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The surface current method in gray Dancoff factor calculation

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Abstract The Dancoff correction is important in the calculation of the effective cross section of resonant isotopes in a heterogeneous system. Although the neutron current method is a simple and straightforward approach to estimate the Dancoff factor, its use is limited to the black Dancoff factor. In this paper, we expand the current method used to determine both the black and gray Dancoff factors. The method developed also relies on a neutron transport solver, where a fixed source on a fuel rod surface has an outward direction, a cosine distribution, and a constant shape. The detector is located on the surface of the rods to measure incoming and outgoing currents; therefore, there is no need to calculate the chord length, and the development, validation, and verification of the code can be omitted. The mathematical foundation of the suggested method is derived using the integral transport equation. The effects of the moderator and lattice configuration are followed by a sensitivity analysis of the Dancoff factor for several problems, including pressurized water reactor and cluster fuel assemblies. The maximum and average relative errors of the calculated results are approximately 0.3% and 0.05%, respectively.

Keywords Black Dancoff · Gray Dancoff · Integral transport equation · Interface current · Transport solver

1 Introduction

An accurate estimation of self-shielded cross sections in calculations of lattice physics is vital to the success of reactor design and analysis. The quality of self-shielded cross sections achieved by equivalence theory is strongly connected to Dancoff factor estimation. In equivalence theory, the rational function is used to approximate the neutron escape probability in terms of the total cross section of the resonance isotope. The heterogeneity effect is considered through the escape cross section by the Dancoff correction [1, 2]. The Dancoff factor is also used to establish equivalent one-dimensional models for fuel rods located in the fuel assembly. Then, the ultra-fine group method is carried out to obtain the effective cross section by use of that simplified one-dimensional model [3, 4].

The Dancoff factor is a foundation for improvement of the calculation of multi-group cross sections of resonance absorbers by quantifying the shadowing effect of a fuel rod due to the presence of other rods. The probability of a particle reaching neighboring fuel rods by escaping from one fuel rod without a collision is described by the Dancoff factor.

The effects of non-uniform geometry in reactor cores are major challenges to be measured in Dancoff factor calculation. There are methods to tackle this problem; for example, Carlvik derived an analytic formula for fuel lattices [5, 6]. However, such Dancoff factor evaluations can only express fuel assemblies with rods coordinated in cluster geometry. The DANCOFF-MC and MCDancoff programs were developed using the Monte Carlo method in 2D and 3D, respectively [7]. They start scenarios isotropically and randomly on the fuel surface of the target rod and track the particle path to count its encounters with rods

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with the same material. Finally, the Dancoff factor is obtained as follows:

$$C = \frac{\int \mathrm{d}S \int\limits_{n\cdot\Omega\geq 0} \mathrm{d}\Omega (n\cdot\Omega)\exp[-\Sigma_{\mathrm{tM}} l_{\mathrm{M}}(s,\Omega) - \Sigma_{\mathrm{tC}} l_{\mathrm{C}}(s,\Omega) - \Sigma_{\mathrm{tF}} l_{\mathrm{F}}(s,\Omega)]}{\int \mathrm{d}S \int\limits_{n\cdot\Omega\geq 0} \mathrm{d}\Omega (n\cdot\Omega)}, \tag{1}$$

where l_{F} is the chord length over the fuel, l_{C} is the chord length over the cladding, l_{M} is the chord length over the moderator, Σ_{tF} is the fuel total cross section, Σ_{tC} is the cladding total cross section, and Σ_{tM} is the moderator total cross section. One restriction in the use of DANCOFF-MC is the incorrect distribution of neutrons released on the fuel rod [8]. The chief obstacle with MCDancoff is the bounding regions, which should be cuboids, spheres, or cylinders [9].

These methodologies may, in some cases, increase the accuracy of the Dancoff factor. However, these methods are complex in both the procedures and implementation.

