A new digital Gaussian pulse shaping algorithm based on bilinear transformation*

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Nuclear pulse signal needs to be transformed to a suitable pulse shape to remove noise and improve energy resolution of a nuclear spectrometry system. In this paper, a new digital Gaussian shaping method is proposed. According to Sallen-Key analog Gaussian shaping filter circuits, the system function of Sallen-Key analog Gaussian shaping filter is deduced on the basis of Kirchhoff laws. The system function of the digital Gaussian shaping filter based on bilinear transformation is deduced too. The expression of unit impulse response of the digital Gaussian shaping filter is obtained by inverse z-transform. The response of digital Gaussian shaping filter is deduced from convolution sum of the unit impulse response and the digital nuclear pulse signal. The simulation and experimental results show that the digital nuclear pulse has been transformed to a pulse with a pseudo-Gaussian, which confirms the feasibility of the new digital Gaussian pulse shaping algorithm based on bilinear transformation.

Keywords: Digital shaping, Bilinear transformation, Convolution sum

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I. INTRODUCTION

In a nuclear spectrometry system, the peak amplitude of nuclear pulse from the detector is proportional to energy of the incident particle, so the particle energy can be determined by measuring height of the pulse signal. A major cause of inaccuracy in the energy measurement is electronic noise, ballistic deficit, etc [1]. Shaping techniques are intended to improve the signal-to-noise ratio and extract height of the pulse signal accurately [2–12]. Gaussian pulse shaping, as a general shaping method in the nuclear spectrometry system, performs good in this regard [12, 13]. With the rapid development of digital integrated circuits, and their advantages of flexibility and stability, studies on the digital Gaussian shaping algorithm for nuclear pulse signal has become popular during the past decades [12–21]. Valentin T. Jordanov studied digital Gaussian shaping algorithm of exponential signals [15]. Chen *et al.* made efforts in digital Gaussian shaping algorithm based on wavelet analysis [13, 14]. Zhou *et al.* made advances in digital Gaussian shaping algorithm based on numerical differentiation [19]. The digital nuclear pulse signals are treated as quasi-Gaussian signals, and recursive algorithm is used to achieve the digital Gaussian shaping. In this paper, a digital Gaussian shaping algorithm based on bilinear transform is proposed. The unit impulse response of the digital Gaussian shaping system is deduced, and Gaussian shaping is realized by calculating the convolution sum of unit impulse response and digital nuclear pulse signal.

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II. SYSTEM FUNCTION OF THE ANALOG GAUSSIAN SHAPING SYSTEM

The analog Gaussian shaping in the nuclear spectrometry system, which is often implemented in Sallen-Key filter, as shown in Fig. 1, can shape a nuclear pulse signal as a pseudo-Gaussian signal.

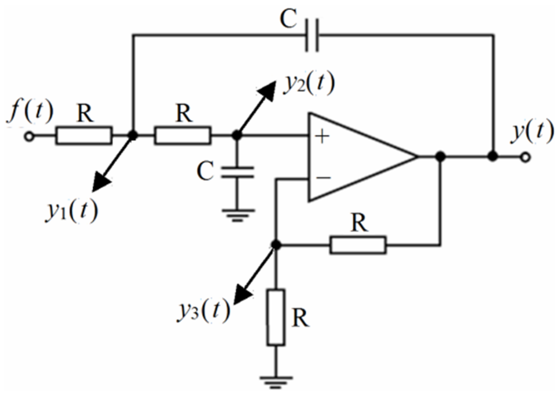

Fig. 1. Scheme of the Sallen-Key filter circuits.

According to Kirchhoff's law, the Eq. (1) can be obtained,

$$[f(t) - y_1(t)]/R = C\mathrm{d}y_2(t)/\mathrm{d}t + C\mathrm{d}(y_1(t) - y(t))/\mathrm{d}t, \quad (1)$$

$$y_2(t) = R\frac{y(t)}{2R}, \quad (2)$$

$$y_1(t) = y_2(t) + RC\mathrm{d}y_2(t)/\mathrm{d}t. \quad (3)$$

Thus, the relationship between input signal $f(t)$ and output signal $y(t)$ can be expressed as

$$R^2C^2y''(t) + RCy'(t) + y(t) = 2f(t). \quad (4)$$

Let the Laplace transform of $f(t)$ and $y(t)$ be $F(s)$ and $Y(s)$, respectively. Using differential properties of Laplace transform, we have the Laplace transform of $y'(t)$ and $y''(t)$ be $sY(s)$ and $s^2Y(s)$, respectively, and we have

