reproduced the results, are presented in Table 1.

It is interesting to elucidate that introducing the effects of γ_{NN} and λ_n ($n = 1, 2$) (solid curves) push the OLA closer to the experimental data, and a quite satisfactory account of the data in the whole scattering angles and proton energies are obtained. It is clear from these results that γ_{NN} plays two important roles as follows: the first one is to help the curves to be smooth. Secondly, it fills the regions of minima, just like the parameter, ϵ_{NN}. This effect is discussed in the work of Dalkarov and Karmanov [51] for p'-nucleus scattering. Table 1 clarifies the values of γ_{NN} in this range of proton energies. The success of introducing γ_{NN} and λ_n in this relatively low proton energy may owe to that both of them modifies the ratio of the real part to the imaginary of the forward elastic scattering amplitude and make the diffraction pattern of $\frac{d\sigma}{d\Omega}$ more shallower.

3.3 $\frac{d\sigma}{dq^2}$ for $200 \leq E_{\mathrm{lab}} \leq 1730$ MeV

Figure 4 illustrates the calculation of $\frac{d\sigma}{dq^2}$ for p-^{4}He in the range of energy from 200 MeV to 1730 MeV. One can notice that the theoretical results with $\gamma_{\mathrm{NN}} = \lambda_n = 0$ (dotted curve) reproduce the available experimental data in the forward directions, $q^2 \leq 0.2$ (GeV/c)^2. Since the aim of this

Table 1 Values of γ_{NN} and λ_n (n = 1 and 2), which gave a better agreement with the available experimental data of elastic scattering differential cross section for p-^{4}He

Energy (MeV)	γ_{NN} (fm^2)	λ_1 (fm^4)	λ_2 (fm^6) $\times 10^{-2}$
19.94	− 0.645	0.2113 + 0.3192i	2.3044 − 25.3217i
21.9	− 0.690	0.1035 + 0.5862i	0.0252 − 31.5153i
23.98	− 0.635	0.1276 + 0.6795i	0.0002 − 30.3013i
25.82	− 0.680	0.0047 + 0.8496i	0.0010 − 33.6456i
28.13	− 1.040	0.3466 + 0.2577i	0.0053 − 24.5361i
30.43	− 1.020	0.3206 + 0.3419i	0.0008 − 26.3360i
32.17	− 0.900	0.2148 + 0.3782i	0.0282 − 22.6792i
34.3	− 0.820	0.0846 + 0.3587i	07.7338 − 18.0956i
36.93	− 0.780	0.0666 + 0.2833i	05.6295 − 12.0034i
39.8	− 0.730	0.0004 + 0.2834i	06.5773 − 09.9711i
45.0	− 0.730	0.00002 + 0.1014i	0.0005 − 04.8729i
100.0	0.210	− 0.0401+0.0004i	0.8642 + 0.0920i
200.0	− 1.600	0.0006 + 0.0082i	−0.0001 − 0.0894i
350.0	− 0.200	0.00007 + 0.0084i	−0.0097 − 0.0392i
500.0	− 0.380	0.0001 + 0.0214i	−0.0801 − 0.1548i
560.0	− 0.220	0.0001 + 0.0165i	−0.0872 − 0.0938i
800.0	− 0.340	0.00003 + 0.0149i	−0.0807 − 0.0505i
1030.0	0.100	0.0002 − 0.0016i	−0.0188 − 0.0186i
1240.0	0.100	0.0002 − 0.0016i	−0.0188 − 0.0186i
1730.0	0.220	0.00005 − 0.0044i	−0.0479 − 0.0355i
45 000	− 0.49		
97 000	0.12		
145,000	− 0.01		
200,000	0.04		
259,000	0.045		
301,000	0.052		
393,000	0.050		

work is to establish the suitability of the NN elastic scattering amplitude Eq. (7) in different situations, the phase variation parameter is considered only as seen in Fig. 4 (dashed curve). This highly improves the fitting with the experimental data [52, 53] in the whole range of momentum transferred square, except at $E_{\mathrm{lab}} = 200$ MeV. Moreover, we also performed calculations for $\frac{d\sigma}{dq^2}$ by considering only λ_n (solid curve). It is found that this higher-order momentum transfer components provided a more satisfactory explanation of the experimental data. Indeed, one can say that both γ_{NN} and λ_n played almost the same role in describing the elastic scattering angular distributions in this range of proton energies. The values of γ_{NN} and λ_n are listed in Table 1.