The neutron current method is presented as a simple use of a neutron transport solver that does not require programming new code [10, 11]. In this approach, the Dancoff factor is estimated using the average flux of the target fuel rod in the isolated system in which a satisfactorily large moderator is used to surround the fuel pin and the average regional flux of the target fuel pin in lattice geometry. In other words, the Dancoff correction was obtained by

$$C = \frac{\phi_0 - \phi}{\phi_0}, \tag{2}$$

where ϕ_0 is the target fuel rod average flux in the large moderator system and ϕ is the target fuel rod average flux in lattice geometry. This approach was presented to only calculate the black Dancoff coefficient.

In this study, we improved the neutron current method used to compute both the black and gray Dancoff factors in fuel assemblies with arbitrary geometries. This approach is based on a neutron transport solver that calculates the inward and outward currents over the surfaces of fuels. Therefore, there is no need for new programming or to derive analytical equations. In the proposed approach, the transport calculation is implemented in lattice geometry by locating the source on the surface of the fuel region and calculating the outward and inward currents on the surfaces of the fuels.

2 Methodology

The fuel absorbers must be brightened from black to gray to provide true representations of actual fuel rods. Therefore, the gray Dancoff factor is introduced for partially absorbing fuel rods. This factor, known as the probability of a neutron traversing one or more fuel rods without interaction, is used in equivalence theory based on an initial estimate of fuel cross section and an iteration procedure that finally calculates the effective cross sections of the fuel regions [12, 13]. In this section, we derive a formula for evaluating the Dancoff factor from the Boltzmann equation that can be applied to a transport solver. For the sake of simplicity, the one-speed, steady-state, integral form of the equation can be given as [14]

$$\phi(r,\Omega) = \phi_{-}(r_{s'},\Omega)\mathrm{e}^{-\tau(r,r_{s'})} + \int_0^{R_{s'}} \mathrm{d}R' Q(r',\Omega)\mathrm{e}^{-\tau(r,r')}, \tag{3}$$

where ϕ is the angular flux, r is a defined point on the computational domain, Ω is the angle of travel, $r_{s'}$ and $R_{s'}$ are points on the system boundary as shown in Fig. 1, τ is the optical length, and ϕ_{-} is the inward angular flux. The neutron source is written as follows:

$$\begin{aligned} Q(r,\Omega) &= \int\limits_{4\pi} \mathrm{d}\Omega' \Sigma_s(r,\Omega'\rightarrow\Omega) \\ &\quad \phi(r,\Omega') + \frac{\nu\Sigma_{\mathrm{f}}(r)}{4\pi k_{\mathrm{eff}}} \int\limits_{4\pi} \mathrm{d}\Omega' \phi(r,\Omega') + S(r,\Omega), \end{aligned} \tag{4}$$

where $\nu\Sigma_{\mathrm{f}}$ represents the production cross section, $S(r,\Omega)$ is the fixed volumetric neutron source, k_{eff} is the multiplication factor, and scattering cross section is denoted by Σ_{s}.

Figure 1 shows the angular flux that can be obtained by considering all uncollided neutrons flying from the

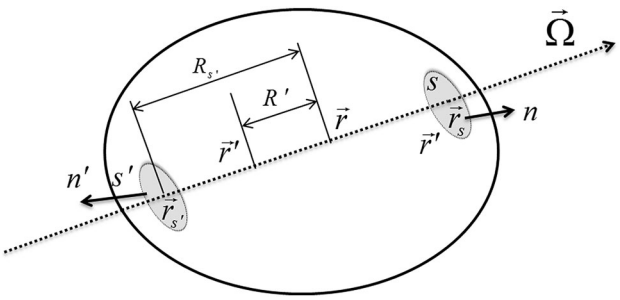

Fig. 1 Coordinate characterization of particle transport

boundary of the volume and summing the uncollided neutrons generated in the volume.

