

# Estimation method for parameters of overlapping nuclear pulse signal

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Abstract Identification of nuclear pulse signal is of importance in radioactive measurements, especially in recognizing adjacent overlapping nuclear pulses. In this article, we propose an estimation method for parameters of typical overlapping nuclear pulse signals. First, the nuclear pulses are regarded as individual genes and the norm is set as the fitness function. Second, the global optimal solution is found by searching the population of genetic algorithm, so as to estimate the parameters of nuclear pulse. With high precision, this method can identify parameters of overlapping nuclear pulses in the Sallen–Key Gaussian signal decomposition experiments. This pulse recognition method is of great significance to improve the precision of radioactive measurement and is suitable for serious overlap of nuclear pluses.

 $\label{eq:constraint} \textbf{Keywords} \ \ \ Nuclear \ \ pulse \ \cdot \ \ Overlapping \ \cdot \ \ Parameter \\ identification$ 

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# **1** Introduction

The acquisition and processing of nuclear pulse signals are important for radioactive measurements. With the development of high-speed integrated circuit, digital shaping has become an important technology in nuclear pulse signal processing [1-3] and a useful technology to improve the performance of nuclear instruments greatly.

In fact, overlapping of adjacent nuclear pulses is inevitable, especially under high count rates, which is still a problem for decomposition and identification of waveform shaping technology [4–7]. So, collection, identification and decomposition of nuclear pulses have been carried out extensively [8–13], and most of the methods tended to reject overlapped pulses.

The fluctuation in detector response and subsequent circuits may affect consistency and stability of the signal parameters. For example, the fast or slow time constant of index or double-exponential pulse signal is helpful to understand the response characteristics of detector and subsequent circuits and is important for waveform shaping of nuclear instruments and the research of spectrum shifting. The decomposition and identification of overlapping signals after Gaussian shaping are important too [14–18].

Using typical nuclear pulse signals and a particular example to Gaussian pulse, in this article we propose a genetic algorithm for parameter estimation of overlapping nuclear pulse signals. By population search technology, regarding every overlapping pulse as an individual, constructing the chromosome of corresponding parameter vector, we can make a series of genetic operation, such as selection, crossover, mutation, on the current population to produce a new generation of population and to make the population evolve into the state of global optimal solution

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gradually. This method avoids the problem of local convergence. The optimal results calculated by this method are a global sense of "best match" signal, which means each pulse component of the overlapping pulse is the expected global optimal pulse signal. This method is important to validate forming algorithm, analyze response characteristics of measurement circuits and perform digital processing.

#### 2 Overlapping nuclear pulse models

Nuclear signals are typically in waveforms of the index, double exponential and Gaussian type, with the superposition of noise.

# 2.1 Overlapping with index or double-exponential pulses

The overlapping pulse signals, V(t), with superposition of N index pulses and N double-exponential pulses can be expressed by Eqs. (1) and (2), respectively:

$$V(t) = \sum_{i=1}^{N} \left[ u(t - T_i) A_i e^{-(t - T_i)/\tau} \right] + v(t),$$
(1)  
$$V(t) = \sum_{i=1}^{N} \left[ u(t - T_i) A_i (e^{-(t - T_i)/\tau_1} - K e^{-(t - T_i)/\tau_2}) \right] + v(t),$$
(2)

where u(t) is the step function; *K* is the proportion coefficient;  $A_i$  is associated with pulse amplitude;  $\tau_1$  and  $\tau_2$  are slow and fast time constant of the double-exponential signal, respectively; and v(t) is the noise.

Discretize Eqs. (1) and (2) with sampling period  $T_s$ , and then, the overlapping pulse signals can be written as:

$$V(kT_{\rm s}) = \sum_{i=1}^{N} \left[ u(kT_{\rm s} - T_i)A_i e^{-(kT_{\rm s} - T_i)/\tau} \right] + v(kT_{\rm s}), \tag{3}$$

$$V(kT_{s}) = \sum_{i=1}^{N} \left[ u(kT_{s} - T_{i})A_{i}(e^{-(kT_{s} - T_{i})/\tau_{1}} - Ke^{-(kT_{s} - T_{i})/\tau_{2}}) \right] + v(kT_{s}).$$
(4)