3.4 $\frac{d\sigma}{dq^2}$ for $45 \le E_{\mathrm{lab}} \le 393$ GeV

It is well known that the investigations within GMSM are physically meaningful when one could consistently have a satisfactory account of the available scattering data for the same target nucleus, but at different ranges of incident proton energy. Therefore, the elastic scattering differential cross section for p-^{4}He is extended to the range of proton energy from 45 to 393 GeV, as shown in Fig. 5. It is apparent that when $\gamma_{NN} = \lambda_n = 0$ a satisfactory agreement is obtained in the whole range of $|t| = q^2$ (dashed curve), in comparison with the experimental data [47]. However, some discrepancies still exist in the regions minima. Introducing the phase variation parameter γ_{NN} pushed the theoretical results (dotted curve), to agree with the data in these regions. It is possible to say that the higher-order momentum transfer components of the NN elastic scattering amplitude have no announced effect in this energy range, where q^2 extends only to 0.4 (GeV/c)^2. Furthermore, it is observed that this approach gives relatively better results than the conventional Glauber model at 45 GeV [54], the work of Bujak et al [47] at 393 GeV, and the work of Mosallem et al. [55] within Glauber model at $E_{\mathrm{lab}} = 96, 145, 259,$ and 301 GeV, where the target nucleus, ^{4}He, is described by a collective 12-quark bag. The values of γ_{NN} are shown in Table 1. The values of γ_{NN}