The optical distance is written as

$$\tau(r, r_{s'}) = \int_{r}^{r_{s'}} \sum_{\mathrm{t}}(r)\mathrm{d}l, \tag{5}$$

where dl is the differential length of neutron flight path at direction Ω. Equation (3) can be written for a purely absorbing and not multiplying media with no volumetric source as follows.

$$\phi(r, \Omega) = \phi_{-}(r_{s'}, \Omega)\mathrm{e}^{-\tau(r, r_{s'})} \tag{6}$$

The inward current on surface s for a neutron emitting from location $r_{s'}$ with direction Ω, as shown in Fig. 1, is determined as follows:

$$J_{-s} = (n \cdot \Omega)\phi(r_s, \Omega), \tag{7}$$

where n represents the inward normal vector of surface s. The inward current to surface s by a neutron source located at $r_{s'}$ emitting at all solid angles is achieved through integration over the angle.

$$J_{-s} = \int_{n\cdot\Omega>0} (n \cdot \Omega)\phi(r_s, \Omega)\mathrm{d}\Omega \tag{8}$$

Then, the total inward current is obtained by integrating Eq. (8) over the source surface area.

$$J_{-s} = \int_{s'} \mathrm{d}s \int_{n\cdot\Omega>0} (n \cdot \Omega)\phi(r_s, \Omega)\mathrm{d}\Omega \tag{9}$$

The following equation is found by substituting Eq. (6) into Eq. (9).

$$J_{-s} = \int_{s'} \mathrm{d}s \int_{n\cdot\Omega>0} (n \cdot \Omega)\phi_{-}(r_{s'}, \Omega)\mathrm{e}^{-\tau(r_s, r_{s'})}\mathrm{d}\Omega \tag{10}$$

By assuming isotropic incoming angular flux on surface s', in other words, isotropic angular distribution of the source, Eq. (10) can be rewritten as

$$J_{-s} = \int_{s'} \mathrm{d}s \int_{n\cdot\Omega>0} \frac{(n \cdot \Omega)}{4\pi}\mathrm{e}^{-\tau(r_s, r_{s'})}\mathrm{d}\Omega \tag{11}$$

In the same way, the total outward current at surface s', where the source is located, can be obtained by moving r_{s} toward $r_{s'}$, as

$$J_{+s'} = \int_{s'} \mathrm{d}s \int_{n'\cdot\Omega>0} \frac{(n' \cdot \Omega)}{4\pi}\mathrm{d}\Omega \tag{12}$$

where n' is the outward normal vector of surface s'.

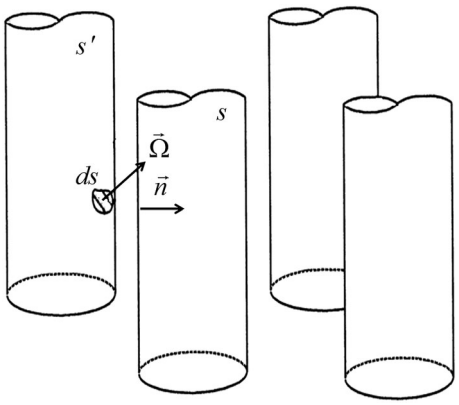

Fig. 2 The neutron flight from a fuel rod surface to other fuel rods

There are many fuel rods in a fuel assembly, as shown in Fig. 2. Therefore, the incoming current of all fuel rods should be considered while estimating the Dancoff factor. Then, the black Dancoff correction is obtained by summing Eq. (11) over all fuel rods and then dividing by Eq. (12) as

$$C = \frac{\sum_{s=1}^{N_s} J_{-s}}{J_{+s'}}, \tag{13}$$

where N_s is the total number of fuel element surfaces. The Dancoff correction is obtained from the probability of a neutron released with isotropic angular distribution from the fuel rod surface to collide with other fuel elements.

The black and gray Dancoff corrections can be obtained by

$$C = \frac{\sum_{s=1}^{N_s} (J_{-s} - J_{+s})}{J_{+s'}}. \tag{14}$$

The Dancoff factor is obtained as

$$D = 1 - C. \tag{15}$$

Therefore, the Dancoff factor can be obtained by placing an outward surface source at the considered fuel element and calculating the incoming and outgoing currents in all fuel elements in a pure absorbing media. The proposed method can be used for complicated geometry as long as the neutron transport solver has the ability to define a fixed neutron source on the surface of a candidate fuel pin and calculate the incoming and outgoing currents on the fuel surfaces.

