



The role of deformations and orientations in an alpha ternary fission of Thorium

A. V. Mahesh Babu¹ · N. Sowmya² · H. C. Manjunatha² · N. Dhananjaya¹

Received: 1 February 2022 / Revised: 19 April 2022 / Accepted: 25 April 2022 / Published online: 29 June 2022

© The Author(s), under exclusive licence to China Science Publishing & Media Ltd. (Science Press), Shanghai Institute of Applied Physics, the Chinese Academy of Sciences, Chinese Nuclear Society 2022

Abstract The alpha ternary fission half-lives of thorium isotopes have been studied using the Coulomb and proximity potential models. The role of the deformation effects and the angle of orientation were included during the evaluation of the total potential. Fragment combinations were identified using cold valley plots of the driving potential. The half-lives and yields were evaluated using the penetration probability. The dependence of the logarithmic half-lives on different angles of orientation was studied. The evaluated alpha ternary fission yield was compared with that of the available experiments with and without deformations. The half-lives obtained in the present work were compared with those of the available data. Possible alpha ternary fission fragments were identified in the isotopes of thorium. The alpha ternary fission half-lives were compared to the binary fission half-lives. The binary fission half-lives are dominant in the $^{209-225}\text{Th}$ nuclei, and the ternary fission half-lives are dominant in the isotopes of the $^{226-238}\text{Th}$ nuclei.

Keywords Alpha ternary fission · Yield · Half-lives · Driving potential

1 Introduction

The splitting of heavy and superheavy nuclei into three fission fragments is referred to as ternary fission (TF). The long-range alpha particle experimentally observed by San Tsiang et al. [1] and later by Tsien et al. [2] discovered ternary fission. Manjunatha et al. [3] theoretically investigated ternary fission, binary fission, cluster, and alpha radioactivity in superheavy nuclei $Z=126$. Light-charged ternary fission has been studied experimentally [4–7] and theoretically [8–10]. Raisbeck [11] measured the angular distribution and energy spectrum of light particles that were accompanied by ternary fission of ^{252}Cf and showed maximum relative probability when the angular distribution was at $\theta = 90^\circ$. Balasubramaniam et al. [12] studied the ternary fission of ^{252}Cf using a level-density approach by keeping ^{48}Ca as a fixed third particle using a three-cluster model [13], alpha ternary fission in ^{252}Cf .

Triple correlation has been observed in ternary fission [14]. Probable alpha ternary fission fragments of ^{257}Fm have been studied [15]. In addition to ternary fission, binary, cluster, and alpha radioactivity has been studied in superheavy nuclei $Z = 122, 124$, and 126 [3, 16, 17]. Ref. [6] observed enhancement of neutron intensity along the direction of alpha ternary particles when compared to the same in the opposite direction. Furthermore, Ref. [18] measured correlated alpha-particle energies and neutron velocities from ^{252}Cf at the relative angles 0° and 180° . In addition, many theoretical studies have focused on binary fission, cluster radioactivity, heavy-particle radioactivity, and alpha decay [19–22].

The lifetimes of ^{234}U and ^{230}Th [23] were studied using high-precision thermal ionization mass spectrometry

✉ N. Sowmya
sowmyaprakash8@gmail.com

✉ H. C. Manjunatha
manjunathhc@rediffmail.com

¹ Department of Physics, BMSIT &M, Affiliated to VTU, Bangalore, Karnataka 560064, India

² Department of Physics, Government College for Women, Kolar, Karnataka 563101, India

(TIMS). Häusser et al. [24] measured the lifetimes and α -decay energies of isotopes of thorium ($^{218-220}\text{Th}$) using the recoil technique. Yahya and Falaye [25] evaluated α -decay half-lives of Thorium isotopes using the double folding potential model. Pahlavani and Karamzadeh [26] studied the partial α -decay half-lives of the thorium family. Ghorbani et al. [27] studied the temperature-dependent α -decay halves of isotopes of thorium. In the case of cold fusion reactions, isotopes of thorium are used as targets [28, 29]. Many studies [31–35] have shown the possible decay modes of superheavy nuclei by investigating binary fission, alpha ternary fission, spontaneous fission, α -decay, and cluster decay. All of these studies revealed a lack of information on the deformation parameter and angle of orientation in the isotopes of thorium nuclei. This has motivated us to investigate possible alpha ternary fission half-lives in $^{209-239}\text{Th}$ and to compare the alpha ternary fission half-lives with the binary fission half-lives.

The structure of the paper is as follows: the theory of evaluation of half-lives and yield by the inclusion of the deformation parameter and angle of orientation in the Coulomb and proximity potential model are presented in Sect. 2. The results and discussion of the present work are presented in Sect. 3. The conclusions are presented in Sect. 4.

2 Theory

An alpha accompanied ternary fission fragmentation potential is defined as follows:

$$V(R) = V_C(R) + V_N(R) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}, \quad (1)$$

here $V_C(R)$ and $V_N(R)$ are the Coulomb and nuclear potentials, respectively. The last term in Eq. (1) is the centrifugal potential. ℓ is the angular momentum and μ is the reduced mass of fission fragments and it is as follows:

$$\mu = \left(\frac{\mu_{12} A_3}{\mu_{12} + A_3} \right) m, \quad (2)$$

where m is the nucleon mass and $\mu_{12} = \frac{A_1 A_2}{A_1 + A_2}$. However, in the present calculations, we considered the ground state-to-ground state transitions. The Coulomb potential V_C is expressed as

$$V_C = \left[\frac{Z_1 Z_2 e^2}{r} \right] [(1 + n_1(R_{0,1}^2 \beta_1 + R_{0,2}^2 \beta_2)) + n_2(R_{0,1}^2 \beta_1 + R_{0,2}^2 \beta_2) + n_3(R_{0,1}^4 \beta_1 + R_{0,2}^4 \beta_2) + n_4 \beta_1 \beta_2], \quad (3)$$

where $r = s + R_1 + R_2$, $n_1 = \frac{3}{2\sqrt{5}\pi r^2}$, $n_2 = \frac{3}{7\pi r^2}$, $n_3 = \frac{9}{14\pi r^4}$

and $n_4 = \frac{27R_1^2 R_2^2}{10\pi r^4}$. Where R_i is the radius of fission fragments, which is evaluated as follows:

$$R_i(\theta) = R_{0,i}(1 + \beta_i Y_{20}(\theta)), \quad (4)$$

where β_i and θ are the quadruple deformation parameter and the angle created by the axis of symmetry with the fission axis, respectively. The net radius of each fission fragment is given by

$$R_{0,i}(\text{fm}) = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}. \quad (5)$$