$$R^2C^2s^2Y(s) + RCsY(s) + Y(s) = 2F(s). \tag{5}$$

Thus, the system function of the analog Gaussian shaping system is

$$H(s) = Y(s)/F(s) = 2/(R^2C^2s^2 + RCs + 1). \tag{6}$$

III. SYSTEM FUNCTION OF THE DIGITAL GAUSSIAN SHAPING SYSTEM

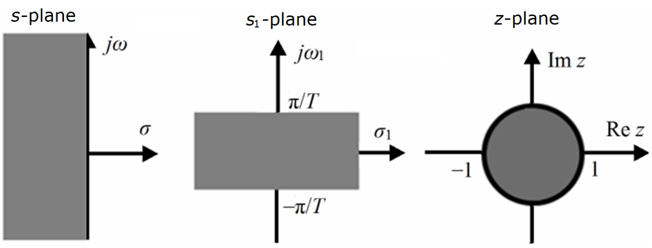

Fig. 2. Mapping relationship of bilinear transformation.

In the process of transforming Gaussian shaping system from analog domain to digital domain, bilinear transformation is used for conversion in order to maintain the basic properties of the system function unchanged and overcome the spectrum aliasing phenomenon. The principle of bilinear transformation is secondary mapping method, as shown in Fig. 2.

The mapping process includes three steps,

(1) The s-plane is mapped onto the strip-like region from $-\pi/T$ to π/T of the s_1-plane. A conversion can be done as

$$s = 2\tan(s_1T/2)/T = (1 - \mathrm{e}^{-s_1T})/(1 + \mathrm{e}^{-s_1T})[2/T]. \tag{7}$$

(2) Mapping the s_1-plane onto the z-plane; the conversion relationship can be expressed as

$$z = \mathrm{e}^{s_1T}. \tag{8}$$

(3) Mapping the s-plane onto the z-plane; the following transform equation between s and z domain based on the Eqs. (7) and (8) can be obtained,

$$s = (1 - z^{-1})/(1 + z^{-1})[2/T]. \tag{9}$$

This allows the one-to-one mapping relationship from the s-plane to the z-plane. Thereby spectrum aliasing phenomenon is eliminated. The system function of the digital Gaussian shaping system can be obtained by Eqs. (6) and (9) when letting $M = RC/T$

$$H(z) = (2 + 4z^{-1} + 2z^{-2})/[4M^2 + 2M + 1 + (2 - 8M^2)z^{-1} + (4M^2 - 2M + 1)z^{-2}]. \tag{10}$$

Let $a = 4M^2 + 2M + 1$, $b = 2 - 8M^2$, and $c = 4M^2 - 2M + 1$, the inverse z transform is carried out to Eq. (10). Then the unit impulse response of the digital Gaussian shaping system is

$$\begin{aligned} h(n) =&(2k_1/a)\{[-b - (b^2 - 4ac)^{1/2}]/(2a)\}^n u(n) + (2k_2/a)\{[-b + (b^2 - 4ac)^{1/2}]/(2a)\}^n u(n) \\ &+ (2(k_3/a)\{[-b - (b^2 - 4ac)^{1/2}]/(2a)\}^{n-1} u(n-1) + ([2k_4/a)\{[-b + (b^2 - 4ac)^{1/2}]/(2a)\}^{n-1} u(n-1), \end{aligned} \tag{11}$$

where, $k_1 = (b - 4a)/[2(b^2 - 4ac)^{1/2}] + 1/2$, $k_2 = (-b + 4a)/[2(b^2 - 4ac)^{1/2}] + 1/2$, $k_3 = -a/(b^2 - 4ac)^{1/2}$, and $k_4 = a/(b^2 - 4ac)^{1/2}$.

Let $z = e^{j\omega}$, the discrete-time Fourier transform of $h(n)$ can be obtained by Eq. (10),

$$H(\mathrm{e}^{j\omega}) = (2 + 4\mathrm{e}^{-j\omega} + 2\mathrm{e}^{-2j\omega})/(a + b\mathrm{e}^{-j\omega} + c\mathrm{e}^{-2j\omega}). \tag{12}$$

The amplitude spectrum of $H(\mathrm{e}^{j\omega})$ is shown in Fig. 3. The spectral width of the digital Gaussian shaping system increases with M decreasing. This makes the high-frequency noise filtration reduce.