We utilized the following procedure to calculate the interface currents on the surfaces of fuels.

1. A one-energy-group neutron library is prepared for each problem. In this library, the total cross section is set to be equal to the absorption cross section. In other words, the scattering cross section is zero and the neutron transport is done in a pure absorbing media.

2. We set the outward neutron source on the outer surface of the fuel for which we want to calculate the Dancoff factor. In other words, the fixed neutron source is zero in inner angular direction of the outer surface of the fuel.
3. After performing a one-group transport calculation, we can evaluate the total inward and outward neutron currents in all fuel regions.
4. The Dancoff correction is evaluated using Eqs. (14)–(15), in which the denominator is the outward neutron current of the rod for which we want to calculate the Dancoff factor.

In the next section, we evaluate the proposed method against the collision probability (PIJ) method. The PIJ method produces accurate results, but execution time and memory requirements increase quadratically with respect to the spatial mesh. Therefore, there are inherent limits to applying the PIJ method in large geometries. Additionally, implementing the PIJ method is considerably complex for non-uniform geometries. The merit of the proposed method over the PIJ method is that there is no need to design and program new computer code to calculate collision probabilities. Therefore, we can use the current neutron transport solver to compute the black and gray Dancoff factors. Furthermore, the computational cost of the proposed method can be significantly reduced with a more efficient transport solver.

3 Results

We first determine the Dancoff factors for a typical light water reactor (LWR) lattice. The lattice geometry is depicted in Fig. 3. The fuel rod pitch is 1.26 cm, and the outer radius of the fuel is 0.4025 cm. The outer radius of the cladding is 0.475 cm. We assume that there is no space between the fuel and cladding. The macroscopic total cross sections of the moderator, cladding, and fuel are 1.035, 0.2943, and 10^4 cm^{-1}, respectively.

Table 1 The Dancoff factors of an LWR lattice

Rod	Dancoff factor	
	Proposed method	PIJ method
1	0.7343	0.7345
2	0.6509	0.6512
3	0.7915	0.7916
4	0.6750	0.6751
5	0.6508	0.6511
6	0.7343	0.7345
7	0.6750	0.6751
8	0.6536	0.6539
9	0.6478	0.6480
10	0.6509	0.6512
11	0.6508	0.6511
12	0.6478	0.6480
13	0.6476	0.6479

The calculated fuel rod Dancoff factors are shown in Table 1. These results are assessed based on those found using the PIJ method [7]. The results indicate that the black Dancoff factors obtained by the proposed method are in good agreement with the conventional method. We used the MCNP code as a transport solver to calculate the incoming and outward currents.

In the second problem, the Dancoff factor of a PWR fuel assembly is computed by the proposed method. The assembly is a 17 × 17 square lattice, as shown in Fig. 4. The lattice pitch is 1.26 cm. The rod radii and cross sections are presented in Table 2. The boundary condition is specified as specular reflection.

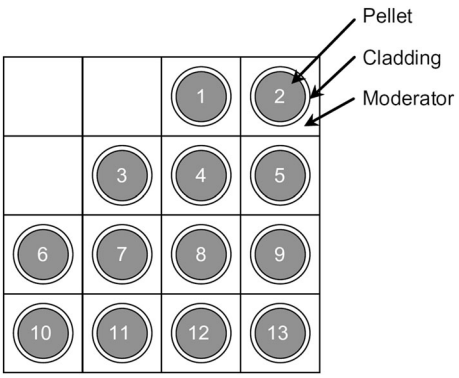

Fig. 3 Typical LWR lattice geometry [11]