#### 2.2 Overlapping with standard Gaussian pulses

For the overlapping pulse signal of V(t) with the superposition of N standard Gaussian pulses, the discretized V(t) can be expressed by Eqs. (5) and (6), respectively:

$$V(t) = \sum_{i=1}^{N} \left[ u(t - T_0(i)) \frac{A_i}{\sqrt{2\pi\sigma}} e^{\frac{-(t - T_i)^2}{2\sigma^2}} \right] + v(t),$$
(5)

$$V(kT_{\rm s}) = \sum_{i=1}^{N} \left[ u(kT_{\rm s} - T_0(i)) \frac{A_i}{\sqrt{2\pi\sigma}} e^{\frac{-(kT_{\rm s} - T_i)^2}{2\sigma^2}} \right] + v(kT_{\rm s}),$$
(6)

where  $u(\cdot)$ ,  $v(\cdot)$ ,  $T_s$  and  $A_i$  are the same as those in Eqs. (1)–(4);  $T_0(i)$  and  $T_i$  is the start time and the time to large amplitude of the *i*th Gaussian pulse, respectively; and  $\sigma$  is the standard deviation. Actually, in purely mathematical sense, the start time of Gaussian pulse is infinite, and it is not in full accordance with the actual physical circuit. So,  $T_0(i)$  is used to cut off the insignificant front part of Gaussian pulse.

# 2.3 Overlapping with Sallen–Key Gaussian shape pulse

Under high count rate, the pulse overlapping phenomenon occurs and becomes serious after the Gaussian shaping. With the example of common Sallen–Key (S–K) waveform shaping circuit (Fig. 1a) [18], each resistance is R, each capacitance is C, and RC = 400 ns. Input an exponential decay signal with time constant of  $\tau = 100$  ns. In Fig. 1b, due to the short interval between Input 1 and Input 2, Output 1 and Output 2 overlap so seriously that they merge into one Gaussian peak. This overlapping peak may be processed as one pulse, or eliminated for not being distinguished in the measurement; bring adverse impact to the extraction of pulse amplitude and time constant, with reduced measurement efficiency and accuracy.

Digital model of S-K shaping circuit is [18]:

$$V(nT_{\rm s}) = [(K + 2K^2)V((n-1)T_{\rm s}) - K^2V((n-2)T_{\rm s}) + 2X(nT_{\rm s})]/(1 + K + K^2),$$
(7)

where  $X(nT_s)$  and  $V(nT_s)$  is Input and Output, respectively;  $X(nT_s) = V(nT_s) = 0, n < 0; K = RC/T_s.$ 

 $X(nT_s)$  is often the superposition of index pulse and noise:

$$X(nT_{\rm s}) = \sum_{i=1}^{N} \left[ u(nT_{\rm s} - T_i) A_i e^{-(nT_{\rm s} - T_i)/\tau} \right] + v(nT_{\rm s}).$$
(8)

So Eq. (7) becomes:

V

$$(nT_{s}) = [(K + 2K^{2})V((n - 1)T_{s}) - K^{2}V((n - 2)T_{s}) + 2X(nT_{s})]/(1 + K + K^{2}) = \left\{ (K + 2K^{2})V((n - 1)T_{s}) - K^{2}V((n - 2)T_{s}) + 2\sum_{i=1}^{N} [u(nT_{s} - T_{i})A_{i}e^{-(nT_{s} - T_{i})/\tau}] + 2v(nT_{s}) \right\} / (1 + K + K^{2}),$$
(9)



Fig. 1 S-K shaping circuit (a) and the Gaussian shaping signal (b)

 $V(nT_s)$  is the overlapping signal of Gaussian pulse calculated by S–K digital shaping arithmetic;  $v(nT_s)$  is the noise. The deduction of digitalized model of the S–K shaping circuit can be found in Ref. [18].

#### **3** Parameter estimation of overlapping pulses

Finding the optimal parameter  $\theta_{opt}$  in the sense of an objective function, for the pulse signal  $V(\cdot)$  overlapped by N pulse components, we are able to realize the decomposition of overlapping pulse. When the pulse signal  $V(\cdot)$  is overlapped by index pulses, double-exponential pulses and standard Gaussian pulses, we have, respectively,  $\theta = [A_1]$  $A_2...A_M \tau T_1 T_2...T_M, \theta = [A_1 A_2...A_M \tau_1 \tau_2 K T_1 T_2...T_M]$ and  $\theta = [A_1 A_2 \dots A_M \sigma T_1 T_2 \dots T_M]$ , where  $A_i$  is related to the pulse amplitude;  $\tau$ ,  $\tau_1$  and  $\tau_2$  are time constants,  $\sigma$  is the standard deviation, and  $T_i$  is the correlation coefficient with pulse time. For the overlapping signal generated by S-K digital Gaussian shaping algorithm, we have  $\theta = [A_1]$  $A_2...A_M \tau K T_1 T_2...T_M$ , being the same as Eqs. (7)–(9). For the overlapping problem, this method can reduce the possibility of forced counting or abandoning treatment and incorrect counting of pulse. Also, the effectiveness of decision and parameter identification of pulse can be achieved by multiple waveform parameters of the "best match", with "minimum damage" of the original pulse signal.