IV. GAUSSIAN SHAPING OF THE NUCLEAR PULSE SIGNAL

A. Mathematical description of the nuclear pulse signal

The structure of a nuclear spectrometer system with digital shaping is shown in Fig. 4. If the charge generated by an impinging particle is instantaneously collected by the detector, the output signal from the preamplifier can be approximately expressed by a continuous-time exponential signal

$$f(t) = B(\mathrm{e}^{-t/\tau_1} - B_1\mathrm{e}^{-t/\tau_2})u(t), \tag{13}$$

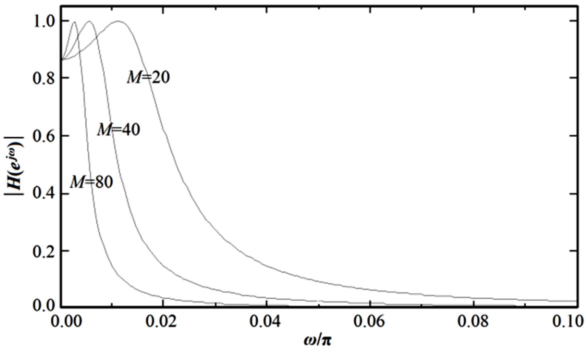

Fig. 3. The amplitude spectrum of $H(e^{j\omega})$.

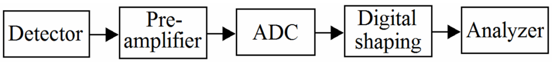

Fig. 4. Scheme of a spectroscopic system with digital shaping system.

where, B is magnitude of the signal; $u(t)$ is a unit step signal; τ_1 and τ_2 are time constants of the exponential signal; and B_1 is proportionality coefficient. To simplify the calculation, let $B_1 = 1$. If the ADC sampling period is T, the sampled discrete-time signal $f(n)$ is

$$f(n) = B(e^{-nT/\tau_1} - B_1 e^{-nT/\tau_2})u(n). \tag{14}$$

B. Digital Gaussian shaping of the simulated double exponential signal

The response of the input signal $f(n)$ after the digital Gaussian system is described as

$$\begin{aligned} y(n) &= f(n) * h(n) \\ &= \sum_{m=-\infty}^{\infty} f(m)h(n-m), \end{aligned} \tag{15}$$

where, $h(n)$ is the unit impulse response of the system; and $y(n)$ is the zero state response of the system.

Take double exponential Gaussian signal as input of the digital Gaussian shaping system, and the sampling period $T = 0.01\,\mu$s, $B = 1.6$, $\tau_1 = 0.23\,\mu$s. Fig. 5 shows the response $y(n)$ of digital Gaussian shaping system, calculated and plotted by Matlab at $\tau_2 = 0.4$ and $1.8\,\mu$s for different of M values. Width of the shaping waveform increases with M, and the symmetry improves with larger M. An advantages of the digital Gaussian shaping system is that one needs just changing the M value of the parameter, rather than adjusting the hardware.

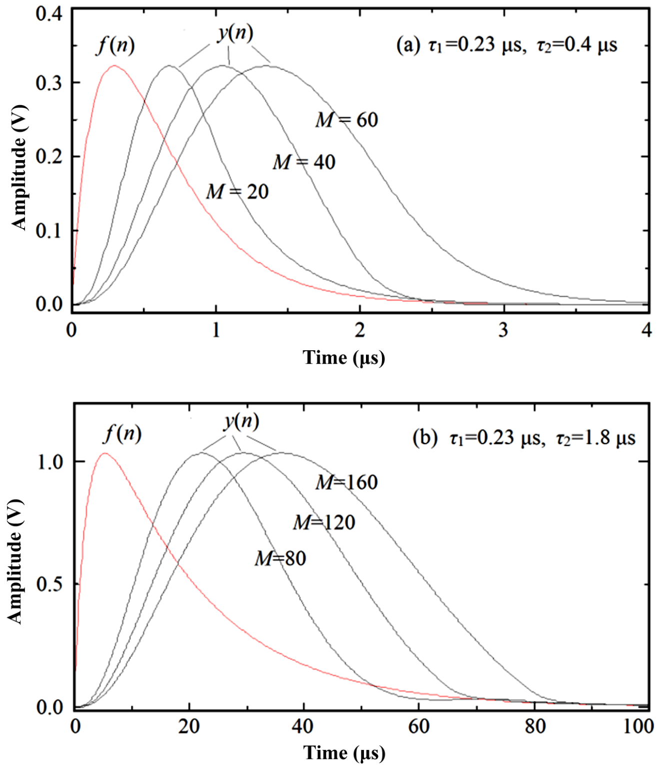

Fig. 5. (Color online) Response of the digital Gaussian shaping system with simulation exponential signals.