The spherical harmonic function is denoted as Y_{j0} . The nuclear proximity potential [36] is given by

$$V_N(R) = 4\pi\gamma\bar{R}\Phi(\xi). \quad (6)$$

Here, the mean curvature radius is $\bar{R} = \frac{C_1 C_2}{C_1 + C_2}$ and C_i is the centre of the matter radii, which is evaluated as follows:

$$C_i = c_i + \frac{N_i}{A_i} t_i \quad (i = 1, 2). \quad (7)$$

where c_i is the half-density radius of the charge distribution, evaluated as follows:

$$c_i = R_{00i} \left(1 - \frac{7b^2}{R_{00i}^2} - \frac{49b^4}{8R_{00i}^4} \right) \quad (i = 1, 2), \quad (8)$$

where R_{00i} is the radius of nuclear charge.

$$R_{00i} = 1.256 A_i^{1/3} \left(1 - 0.202 \left(\frac{A_i - 2Z_i}{A_i} \right) \right) \quad (9)$$

The neutron skin t_i of the nucleus is evaluated as follows:

$$t_i = \frac{3}{2} r_0 \left[\frac{J_i - \frac{1}{12} g Z_i A_i^{-1/3}}{Q + \frac{9}{4} A_i^{-1/3}} \right] \quad (i = 1, 2), \quad (10)$$

with $r_0 = 1.14$ fm, $J = 32.65$ MeV is the nuclear symmetric energy coefficient, $g = 0.757895$ MeV, $Q = 35.4$ MeV is the neutron skin stiffness coefficient, $I_i = (N_i - A_i)/A_i$ is the neutron mass excess and surface energy coefficient γ is evaluated as follows:

$$\gamma = \frac{1}{4\pi r_0^2} \left[18.63 - Q \frac{(t_1^2 + t_2^2)}{2r_0^2} \right]. \quad (11)$$

The universal function of the nuclear potential is given by:

$$\Phi(\xi) = \begin{cases} -0.1353 + \sum_{n=0}^5 \left[\frac{\alpha_n}{(n+1)} \right] (2.5 - \xi)^{n+1} \\ -0.0955 \exp\left(\frac{2.75 - \xi}{0.7176}\right). \end{cases} \quad (12)$$

Here $\xi = (r - C_1 - C_2)/b$ and the width parameter $b \approx 1$. The different values of α_n constant are $\alpha_0 = -0.1886$, $\alpha_1 = 0.2628$, $\alpha_2 = -0.15216$, $\alpha_3 = -0.04562$, $\alpha_4 = 0.069136$ and $\alpha_5 = -0.011454$.

The alpha ternary fission half-lives of isotopes of thorium is evaluated as follows:

$$T_{1/2} = \left(\frac{\ln 2}{\lambda} \right) = \left(\frac{\ln 2}{\nu P} \right), \quad (13)$$

where λ is the decay constant and ν is the assault frequency of the zero-point vibration of the nucleus in the fission mode direction, which is given by $\nu = \left(\frac{\omega}{2\pi} \right) = \left(\frac{2E_v}{h} \right)$. Hence, the fissioning nuclei half-life is inversely proportional to the empirical vibrational energy. This is the vibrational energy that molecules retain, even at an absolute zero temperature. The term E_v is expressed as:

$$E_v = Q \left[0.056 + 0.039 \exp \left(\frac{4 - A_e}{2.5} \right) \right] \text{ MeV}, \quad (14)$$

A_e is the mass of emitted particles. P is the penetration probability, which is evaluated using WKB approximation.

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu[V(r) - Q]} dr \right], \quad (15)$$

where R_{in} and R_{out} are classical turning points. These were evaluated using boundary conditions such that $V(r = R_{in}) = V(r = R_{out}) = Q$. Here, μ denotes the reduced mass. The relative yield is the ratio between the probabilities of a given fragmentation and the sum of the probabilities of all possible fission fragments are as follows:

$$Y(A_i, Z_i) = \frac{P(A_i, Z_i)}{\sum P(A_i, Z_i)}. \quad (16)$$

3 Results and discussion

The total potential $V(R)$ is evaluated for each fragmentation from the isotopes of thorium by with and without deformation the deformation parameter. Figure 1 shows the variation of an alpha accompanied by fragmentation potential as a function of the orientation degrees of freedom (θ). The figure clearly shows the dependence of the total potential on the angle of orientation. As the angle of orientation increased, there was a gradual increase in the total potential, and the maximum value of the potential was observed when $\theta = 90^\circ$, that is, for the equatorial configuration. Later, it gradually decreased with an increase in the orientation angle. The maximum value of the total potential plays a key role in the evaluation of the half-lives. The greater the value of the total potential, the lower the half-life. Hence, to predict the smaller half-lives in the alpha accompanied by ternary fission, we considered the equatorial configuration, that is, $\theta = 90^\circ$.