Using population search technology, every overlapping pulse  $V_l(\cdot)$  (l = 1...PopSize) is regarded as an individual, where PopSize is the population size. The corresponding parameter vector  $\theta_i[\cdot]$  constitutes the chromosome. Making a series of genetic operation, such as selection, crossover, mutation, on the current population  $V_1(\cdot)$ ,  $V_2(\cdot),...,$  $V_{PopSize}(\cdot)$ , we are able to produce a new generation of population and to make the population evolve into the state of global optimal solution gradually. This avoids the problem of local convergence. The calculated optimal overlapping pulse  $V_{opt}(\cdot)$  is a global sense of "best match" signal, which means each pulse component of the overlapping pulse  $V_{opt}(\cdot)$  is the expected global optimal pulse signal. The procedures are as follows:

- 1. Set the initial value of pulse number as  $M \ (M \ge N)$ , and regard every  $V_l(\cdot)$  as an individual. For the index, double exponential and standard Gaussian overlapping, make the parameter combinations of  $[A_1 A_2...A_M \tau T_1 T_2...T_M]$ ,  $[A_1 A_2...A_M \tau_1 \tau_2 K T_1 T_2...T_M]$  and  $[A_1 A_2...A_M \sigma T_1 T_2...T_M]$  as their own chromosome, respectively. For the overlapping Gaussian pulse signal generated by S–K digital forming algorithm, make the parameter combination of  $[A_1 A_2...A_M \tau K T_1 T_2...T_M]$ as the chromosome.
- Create an initial population of uniform distribution, with the scope of each gene depending on the pulse signal characteristics.
- 3. Construct the objective function  $f(\theta_l)$  according to the matching degree of individual  $V_l(\cdot)$  with original overlapping pulse signal  $V(\cdot)$ .

$$f(\theta_l) = \left\{ \sum_{k} \left[ V(kT_{\rm s}) - V_l(kT_{\rm s}) \right]^2 \right\}^{1/2} / N_{\rm V}, \qquad (10)$$

where  $N_V$  is the discrete points. For overlapping of index pulse:

$$V_l(kT_s) = \sum_{j=1}^{M} \left[ u(kT_s - T_j) A_j e^{-(kT_s - T_j)/\tau} \right],$$
 (11)

for overlapping of double-exponential pulse:

$$V_{l}(kT_{s}) = \sum_{j=1}^{M} \left[ u(kT_{s} - T_{j})A_{j}(e^{-(kT_{s} - T_{j})/\tau_{1}} - Ke^{-(kT_{s} - T_{j})/\tau_{2}}) \right],$$
(12)





for overlapping of standard Gaussian pulse:

$$V_l(kT_s) = \sum_{j=1}^{N} \left[ u(kT_s - T_0(i)) \frac{A_j}{\sqrt{2\pi\sigma}} e^{\frac{-(kT_s - T_j)^2}{2\sigma^2}} \right], \quad (13)$$

and for overlapping of S-K digital Gaussian shaping:

$$V_l(kT_s) = [(K + 2K^2)V_l((k-1)T_s) - K^2V_l((k-2)T_s) + 2X_l(kT_s)]/(1 + K + K^2),$$
(14)

where  $X_l(kT_s)$  can be obtained by:

$$X_{l}(kT_{s}) = \sum_{j=1}^{N} \left[ u(kT_{s} - T_{j})A_{j}e^{-(kT_{s} - T_{j})/\tau} \right].$$
 (15)

The other parameters in Eqs. (10)–(15) are defined the same as those in Eqs. (3) and (4).

Ordering  $f(\theta_l)$  from small to large, numbered as 1,2,..., PopSize consecutively, calculating each individual scaling function as follows:

$$SFValue(j) = \varepsilon (1 - \varepsilon)^{j-1}$$
  $j = 1, 2, \dots, PopSize,$  (16)

where  $\varepsilon = 0-1$ . Record the best and worst individual in the current population.