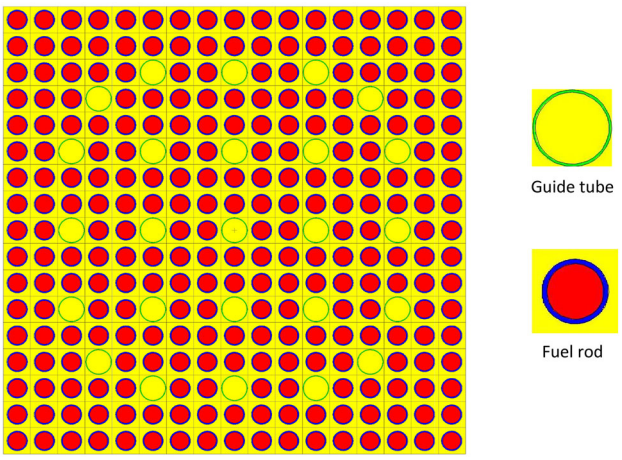

Fig. 4 (Color online) Geometry of the second problem

Table 2 Physical and geometric properties of the second problem

Total cross section		Rod radius	
Material	Value (cm^{-1})	Region	Value (cm)
Fuel	10^5	Fuel	0.41
Cladding	0.30	Cladding	0.48
Moderator	1.05	Guide tube (inner)	0.57
Guide tube	0.30	Guide tube (outer)	0.61

The computed Dancoff factors for 1/8-symmetry are depicted in Fig. 5. The reference values are found using the collision probability method [10]. The deviation from the previously accepted method is small. The maximum error is about 0.12%. The average error is about 0.05%. From Fig. 5, the proposed method delivers precise Dancoff factors for complex geometries.

The third problem consists of four cells made of fuel, guide, moderator, and baffle, in a configuration as illustrated in Fig. 6. This lattice is assembled as a mini PWR core with reflector and baffle regions. The lattice pitch and the rod radii are the same as those in the second problem. The macroscopic cross section specifications of the fuel, guide, baffle, and moderator are specified in Table 3. The boundary condition is specified as specular reflection.

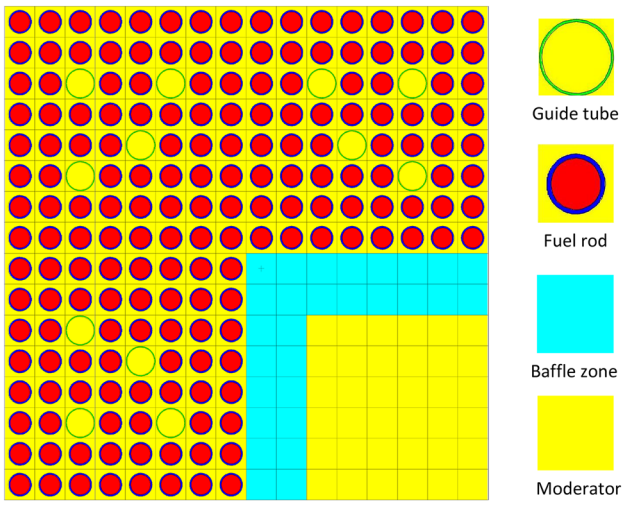

Fig. 6 (Color online) Geometry of the third problem

The calculation results are shown in Fig. 7, which are the same as those of the reference values achieved by the PIJ method that is implemented in AEGIS [10]. The deviations of the black Dancoff factors from those of the previously accepted method are minimal. The average