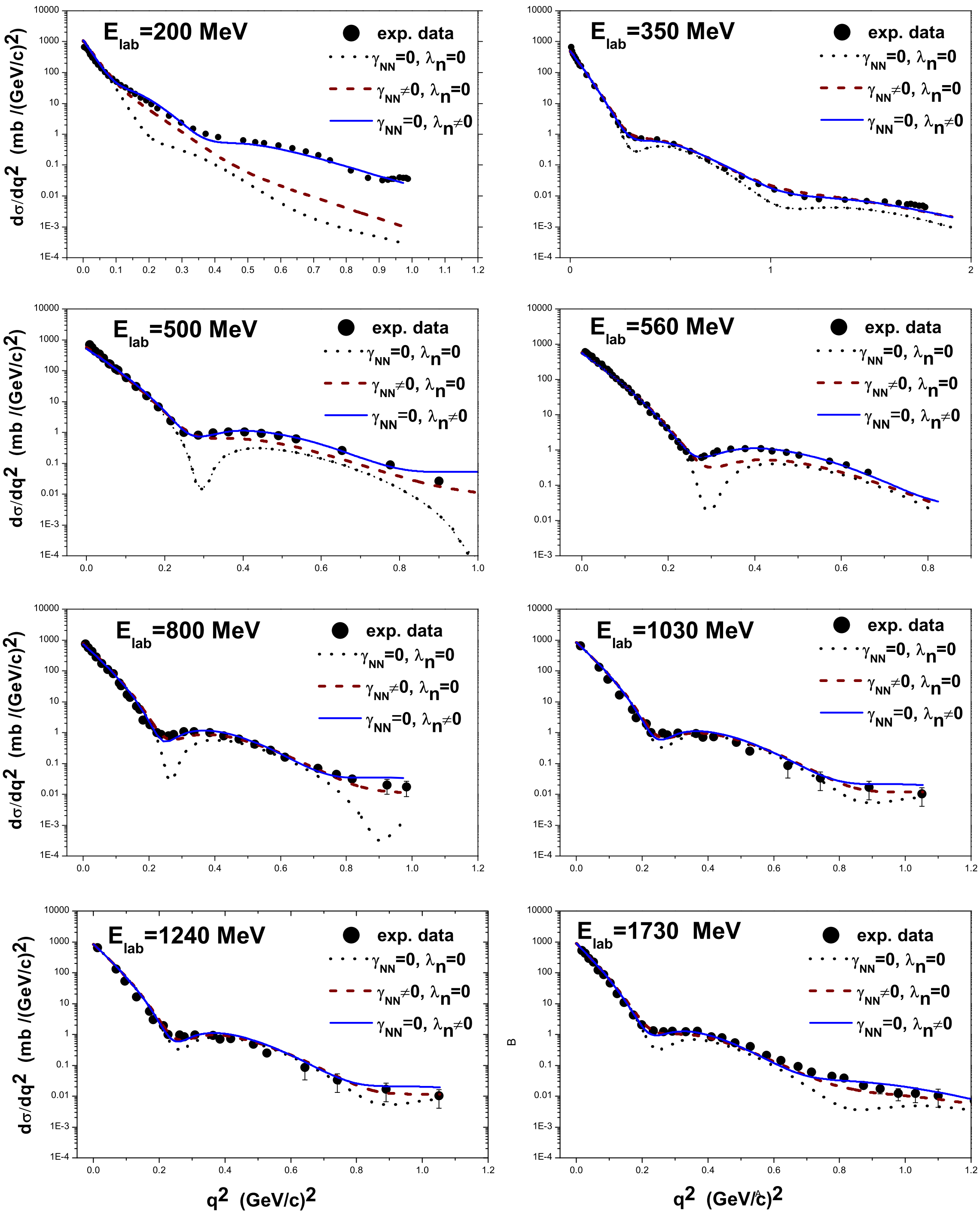

Fig. 4 $\frac{d\sigma}{dq^2}$ for proton energies $200 \leq E_{\mathrm{lab}} \leq 1730$ MeV

and λ_n (n = 1 and 2) in Table 1 declared that these parameters are very sensitive to the proton energy.

It is well known that NN scattering measurements leave an overall phase of the amplitude undetermined. The phase factor $e^{-\gamma_{\mathrm{NN}}q^2/2}$ in Eq. (7) is to take care of this fact. The phase variation parameter could not be detected experimentally.

The phase parameter, γ_{NN}, may be positive or negative [33, 36, 56], and it has been shown that in some situations inclusion of the phase variation significantly affects the