- 4. Selection, crossover and mutation operator are adopted to make the genetic operation.
  - a. Selection operation: regarding each individual scaling function value SFValue(j) as the value of discrete probability density function, after the normalization, and then generating the intermediate population by the direct sampling method.
  - b. Crossing the middle population: determining the source of gene (from the first or the second parents) by the vector components (1 or 0) created in a random binary.
  - c. Mutation operation: taking Gaussian function as the mutation function and adding random numbers to each component of parent vector.

The above operations continue until the stop condition is satisfied. The searched optimal individual is the solution when the pulse number is M. Figure 2 shows the process of searching for the optimal individual.

Searching for the optimal individual at pulse number of M - 1, M - 2, M - 3,... From the optimal individuals, choosing one with minimum objective function value  $f(\theta)$  as the final solution  $V_{opt}(t)$ . Making chromosome decoding



Fig. 3 True value of index signal (Input x), Gaussian-shaped signal (Output x) and overlapping pulse (Output 1 +Output 2)

of optimal individual  $V_{opt}(t)$  to realize the decomposition of overlapping pulse and get the parameter values of every pulse component.

# **4** Two application examples

# 4.1 Example 1

Two index pulses Input 1 and Input 2 were inputted into the S–K shaping circuit as shown in Fig. 1a, with the characteristic time of  $\tau = 100$  ns. Standard deviation of white noise is 5 counts for the two input pulses. The Gaussian waveform of the input pulses is Output 1 and Output 2, respectively, as shown in Fig. 3. Inputting the two index pulses into S–K shaping circuit continuously, and assuming the appeared time of index pulses  $T_i$  is 1  $T_s$ and 60  $T_s$ , with amplitude of 300 and 150 counts, respectively. Making sample with the sampling frequency of 200 MHz, namely the sampling period of  $T_s = 5$  ns. Set RC = 150 ns, the S–K digital Gaussian shaping algorithm  $K = RC/T_s = 30$ . In Fig. 3, overlapping pulse "Output 1 + Output 2" is too serious to be distinguished.

Searching for the "global optimum" individual  $V_{opt}(k)$  by the population search technology of genetic algorithm and decoding the genes, the amplitude coefficients  $A_1$  and  $A_2$ , characteristic time  $\tau$ , occurrence time  $T_1$  and  $T_2$  and constant K of S–K digital system can be found. Each maximum Gaussian signal  $V_i(nT_s)$  of index pulse  $X_i(nT_s)$ , refer to Eq. (18), can be calculated by Eq. (17), which is the amplitude of Gaussian shaping signal.  $X_i(nT_s)$  can be calculated from Eq. (18).

$$V_i(nT_s) = [(K + 2K^2)V_i((n-1)T_s) - K^2V_i((n-2)T_s) + 2X_i(nT_s)]/(1 + K + K^2) \quad i = 1, 2,$$
(17)



Fig. 4 Population search process (Example 1)

$$X_i(nT_s) = u(nT_s - T_i)A_i e^{-(nT_s - T_i)/\tau} \quad i = 1, 2.$$
(18)

Usually,  $\tau$  and *K* are known before the search, which means its genes are fixed in population search. In our case,  $\tau = 100$  ns and K = 30. Genetic value adopts double vector type, six variants, initial population size is 200, the scope vector is [0.1 0.1 1 1 100 30; 400 400 300 300 100 30], lower and upper vectors are [0.1 0.1 1 1 100 30] and [400 400 300 300 100 30], respectively. The selection function is uniform random function, with crossover fraction of 0.8. The mutation function is Gaussian function, with the migration interval of 20 and the fraction of 0.2.

The population search process of genetic algorithm (GA) is shown in Fig. 4. The fitness value of optimal individual is 0.018694, which has the gene's corresponding value of [297.80, 148.72, 1.47, 58.14, 100, 30].

Figure 5a shows the index (Input x'), the Gaussianshaped pulse (Output x') and the overlapping pulse (Output 1' +Output 2'), each being optimal individual. Figure 5b shows the index pulse, Gaussian-shaped signal of optimal individual and their true value. Figure 5c shows the contrast between the overlapping pulse of optimal individual and their true pulse. "Input x" and "Output x" stand for the true signal in the figures, while "Input x'" and "Output x'" stand for the signal of optimal individual.

In Table 1, the parameters are between the calculated values of optimal individual and true values, such as Gaussian pulse amplitude  $V_1$  and  $V_2$ , and the corresponding index pulse parameter is  $A_1$ ,  $A_2$ ,  $T_1$  and  $T_2$ . Relative error of  $A_1$  and  $A_2$  is 0.73 % and 0.85 %, respectively. The error of  $T_1$  and  $T_2$  is 0.47  $T_s$  and 1.86  $T_s$ , respectively, which means higher precision. It is important that relative amplitude error of Gaussian pulse  $V_1$  and  $V_2$  is 0.54 % and 1.18 %, respectively, which means a high precision.