Fig. 5 Dancoff factors for a PWR fuel assembly

0 0 **0.00**								
0.6943 0.6940 **0.05**	0.6655 0.6652 **0.05**							
0.6943 0.6940 **0.04**	0.6655 0.6652 **0.05**	0.6655 0.6651 **0.05**						
0 0 **0.00**	0.6943 0.6940 **0.05**	0.6943 0.6940 **0.05**	0 0 **0.00**					
0.6943 0.6940 **0.05**	0.6655 0.6652 **0.05**	0.6656 0.6652 **0.06**	0.6966 0.6963 **0.05**	0.6840 0.6836 **0.06**				
0.6943 0.6940 **0.04**	0.6655 0.6652 **0.05**	0.6655 0.6651 **0.06**	0.6942 0.6939 **0.04**	0.7149 0.7145 **0.05**	0 0 **0.00**			
0 0 **0.00**	0.6943 0.6939 **0.06**	0.6946 0.6942 **0.06**	0 0 **0.00**	0.7128 0.7124 **0.06**	0.6945 0.6941 **0.06**	0.6609 0.6606 **0.05**		
0.6943 0.6940 **0.05**	0.6633 0.6629 **0.06**	0.6633 0.6630 **0.05**	0.6942 0.6939 **0.05**	0.6633 0.6629 **0.06**	0.6448 0.6444 **0.06**	0.6448 0.6445 **0.04**	0.6425 0.6421 **0.06**	
0.6427 0.6421 **0.09**	0.6451 0.6445 **0.09**	0.6451 0.6445 **0.09**	0.6427 0.6421 **0.09**	0.6451 0.6446 **0.08**	0.6427 0.6421 **0.09**	0.6428 0.6422 **0.09**	0.6427 0.6421 **0.09**	0.6429 0.6421 **0.12**

Reference
Proposed method
Error (%)

Table 3 Physical properties of the third problem

Material	Total cross section (cm^{-1})
Fuel	10^5
Cladding	0.30
Guide tube	0.30
Moderator	1.05
Baffle	0.30

error is 0.06%. The maximum error is 0.13%. As shown, the new method is easily handled for complicated geometry.

The fourth problem is a cluster cell that is deliberated to compute gray Dancoff factors. The cluster consists of five fuel rods with gaps and cladding as portrayed in Fig. 8. The four outer fuel rods are placed symmetrically in the center of the circle, at a radius of 13.5 cm from the center of the cluster and at angles of $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$. Specular reflection is employed for the square cell. The cell features are specified in Table 4.

The Dancoff corrections for this problem are computed by the proposed method and compared with those obtained from the collision probability method [15]. The total cross sections of fuel and cladding are 25.0 and 0.25 cm^{-1}, respectively. Several values for the coolant total cross section are used.

The gray and black Dancoff factors for pin number 1 are given in Table 5 for a square outer boundary and an equivalent cylindrical boundary. The results for pin number 2 are shown in Table 6. The results reveal good agreement with the PIJ method. The relative error ranges from 0.1 to 1.4%.

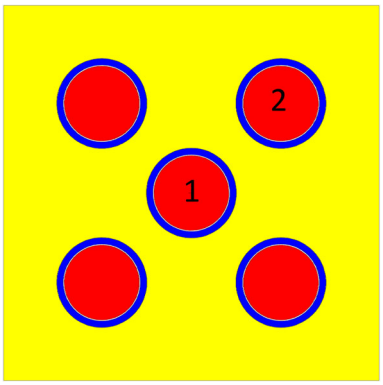

Fig. 8 (Color online) The cluster fuel assembly with a square outer boundary

Table 4 Physical properties of the cell cluster

Region	Value
Fuel radius (cm)	4.0
Cladding thickness (cm)	0.1
Gap thickness (cm)	0.7
Square dimension (cm^2)	40 × 40