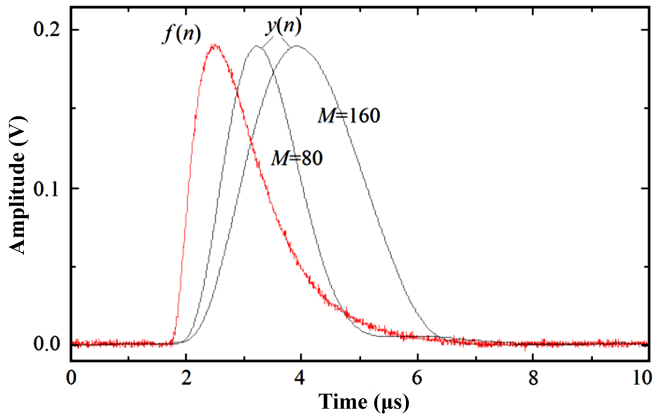

Fig. 6. (Color online) Response of the digital Gaussian shaping system with sampling signals from detecting a ^{60}Co source.

C. Digital Gaussian shaping of the real sampled nuclear pulse signal

Signals from a ^{60}Co source were measured by a NaI(Tl) detector and an Agilent DSO-X 2012A oscilloscope. The response $y(n)$ of the digital Gaussian shaping system is calculated by Eq. (15) in MATLAB, after the digital nuclear pulse signals are stored, with sampling frequency of 200 MHz. Fig. 6 shows the measured pulse signal and its corresponding waveform of the digital shaping signal at $M = 80$ and 160. The

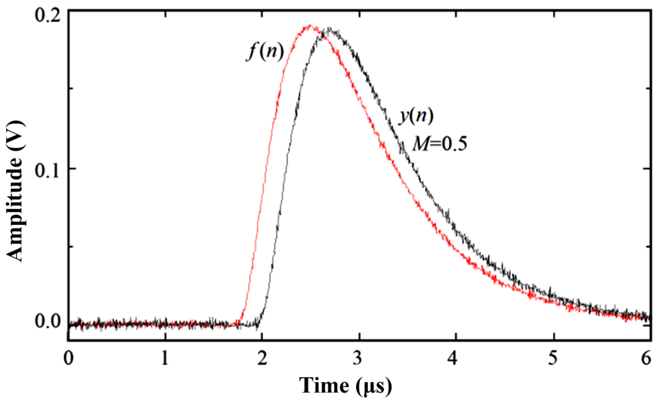

Fig. 7. (Color online) Response of the digital Gaussian shaping system with actual sampling signals at $M = 0.5$.

shaping waveform at $M = 160$ is wider, and the symmetry is better, than those at $M = 80$. The shaping waveform is preferably Gaussian waveform. Decreasing the M value, the shaping waveform will be narrowed in time domain. Also, a low shaping time reduces the noise filtration and resulted in a poor energy resolution. At $M = 0.5$ (Fig. 7), significant noises in shaping waveform can be seen.

V. DISCUSSION AND CONCLUSION

Pulse shaping digitization is the key to digitization of nuclear instruments. After deducing unit impulse response of the digital Gaussian shaping system by bilinear transformation, the convolution sum of the digital nuclear pulse signal and unit impulse response of the derived digital Gaussian system can be carried out. The digital nuclear pulse signal is shaped as pseudo-Gaussian signal and the noises are removed. A higher M leads to better symmetry of the shaping waveform, wider pulse, and better effect of noise filtering, with flat top pulse and small ballistic deficit. But a wider pulse will increase the pulse pile-up. Pile-up distortion is a major drawback in nuclear spectrometry system at high count rate. Narrowing the width of the pulse can lead to the reduction of the pulse pile-up distortion, but a low shaping time reduces the noise filtration and results in a poor energy resolution. In practical applications, the shaping parameter values can be changed neatly to meet different shaping needs without adjustment of the hardware.

The study provides a new method for achieving the digital Gaussian shaping of the nuclear pulse signal.

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