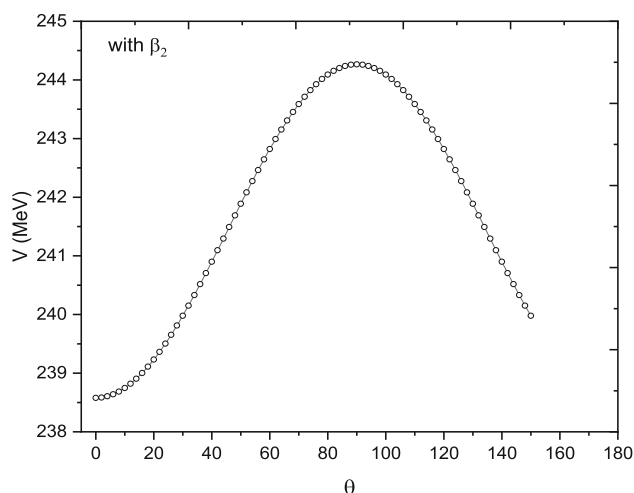


Fig. 1 Variation of total alpha accompanied fragmentation potential as a function of orientation degrees of freedom (θ)

The driving potential is the difference between total potential and amount of energy released during alpha ternary fission (i.e., $(V-Q)$). The amount of energy released during an alpha ternary fission (i.e., keeping $A_3 = {}^4\text{He}$ as constant) and the mass excess values of parent and fission fragment combinations have been calculated from the latest mass excess table [30].

The role of deformations β_2 and orientation degrees of freedom were considered during the evaluation of alpha ternary fission. Figure 2 shows a plot of the driving potential with respect to the mass number of fission fragments A_1 with β_2 and without β_2 deformation in the equatorial configuration. The driving potential increases with an increase in the mass number of fission fragments, A_1 . The figure represents cold valley minima which occurs

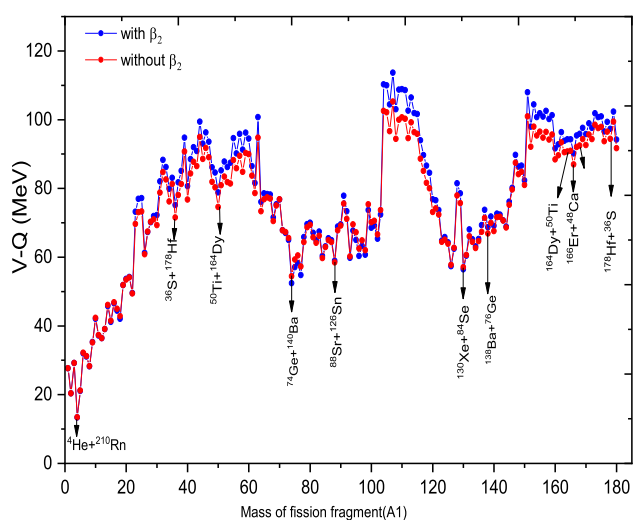


Fig. 2 Plot of driving potential with respect to mass number of fission fragment A_1 with β_2 and without β_2 deformation from the ${}^{218}\text{Th}$ at $\theta = 90^\circ$.

at ${}^4\text{He}+{}^{210}\text{Rn}$, ${}^{36}\text{S}+{}^{178}\text{Hf}$, ${}^{50}\text{Ti}+{}^{164}\text{Dy}$, ${}^{74}\text{Ge}+{}^{140}\text{Ba}$, ${}^{88}\text{Sr}+{}^{126}\text{Sn}$, ${}^{130}\text{Xe}+{}^{84}\text{Se}$, ${}^{138}\text{Ba}+{}^{76}\text{Ge}$, ${}^{164}\text{Dy}+{}^{50}\text{Ti}$, ${}^{166}\text{Er}+{}^{48}\text{Ca}$ and ${}^{178}\text{Hf}+{}^{36}\text{S}$. From these fission fragment combinations, it is clear that either fission fragment combinations are observed as magic or doubly magic nuclei. Among all these fission fragment combinations, the fission fragment combination ${}^{88}\text{Sr}+{}^{126}\text{Sn}$ is observed to be the minimum with a shorter driving potential owing to the presence of doubly magic nuclei of ${}^{122}\text{Sn}$ and ${}^{88}\text{Sr}$, which are near the doubly magic nuclei.

Further, the penetration probability is evaluated for each ternary fission fragment combinations using WKB integral method. The half-lives of an alpha-accompanied ternary fission fragment were evaluated, as explained in Eq. (13) (Fig. 3).

Furthermore, the role of the angle of orientation of fission fragments on the half-lives was studied. For example, an alpha ternary fission fragment of ${}^{218}\text{Th}$ was studied as a function of theta during the evaluation of the total potential. Because a shorter driving potential is observed in the case of ${}^{88}\text{Sr}+{}^{126}\text{Sn}$ fission fragment combination from the parent nuclei ${}^{218}\text{Th}$, the same fission fragment combination is used to evaluate the angle-dependent half-lives. The plot of logarithmic half-lives as a function of orientation angle, the logarithmic half-lives increase with an increase in the orientation angle, and a larger $\log T_{1/2}$ is observed when $\theta = 90^\circ$. Again, it gradually decreased with an increase in θ . Even though larger half-lives were observed when $\theta = 90^\circ$, when compared to their neighboring fission

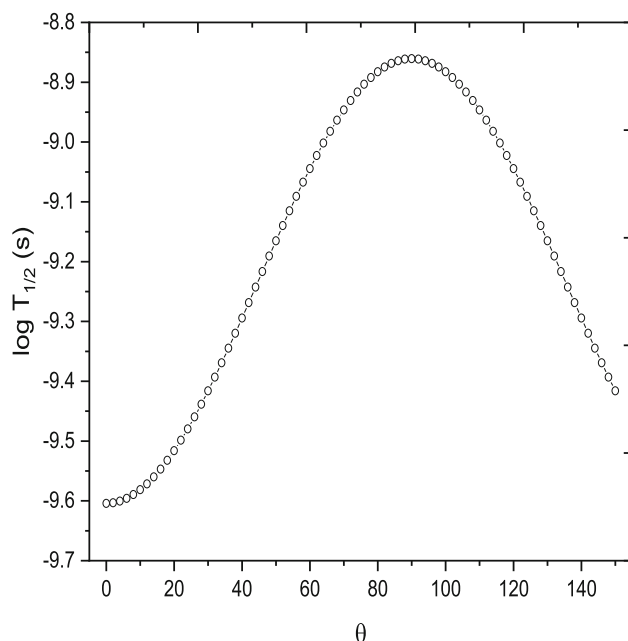


Fig. 3 A plot of logarithmic half-lives as a function of θ for the fission fragment combination ${}^{88}\text{Sr}+{}^{126}\text{Sn}+{}^4\text{He}$ from ${}^{218}\text{Th}$

fragment combinations, the value corresponding to $\theta = 90^\circ$ was shorter. Hence, in further analysis, we considered $\theta = 90^\circ$ for the evaluation of the half-lives and yield.