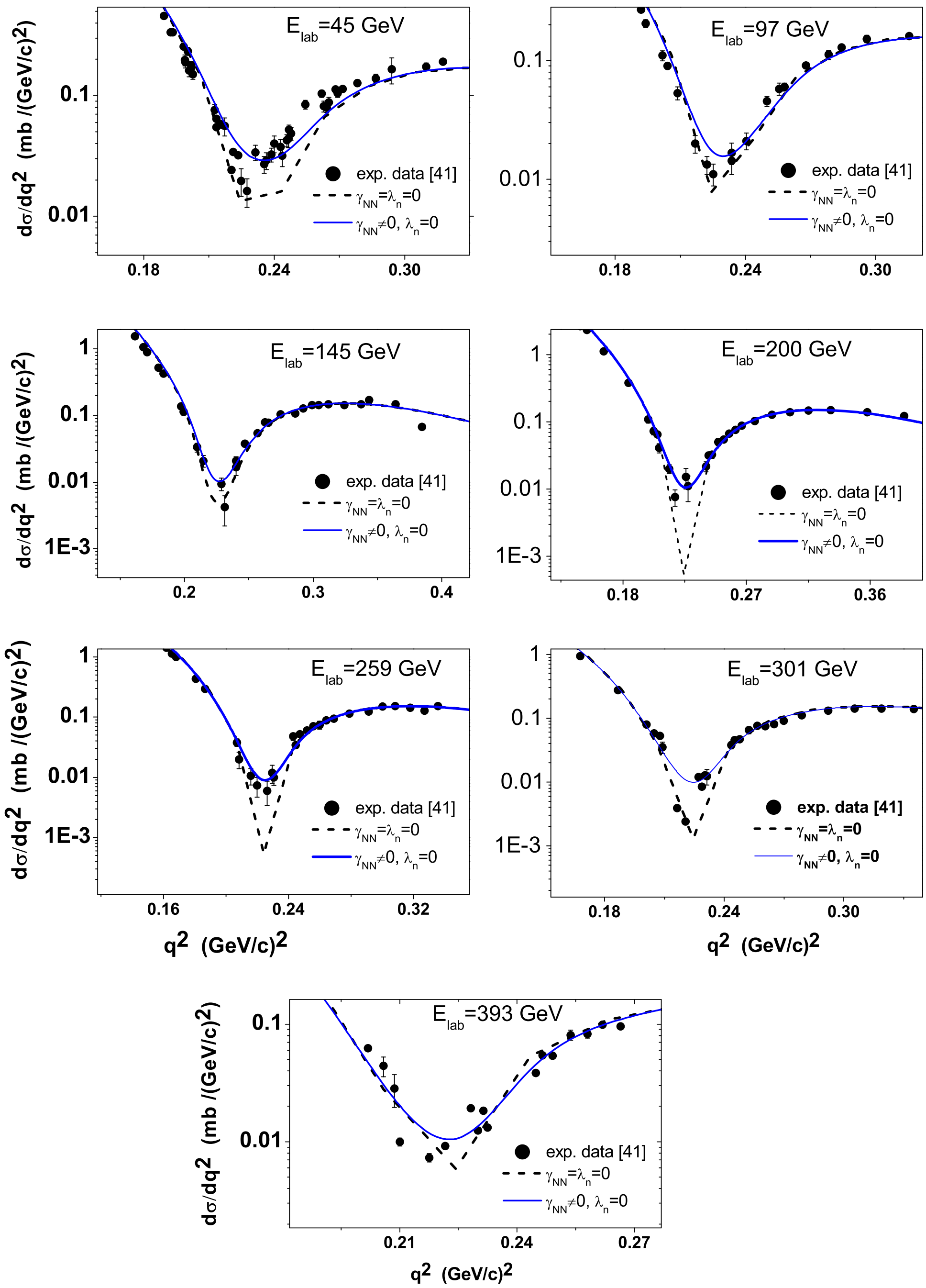

Fig. 5 $\frac{d\sigma}{dq^2}$ for proton energies $45 \leq E_{\mathrm{lab}} \leq 393$ GeV

calculated cross section [31, 33, 36, 56]. On the other hand, for the given value of ϵ_{NN}, the variation of γ_{NN} leads to either an overall increase or decrease in the estimated values of the cross section [33, 57].

Moreover, many efforts are made to determine the phase variation parameter, γ_{NN}, using different NN potentials [56, 58, 59]. They obtained different values for γ_{NN} at $E_{\mathrm{lab}} = 1$ GeV. Then, in our analysis, γ_{NN} is taken as an adjustable unknown energy-dependent parameter.

In the work of Chauhan and Khan for ^{4}He–nucleus elastic scattering in the energy range 25–70 MeV/nucleon [36] and for ^{4}He–nucleus total reaction cross section in the energy range from 69.62 to 192.4 MeV [37], the parameters λ_1 and λ_2 for both p–p(n–n) and p–n(n–p) collisions are estimated. It is difficult to compare the present result with theirs. So, in this study, λ_n ($n = 1$ and 2) are treated as a free energy-dependent parameters.

Also, the values of γ_{NN} have a negative sign in the range of energy $19.94 \leq E_{\mathrm{lab}} \leq 45$ MeV. This is compatible with the work of Deeksha and Khan [36] for α-nucleus total reaction cross sections.

4 Conclusion

The main objective of this paper clarifies how the NN elastic scattering amplitude with its in-medium parameters (Eq. 7 with Eq. 8) can behave in accounting the angular distributions of elastic scattering differential cross section for p-^{4}He in relatively law and intermediate and high proton energies within OLA of GMSM. It is found that reducing the NN total cross section according to Pauli blocking with $(\lambda_n = \gamma_{\mathrm{NN}} = 0)$ in the energy range $19 < E_{\mathrm{lab}} < 100$ MeV cannot reproduce the data. However, for $100 \leq E_{\mathrm{lab}} < 2000$ MeV, an improved agreement is achieved in the forward directions and a qualitative agreement is noticed after these directions. Introducing both λ_n ($n = 1$ and 2) and γ_{NN} supported the theoretical results in comparison with the experimental data over the whole ranges of $\Theta_{\mathrm{c.m.}}$ and q^2 (GeV/c)^2 in the energy range $19 < E_{\mathrm{lab}} < 2000$ MeV. On the other hand, considering only γ_{NN} ($\lambda_n = 0$) led to improve the calculations, especially in the region of minima for $45 \leq E_{\mathrm{lab}} \leq 393$ GeV. In this high range of proton energies, where [σ_t^{NN} (free) $\simeq \sigma_t^{\mathrm{NN}}$ (bound)], it is apparent that Pauli blocking and λ_n played a negligible role in describing the data.

In addition, it is concluded that the consideration of two terms in NN elastic scattering amplitude (λ_n, $n = 1$ and 2) with γ_{NN} provides a more satisfactory explanation of the data throughout the available ranges of momentum transfer (or scattering angles) than does in one Gauss ($\lambda_n = 0$). Also, this NN elastic scattering amplitude may not only cover the relatively large scattering angles, but also describe the non-diffractive behavior of proton–nucleus scattering at relatively low energies. Unfortunately, there is no obvious systematic variation for these parameters with the proton energy. This point needs more investigations, specially when one could have a consistently as satisfactory account of the available scattering data for different target nuclei at the same incident proton energies.

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