For digital Gaussian shaping,  $\tau$  is often a known value, with slight fluctuations though, such as  $\pm 5$  %. The scope vector of initial population is [0.1 0.1 1 1 95 30; 400 400 300 300 105 30]; lower and upper vectors are [0.1 0.1 1 1 95 30] and [400 400 300 300 105 30], respectively. Figure 6 shows the index pulses, Gaussian pulses, the overlapping pulses and their true values, being the optimal



Fig. 5 Optimal individual and their true values ( $\tau = 100$  ns, K = 30)

Table 1       Comparison of the calculated value of optimal individuals and true values         (1)       (1)         (1)       (2)	Items	$A_1$ (counts)	$A_2$ (counts)	$T_1 (T_s)$	$T_2 \left( T_{\rm s} \right)$	$V_1$ (counts)	$V_2$ (counts)
	True value	300	150	1	60	183.53	92.36
$(\tau = 100 \text{ ns}, K = 30)$	Calculated value	297.80	148.72	1.47	58.14	182.54	91.27
	Error	2.2 (0.73 %)	1.28 (0.85 %)	0.47	1.86	0.99 (0.54 %)	1.09 (1.18 %)

individuals. The parameters are compared in Table 2. The relative errors of  $A_1$  and  $A_2$  are 6.05 and 2.55%, respectively; the errors of  $T_1$  and  $T_2$  are 0.20  $T_s$  and 0.64  $T_s$ , respectively; the absolute error and relative deviation of  $\tau$  are 3.84 ns and 3.84%, respectively. The relative errors of Gaussian pulse amplitude are 3.19 and 0.52 %, respectively, which are still of high precision, obviously.

#### 4.2 Example 2

Index pulses Input 1, Input 2, Input 3 and Input 4 were inputted into a S–K shaping circuit (Fig. 1), which had the



**Fig. 6** Index signal (Input x'), Gaussian-shaped signal (Output x') and overlapping pulse (Output 1' + Output 2') of optimal individual and their true values (Input x, Output x, Output 1 + Output 2),  $\tau$  is indefinite, K = 30

same parameter  $\tau$  and white noise as those of Example 1. The Gaussian waveforms of Output 1, Output 2, Output 3 and Output 4 are shown in Fig. 7a. Assuming that the four index pulses were inputted into the S–K shaping circuit continuously, the appearing time of index pulse  $T_i$  was 1, 100, 200 and 300  $T_s$ , and the amplitude of index pulse  $A_i$ was 300,150, 200 and 250 counts, respectively. The sampling period of Example 2 was the same as that of Example 1. Set RC = 250 ns, the parameter of  $K = RC/T_S = 50$  of the S–K digital Gaussian shaping circuit could be obtained. In Fig. 8b, the overlapping pulse "Output 1 + Output 2 + Output 3 + Output 4" is serious and is hard to distinguish.

The scope vector of initial population was set to be [0.1 0.1 0.1 0.1 1 1 1 1 100 50; 400 400 400 400 300 300 300 300 100 50]; the lower and upper vectors were [0.1 0.1 0.1 0.1 0.1 1 1 1 1 100 50] and [400 400 400 400 300 300 300 300 100 50], respectively, while the  $\tau$  and *K* were known, whose genes were fixed in population search. The selection, the crossover, the mutation function and its parameter of Example 2 were the same as those of Example 1.

Figure 8 shows the population search process. The fitness value of optimal individual is 0.026556, with corresponding gene parameters in the following order (after adjustment):

[305.62,150.85, 189.33, 256.69,1.17, 99.52, 198.63, 299.90, 100,50]

**Table 2** Comparison of the calculated value of optimal individuals and true values (scope of  $\tau$ : (1 ± 5 %) × 100 ns; K = 30)

Items	$A_1$ (counts)	$A_2$ (counts)	$T_1 (T_s)$	$T_2 \left( T_{\rm s} \right)$	$V_1$ (counts)	$V_2$ (counts)	T (ns)
True value	300	150	1	60	183.53	92.36	100
Calculated value	281.86	146.18	1.20	59.36	177.68	91.88	103.84
Error	18.14 (6.05 %)	3.82 (2.55 %)	0.20	0.64	5.85 (3.19 %)	0.48 (0.52 %)	3.84 (3.84 %)