Figure 4 shows the variation in the logarithmic half-lives of alpha ternary fission with the mass number of fission fragments, A_1 . Shorter half-lives were observed for fission fragment combinations such as ${}^4\text{He}+{}^{210}\text{Rn}$, ${}^{36}\text{S}+{}^{178}\text{Hf}$, ${}^{50}\text{Ti}+{}^{164}\text{Dy}$, ${}^{74}\text{Ge}+{}^{140}\text{Ba}$, ${}^{88}\text{Sr}+{}^{126}\text{Sn}$, ${}^{130}\text{Xe}+{}^{84}\text{Se}$, ${}^{138}\text{Ba}+{}^{76}\text{Ge}$, ${}^{164}\text{Dy}+{}^{50}\text{Ti}$, ${}^{166}\text{Er}+{}^{48}\text{Ca}$, and ${}^{178}\text{Hf}+{}^{36}\text{S}$ when compared to their neighbouring fission fragment combinations with the fission fragments, and the effect of deformations was also included. Among all the mentioned fission fragments with shorter half-lives, it was noticed that either of the fission fragment combinations possesses spherical nuclei, as seen in Fig. 4. The only exception was the fission fragment combination of ${}^{130}\text{Xe}+{}^{84}\text{Se}$ which possess both oblate and prolate combinations.

The relative yield is evaluated as the ratio of the penetration probability of a given alpha ternary fission fragment to the sum of the penetration probabilities of all possible fission fragments, as in Eq. (5). The evaluated yield for the alpha ternary fission of ${}^{218}\text{Th}$ as a function of the mass number A_1 is shown in Fig. 5. From the comparison shown in Figs. 4 and 5, it shows that each shorter logarithmic half-life is comparable to a maximum value of relative yield. The maximum value of the relative yield is due to the presence of shell closure in fission fragment combinations of ${}^{74}\text{Ge}+{}^{140}\text{Ba}$, ${}^{88}\text{Sr}+{}^{126}\text{Sn}$, ${}^{130}\text{Xe}+{}^{84}\text{Se}$, and ${}^{138}\text{Ba}+{}^{76}\text{Ge}$, where the maximum peak is due to the presence of even nearly closed shells of ${}^{140}\text{Ba}$ ($Z = 56$, $N = 84$). Similarly,

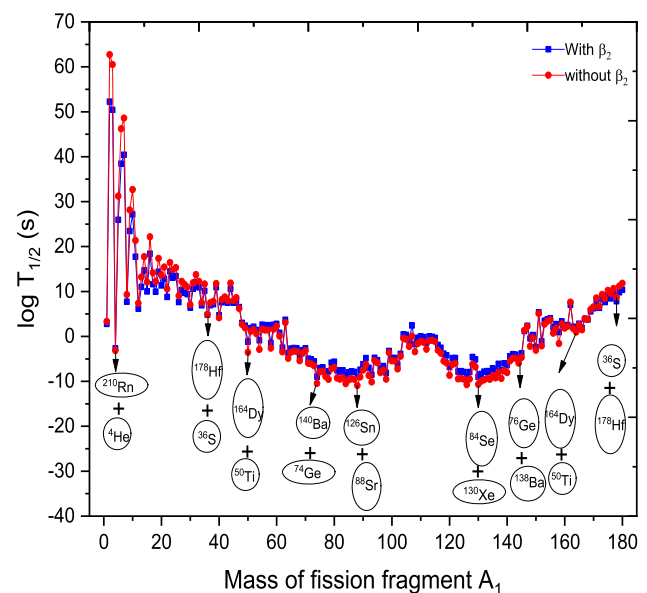


Fig. 4 Plot of $\log T_{1/2}$ with respect to a mass number of fission fragment A_1 with β_2 and without β_2 deformation from ${}^{218}\text{Th}$ at $\theta = 90^\circ$

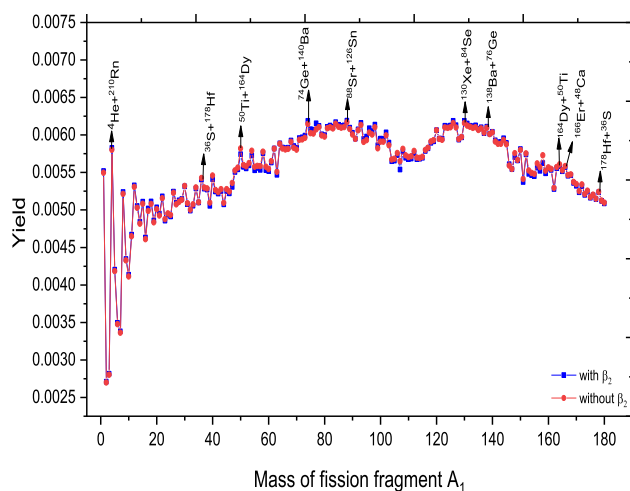


Fig. 5 Variation of yield with respect to mass number of fission fragment A_1 from the parent nuclei ^{218}Th at $\theta = 90^\circ$

the larger peak was attributed to magic nuclei with $Z = 50$ and $N = 50$ for ^{126}Sn and ^{84}Se , respectively. Furthermore, ^{138}Ba ($Z = 56$, $N = 82$) was attributed to maximum yield due to the presence of magic nuclei.

Because there are no experimental data available for α -ternary fission of thorium, to validate the present work, we have reproduced the α -ternary fission yield of ^{252}Cf for which the experimental data are available [15]. The figure shows the prediction of alpha ternary fission fragments of ^{252}Cf with and without deformations, along with the experimental values available in the literature [15], as

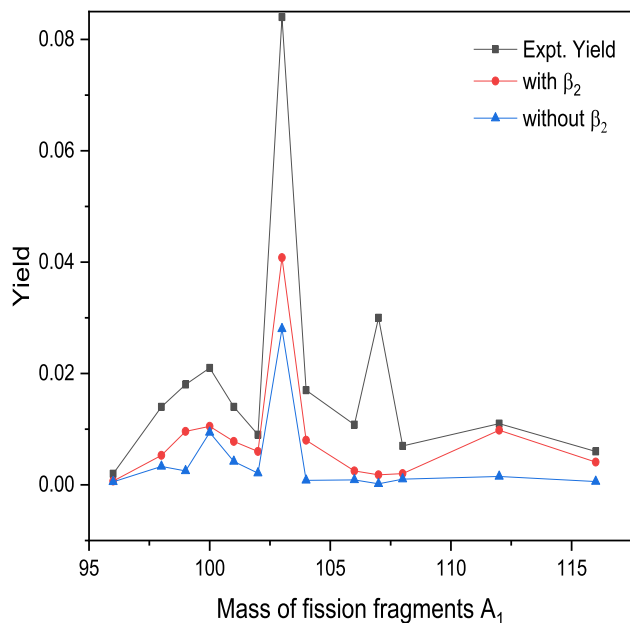


Fig. 6 Comparison of ternary fission yields of ^{242}Cf calculated using present work with that of experiments [15] for excluding and including deformations

shown in Fig. 6. From this comparison, it is noticed that the relative yield obtained by including the deformation effects is in close agreement with the available experiments when compared to that without the inclusion of the deformation parameter. After a detailed literature survey, it was found that there were no experimental α ternary fission half-lives. The only way to validate the evaluated α ternary fission half-lives is to compare them with the available theory. Poenaru et al. [37] evaluated the ^{10}Be ternary fission of a few heavy and superheavy nuclei within a three-center phenomenological model. Thus, to validate the present work, we also evaluated the ^{10}Be ternary fission half-lives and compared them with those of the Poenaru et al. [37] data. Table 1 shows a comparison between the present study and the available data. A close examination reveals that the values obtained in the present work are in good agreement with those of Poenaru et al. [37].

From a detailed study, it is observed that both the angle of orientation and the effect of deformations during the

Table 1 Comparison of ^{10}Be ternary fission fragment from the present work with that of Poenaru et al. [37]

Parent nuclei	Fission fragments	Q (MeV)	$\log T_{1/2}$	
			Ref.	PW
^{236}U	$^{132}\text{Sn} + ^{94}\text{Sr}$	185.30	-11.53	-10.27
	$^{134}\text{Te} + ^{92}\text{Kr}$	181.02	-9.10	-9.46
	$^{128}\text{Cd} + ^{98}\text{Zr}$	178.40	-7.62	-1.49
	$\text{Pd} + \text{Mo}$	176.49	-5.80	-3.43
^{240}Pu	$^{132}\text{Sn} + ^{98}\text{Zr}$	195.41	-12.05	-8.38
	$^{136}\text{Te} + ^{94}\text{Sr}$	190.78	-9.26	-9.10
	$^{126}\text{Cd} + ^{104}\text{Mo}$	190.18	-7.84	-5.12
^{246}Cm	$^{120}\text{Pd} + ^{110}\text{Ru}$	188.42	-6.70	-4.80
	$^{132}\text{Sn} + ^{104}\text{Mo}$	206.96	-11.75	-9.48
	$^{136}\text{Te} + ^{100}\text{Zr}$	201.03	-9.00	-5.55
	$^{128}\text{Cd} + ^{108}\text{Ru}$	200.96	-8.03	-2.41
^{252}Cf	$^{120}\text{Pd} + ^{116}\text{Pd}$	200.74	-6.84	-3.64
	$^{132}\text{Sn} + ^{110}\text{Ru}$	220.18	-11.87	-13.21
	$^{146}\text{Ba} + ^{96}\text{Sr}$	201.48	-10.56	-4.54
	$^{136}\text{Te} + ^{106}\text{Mo}$	214.10	-9.49	-7.65
	$^{138}\text{Xe} + ^{104}\text{Zr}$	209.88	-9.23	-6.22
^{252}Fm	$^{128}\text{Cd} + ^{114}\text{Pd}$	214.21	-8.48	-5.10
	$^{136}\text{Xe} + ^{106}\text{Mo}$	226.89	-11.31	-9.55
	$^{126}\text{Cd} + ^{116}\text{Cd}$	225.26	-9.42	-3.67
^{256}Fm	$^{146}\text{Ba} + ^{100}\text{Zr}$	214.58	-11.48	-1.81
	$^{148}\text{Ce} + ^{98}\text{Sr}$	209.93	-11.05	-8.55
^{260}No	$^{142}\text{Ba} + ^{108}\text{Mo}$	232.02	-11.63	-6.57
	$^{136}\text{Xe} + ^{114}\text{Ru}$	240.22	-10.99	-13.53
^{262}Rf	$^{140}\text{Ba} + ^{112}\text{Ru}$	248.93	-12.65	-10.01