**Fig. 7** True value of index signal (Input *x*), Gaussian-shaped signal (Output *x*) and overlapping pulse (Output 1 + Output 2 + Output 3 + Output 4). **a** True value (index signal and Gaussian-shaped signal); **b** true value (Gaussian-shaped signal and overlapping pulse)



Fig. 8 Population search process (Example 2)

Figure 9a shows the index, the Gaussian and the overlapping pulses of optimal individual. Figure 9b shows the index pulse (Input x') of optimal individual and their true value (Input x). Figure 9c shows the Gaussian-shaped signal (Output x') of optimal individual and their true value (Output x).

Table 3 shows the calculated index pulse parameters of  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$ . Table 4 shows the calculated Gaussian pulse amplitude  $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ ; relative errors of  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  are 1.87, 0.57, 5.34 and



Fig. 9 Optimal individual and their true value (Example 2). a Optimal individual (Index signal, Gaussian-shaped signal and overlapping pulse); b Optimal individual and their true value (Index pulse); c Optimal individual and their true value (Gaussian-shaped signal)

**Table 3** Calculated index pulse parameters( $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$ ) and their true values ( $\tau$  and K are fixed;  $\tau$ =100 ns, K=50)

Items	$A_1$ (counts)	$A_2$ (counts)	$A_3$ (counts)	$A_4$ (counts)	$T_1 (T_s)$	$T_2 (T_s)$	$T_3 (T_s)$	$T_4 (T_s)$
True	300	150	200	250	1	100	200	300
Calculated	305.62	150.85	189.33	256.69	1.17	99.52	198.63	299.90
Error	5.62 (1.87 %)	0.85 (0.57 %)	10.67 (5.34 %)	6.69 (2.68 %)	0.17	0.48	1.37	0.1

**Table 4** Calculated Gaussian pulse amplitude of  $V_1$ ,  $V_2$ ,  $V_3$ and  $V_4$  (counts, %) and their true values ( $\tau$ , *K* are fixed;  $\tau = 100$  ns, K = 50)

Items	$V_1$ (counts)	$V_2$ (counts)	$V_3$ (counts)	$V_4$ (counts)
True	122.15	61.83	81.68	102.02
Calculated	124.86	61.61	77.12	104.86
Error	2.71 (2.22 %)	0.22 (0.36 %)	4.56 (5.58 %)	2.78 (2.68 %)

2.68%, respectively. The errors of  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  are 0.17 $T_s$ , 0.48  $T_s$ , 1.37Ts and 0.1 $T_s$ , respectively. Moreover, the relative error of Gaussian pulse amplitude  $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$  is 2.22, 0.36, 5.58 and 2.68%, respectively, which means a high precision.

From the results above, it is clear that using the population search technology to decompose overlapping pulse, one can obtain high-precision parameters of each Gaussian component and even corresponding parameters of inputted index signal. Also, this method is of high accuracy in decomposition and acquisition of the parameters of double-exponential pulse or standard Gaussian overlapping pulse.

# 5 Conclusion

Direct sampling method is commonly used for parameter acquisition of nuclear pulse. When the overlapping of Gaussian signals is serious, however, errors of the extracted parameters would be great, with reduced efficiency of radioactive measurement under high count rates, if abandon the overlapping pulse. By using the population search technology, with a series of genetic operations such as selection, crossover and mutation, the shortcoming of local convergence can be overcome and the optimal solution  $V_{opt}(\cdot)$  can be found.  $V_{opt}(\cdot)$  is the "best match" signal in the global sense. The experimental decomposition of S-K Gaussian signal shows that this method is of high precision and strong anti-jamming capability, in parameter estimation of pulse signals. The parameter estimation of overlapping double-exponential, triangular and trapezoidal pulse can be solved by this method.

In this paper, in order to test the accuracy of the parameter estimation, we select adjacent pulses with serious overlapping, rather than random pulses. In fact, in the process of measuring and counting the pulse, the data usually need to be processed in real time and in line, which is also the problem to be solved. If we combine with multiple digital signal processors (DSPs) to complete data processing, most likely the real-time problem of this method can be solved. In addition, this method can be used as a supplement of data processing that cannot be performed in real time and the following data analysis as well.

This method is still in the preliminary stage. In high count rates (the amount of data is too large to obtain), the entire energy spectra and the comparison of spectra have not been achieved. Greater efforts shall be made, including improvement in the fast algorithm, toward practical application of this method.

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