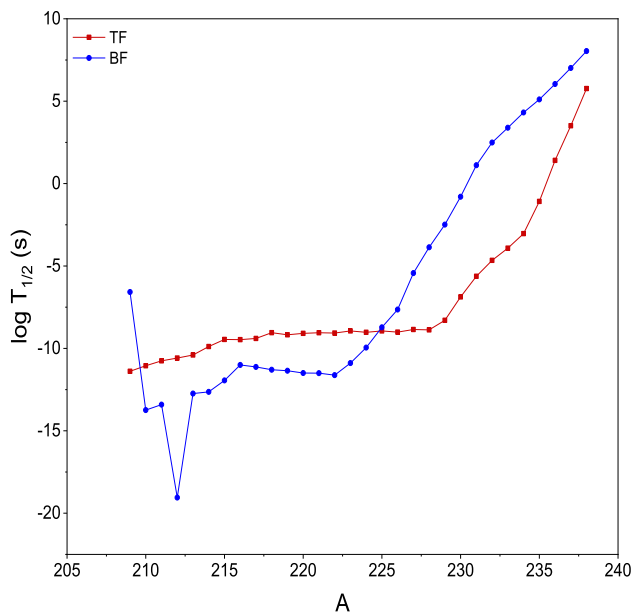
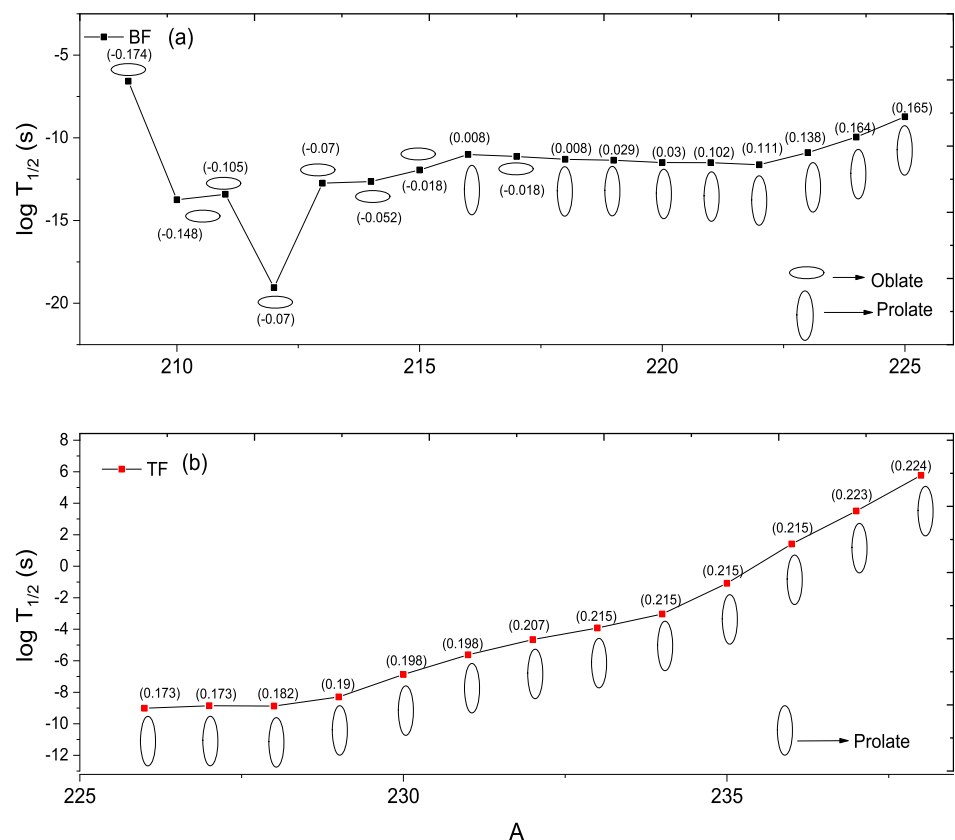


Fig. 7 A comparison of binary and ternary fission half-lives of $^{209-338}\text{Th}$

evaluation of half-lives and relative yield are in good agreement with those of the available experiments. Hence, by including both, we predicted the alpha ternary fission half-lives in the isotopes of ^{209}Th to ^{238}Th nuclei.

Fig. 8 Variation of alpha ternary fission half-lives versus mass number of parent nuclei along with the specification of deformation parameter



Similarly, the binary fission fragments and their half-lives were identified by following the same methodology as in the case of alpha ternary fission. Figure 7 shows a comparison between ternary fission and binary fission half-lives. From a detailed analysis, it is found that the binary fission half-lives are smaller in the $^{209-225}\text{Th}$ nuclei, and the ternary fission half-lives are smaller in the isotopes of $^{226-238}\text{Th}$.

Once the dominant decay mode has been identified in the isotopes of thorium, a plot of alpha ternary fission half-lives versus the mass number of parent nuclei, along with the specification of the deformation parameter within the bracket, is shown in Fig. 8. As can be seen in Fig. 8a a small variation in the deformations from oblate to prolate (i.e., an increase in deformation from negative to positive) will result in the increase of binary fission half-lives in the isotopes of $^{209-225}\text{Th}$. Similarly, an increase in the value of deformation from 0.173 to 0.224 increases the ternary fission half-lives of the isotopes of Th nuclei from ^{226}Th to ^{238}Th . From the detailed analysis, it is observed that a small change in the value of the deformation results in an observed change in the value of half-lives.

Finally, the role of deformations on the half-lives of $^{209-238}\text{Th}$ was studied, as shown in Fig. 9. However, no systematic variation is observed for negative quadrupole deformations in the case of binary fission (Fig. 9a).

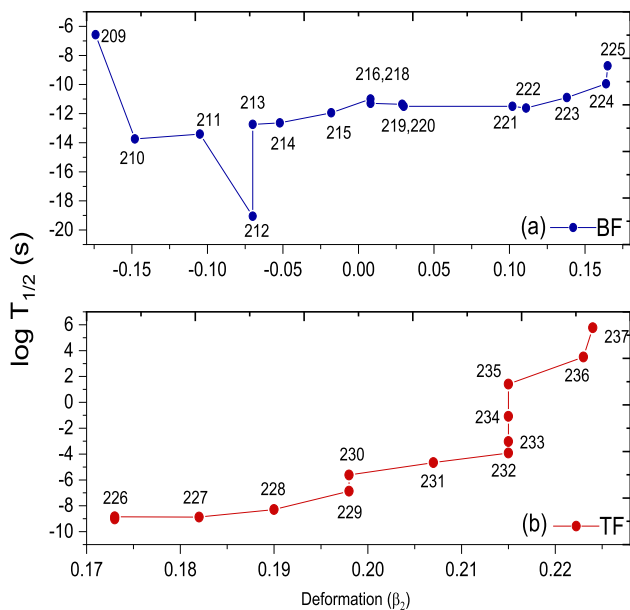


Fig. 9 Variation of **a** binary fission half-lives and **b** alpha ternary fission half-lives as function of deformation parameter of $^{209-238}\text{Th}$ nuclei

However, a close look at the half-lives shows a decrease in the logarithmic half-lives with an increase in the negative quadrupole deformations. Furthermore, an increase in the logarithmic half-lives of binary fission was observed for positive quadrupole deformations. In the case of α -ternary fission Fig. 9b, an increase in $\log T_{1/2}$ is observed with an increase in positive quadrupole deformations. Hence, from the detailed analysis, it is clear that both quadrupole deformations and the angle of orientation influence the logarithmic half-lives.

4 Conclusion

The alpha ternary fission half-lives in the isotopes of thorium were studied using Coulomb and proximity potentials. The role of the angle of orientation and deformation effects was included in the evaluation of the total potential. The penetration probability of alpha ternary fission half-lives was evaluated using the Wentzel–Kramers–Brillouin (WKB) integral. Larger logarithmic half-lives are observed when $\theta = 90^\circ$. The evaluated alpha ternary fission yield was compared with that of available experiments. The evaluated yields with deformation were in close agreement with those of the experiments compared to those without deformations. Based on the remarkable agreement of theoretical yield with that of available experiments, the study has been extended to the isotopes of thorium, and possible fission fragment combinations with shorter half-lives have been identified. The half-lives evaluated using

the present work were compared with those reported by Poenaru et al. [Acta Physica Hungarica Series A, Heavy Ion Physics 14(1),285–295(2001)] Theoretical data. The binary fission half-lives are dominant in $^{209-225}\text{Th}$ nuclei, and the ternary fission half-lives are dominant in the isotopes of $^{226-238}\text{Th}$ nuclei. Hence, the behavior of fission half-lives under the influence of deformation and angle of orientation can help predict half-lives in future experiments.

References

1. S.T. Tsien, Z.W. HO, L. Vigneron et al., Ternary and quaternary fission of uranium nuclei. *Nature* **159**(4049), 773–774 (1947). <https://doi.org/10.1038/159773a0>
2. S.T. Tsien, Sur la bipartition et la tripartition des éléments lourds. *J. Phys. Radium* **9**, 6–19 (1948). <https://doi.org/10.1051/jphysrad:01948009010600>
3. H.C. Manjunatha, N. Sowmya, Competition between spontaneous fission ternary fission cluster decay and alpha decay in the super heavy nuclei of $Z=126$. *Nucl. Phys. A* **969**, 68–82 (2018). <https://doi.org/10.1016/j.nuclphysa.2017.09.008>
4. A.V. Ramayya, J.H. Hamilton, J.K. Hwang et al., Cold (neutronless) α ternary fission of ^{252}Cf . *Phys. Rev. C* **57**(5), 2370 (1998). <https://doi.org/10.1103/PhysRevC.57.2370>
5. A.V. Ramayya, J.K. Hwang, J. Hamilton et al., Observation of ^{10}Be emission in the cold ternary spontaneous fission of ^{252}Cf . *Phys. Rev. Lett.* **81**(5), 947 (1998). <https://doi.org/10.1103/PhysRevLett.81.947>
6. Y.N. Kopatch, M. Mutterer, D. Schwalm et al., $^5\text{He}, ^7\text{He}$, and ^8Li ($E^* = 2.26\text{MeV}$) intermediate ternary particles in the spontaneous fission of ^{252}Cf . *Phys. Rev. C* **65**(4), 044614 (2002). <https://doi.org/10.1103/PhysRevC.65.044614>
7. G.M. Ter-Akopian, A.V. Daniel, A.S. Fomichev et al., New data on the ternary fission of ^{252}Cf from the Gammasphere facility. *Phys. Atom. Nucl.* **67**(10), 1860–1865 (2004). <https://doi.org/10.1134/1.1811191>
8. A. Sandulescu, F. Carstoiu, S. Misicu et al., Neutronless ^{10}Be -accompanied ternary fission of ^{252}Cf . *J. Phys. G Nucl. Partic.* **24**(1), 181 (1998). <https://iopscience.iop.org/0954-3899/24/1/022>
9. D.N. Poenaru, W. Greiner, J.H. Hamilton et al., Multicuster accompanied fission. *Phys. Rev. C* **59**(6), 3457–3466 (1999). <https://doi.org/10.1103/PhysRevC.59.3457>
10. S. Missicu, P.O. Hess, W. Greiner et al., Collective spectra of α -like giant trinuclear molecules. *Phys. Rev. C* **63**(5), 054308 (2001). <https://doi.org/10.1103/PhysRevC.63.054308>
11. G.M. Raisbeck, T.D. Thomas, Light nuclei emitted in the fission of ^{252}Cf . *Phys. Rev.* **172**(4), 1272 (1968). <https://doi.org/10.1103/PhysRev.172.1272>
12. M. Balasubramaniam, C. Karthikraj, S. Selvaraj et al., Ternary-fission mass distribution of ^{252}Cf : a level-density approach. *Phys. Rev. C* **90**(5), 054611 (2014). <https://doi.org/10.1103/PhysRevC.90.054611>
13. K. Manimaran, M. Balasubramaniam, Ternary fission fragmentation of ^{252}Cf for all possible third fragments. *Europ. Phys. J. A* **45**(3), 293–300 (2010). <https://doi.org/10.1140/epja/i2010-11000-7>
14. P. Jesinger, A. Kötzle, A.M. Gagariski et al., Observation of a triple correlation in ternary fission: is time reversal invariance

- violated? Nucl. Instrum. Meth. A **440**(3), 618–625 (2000). [https://doi.org/10.1016/S0168-9002\(99\)01051-7](https://doi.org/10.1016/S0168-9002(99)01051-7)
15. H.C. Manjunatha, N. Sowmya, K.N. Sidhar et al., A study of probable alpha-ternary fission fragments of ^{257}Fm . J. Radio. Nucl. Chem **314**(2), 991–999 (2017). <https://doi.org/10.1007/s10967-017-5450-4>
 16. H.C. Manjunatha, K.N. Sridhar, N. Sowmya et al., Investigations of the synthesis of the superheavy element $Z = 122$. Phys. Rev. C **98**(2), 024308 (2018). <https://doi.org/10.1103/PhysRevC.98.024308>
 17. H.C. Manjunatha, N. Sowmya, Decay modes of superheavy nuclei $Z = 124$. Intern. J. Mod. Phys. E **27**(05), 1850041 (2018). <https://doi.org/10.1142/S0218301318500416>
 18. E. Cheifetz, B. Eylon, E. Fraenkel et al., Emission of the ^5He in the Spontaneous Fission of ^{252}Cf . Phys. Rev. Lett. **29**(12), 805–808 (1972). <https://doi.org/10.1103/PhysRevLett.29.805>
 19. A.M. Nagaraja, H.C. Manjunatha, N. Sowmya et al., Heavy particle radioactivity of superheavy element $Z = 126$. Nucl. Phys. A **1015**, 122306 (2021). <https://doi.org/10.1103/PhysRevLett.29.805>
 20. N. Sowmya, H.C. Manjunatha, Competition between different decay modes of superheavy element $Z = 116$ and synthesis of possible isotopes. Braz. J. Phys. **49**(6), 874–886 (2019). <https://doi.org/10.1007/s13538-019-00710-4>
 21. G.R. Sridhar, H.C. Manjunatha, N. Sowmya et al., Atlas of cluster radioactivity in actinide nuclei. Eur. Phys. J. Plus **135**(3), 1–28 (2020). <https://doi.org/10.1140/epjp/s13360-020-00302-1>
 22. N. Sowmya, H.C. Manjunatha, P.S. Damodara Gupta et al., Competition between cluster and alpha decay in odd Z superheavy nuclei $111 \leq Z \leq 125$. Braz. J. Phys. **51**(1), 99–135 (2021). <https://doi.org/10.1007/s13538-020-00801-7>
 23. H. Cheng, R.L. Edwards, J. Hoffa et al., The half-lives of uranium-234 and thorium-230. Chem. Geo. **169**(1–2), 17–33 (2000). [https://doi.org/10.1016/S0009-2541\(99\)00157-6](https://doi.org/10.1016/S0009-2541(99)00157-6)
 24. O. Häusser, W. Witthuhn, T.K. Alexander et al., Short-lived α emitters of thorium: new isotopes $^{218-220}\text{Th}$. Phys. Rev. Lett. **31**(5), 323–326 (1973). <https://doi.org/10.1103/PhysRevLett.31.323>
 25. W.A. Yahya, B.J. Falaye, Alpha decay study of Th isotopes using a double-folding model with NN interactions derived from relativistic mean field theory. Nucl. Phys. A **1015**, 122311 (2021). <https://doi.org/10.1016/j.nuclphysa.2021.122311>
 26. M.R. Pahlavani, N. Karamzadeh, Partial α -decay half-lives of ground to ground and ground to excited states of Thorium family. Chin. J. Phys. **56**(4), 1727–1733 (2018). <https://doi.org/10.1016/j.cjph.2018.05.014>
 27. F. Ghorbani, S.A. Alavi, V. Dehghani, Temperature dependence of the alpha decay half-lives of even-even Th isotopes. Nucl. Phys. A **1002**, 121947 (2020). <https://doi.org/10.1016/j.nuclphysa.2020.121947>
 28. J.B. Roberto, C.W. Alexander, R.A. Boll et al., Actinide targets for the synthesis of super-heavy elements. Nucl. Phys. A **944**, 99–116 (2015). <https://doi.org/10.1016/j.nuclphysa.2015.06.009>
 29. S. Hofmann, Synthesis of superheavy elements using radioactive beams and targets. Prog. Part. Nucl. Phys. **46**(1), 293–302 (2001). [https://doi.org/10.1016/S0146-6410\(01\)00134-X](https://doi.org/10.1016/S0146-6410(01)00134-X)
 30. M. Wang, W.J. Huang, F.G. Kondev et al., The AME 2020 atomic mass evaluation (II). Tables, graphs, and References. Chin. Phys. C **45**(3), 030003 (2021). <https://doi.org/10.1088/1674-1137/abddaf>
 31. N. Sowmya, H.C. Manjunatha, Investigations on different decay modes of darmstadtium. Phys. Part. Nucl. Lett. **17**(3), 370–378 (2020). <https://doi.org/10.1134/S1547477120030140>
 32. N. Sowmya, H.C. Manjunatha, N. Dhananjaya et al., Competition between binary fission, ternary fission, cluster radioactivity and alpha decay of ^{281}Ds . J. Radio. Nucl. Chem. **323**(3), 1347–1351 (2020). <https://doi.org/10.1007/s10967-019-06706-3>
 33. G.R. Sridhara, H.C. Manjunatha, K.N. Sridhar et al., Systematic study of the α decay properties of actinides. Pramana **93**(5), 1–14 (2019). <https://doi.org/10.1007/s12043-019-1845-9>
 34. H.C. Manjunatha, S. Alfred Cecil Raj, A.M. Nagaraja et al., Cluster radioactivity in superheavy nuclei 299–306122. J. Nucl. Phys. Mater. Sci. Radiat. Appl. **8**(1), 55–63 (2020). <https://doi.org/10.15415/jnp.2020.81007>
 35. N. Sowmya, H.C. Manjunatha, P.S. DamodaraGupta, Competition between decay modes of superheavy nuclei $^{281-310}\text{Og}$. Int. J. Mod. Phys. E **29**(10), 2050087 (2020). <https://doi.org/10.1142/S0218301320500871>
 36. G.L. Zhang, Y.J. Yao, M.F. Guo et al., Comparative studies for different proximity potentials applied to large cluster radioactivity of nuclei. Nucl. Phys. A **951**, 86–96 (2016). <https://doi.org/10.1103/PhysRevC.81.044608>
 37. D.N. Poenaru, W. Greiner, J.H. Hamilton et al., Nuclear molecules in ternary fission. Acta Phys. Hung. A **14**(1), 285–295 (2001). <https://doi.org/10.1556/APH.14.2001.1-4.27>