0.6429	0.645	0.6427	0.6451	0.6451	0.6427	0.6451	0.6427	0.6427	0.6451	0.6427	0.6451	0.6451	0.6427	0.6451	0.6427
0.6421	0.6444	0.6420	0.6444	0.6444	0.6421	0.6445	0.6421	0.6421	0.6445	0.6421	0.6444	0.6444	0.6420	0.6444	0.6421
0.13	**0.09**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
	0.6609	0.6942	0.6633	0.6632	0.6945	0.6611	0.6448	0.6448	0.6611	0.6945	0.6632	0.6633	0.6942	0.6609	0.6448
	0.6605	0.6939	0.6629	0.6629	0.6942	0.6607	0.6444	0.6444	0.6607	0.6942	0.6628	0.6629	0.6939	0.6605	0.6444
	0.06	**0.04**	**0.06**	**0.05**	**0.04**	**0.06**	**0.06**	**0.06**	**0.06**	**0.04**	**0.05**	**0.06**	**0.04**	**0.06**	**0.06**
		0	0.6965	0.6941	0	0.6941	0.6425	0.6425	0.6941	0	0.6942	0.6965	0	0.6942	0.6425
		0	0.6962	0.6938	0	0.6938	0.6421	0.6421	0.6938	0	0.6938	0.6962	0	0.6939	0.6420
		0.00	**0.05**	**0.04**	**0.00**	**0.04**	**0.06**	**0.06**	**0.04**	**0.00**	**0.06**	**0.04**	**0.00**	**0.04**	**0.07**
			0.6839	0.7149	0.7128	0.6633	0.6449	0.6449	0.6633	0.7128	0.7149	0.684	0.6965	0.6633	0.6449
			0.6836	0.7145	0.7124	0.6629	0.6446	0.6446	0.6629	0.7124	0.7145	0.6835	0.6962	0.6628	0.6444
			0.05	**0.05**	**0.05**	**0.06**	**0.05**	**0.05**	**0.06**	**0.05**	**0.05**	**0.07**	**0.05**	**0.08**	**0.07**
				0	0.6945	0.6448	0.6425	0.6425	0.6448	0.6945	0	0.7149	0.6942	0.6632	0.6449
				0	0.6941	0.6444	0.6421	0.6421	0.6445	0.6941	0	0.7145	0.6939	0.6629	0.6444
				0.00	**0.05**	**0.06**	**0.06**	**0.06**	**0.05**	**0.05**	**0.00**	**0.05**	**0.04**	**0.05**	**0.07**
					0.6609	0.6449	0.6427	0.6427	0.6451	0.6612	0.6947	0.7131	0	0.6948	0.6428
					0.6605	0.6445	0.6424	0.6424	0.6448	0.6608	0.6944	0.7127	0	0.6945	0.6423
Reference						**0.06**	**0.05**	**0.05**	**0.05**	**0.06**	**0.05**	**0.06**	**0.00**	**0.05**	**0.08**
Proposed method						0.6425	0.6454	0.6454	0.6475	0.6503	0.6505	0.669	0.6999	0.6668	0.6507
Error (%)						0.6421	0.6450	0.6450	0.6472	0.6500	0.6501	0.6687	0.6996	0.6664	0.6503
						0.05	**0.05**	**0.06**	**0.05**	**0.04**	**0.06**	**0.05**	**0.04**	**0.07**	**0.06**
							0.6703	0.7227	0.7418	0.7509	0.7551	0.76	0.7587	0.7616	0.7595
							0.6700	0.7225	0.7415	0.7506	0.7549	0.7596	0.7585	0.7613	0.7592
							0.05	**0.02**	**0.04**	**0.04**	**0.03**	**0.05**	**0.03**	**0.04**	**0.04**

Fig. 7 The Dancoff factors for the third problem

Table 5 The Dancoff corrections for the cluster cell at position 1

Moderator total cross section (cm^{-1})	PIJ method [15]		Proposed method	
	Black	Gray	Black	Gray
0.3	0.046309	0.034507	0.046290	0.034153
0.4	0.027867	0.020814	0.027862	0.020679
0.6	0.010373	0.007798	0.010369	0.007757
0.8	0.003959	0.002998	0.003958	0.002977
1.0	0.001540	0.001175	0.001540	0.001163
1.2	0.000608	0.000468	0.000608	0.000461

Table 6 The Dancoff corrections for the cluster cell at position 2

Moderator total cross section (cm^{-1})	PIJ method [15]		Proposed method	
	Black	Gray	Black	Gray
0.3	0.015364	0.011531	0.015381	0.011872
0.4	0.008132	0.006110	0.008136	0.006201
0.6	0.002714	0.002047	0.002713	0.002049
0.8	0.001003	0.000761	0.001003	0.000757
1.0	0.000387	0.000295	0.000386	0.000292
1.2	0.000152	0.000117	0.000152	0.000116

4 Conclusion

A new technique has been established and applied to obtain the Dancoff factor. The method relies on a neutron transport solver to compute currents and finally determine the Dancoff factor. Both black and gray Dancoff factors can be computed without any source code development or modification of a transport solver. The maximum relative errors of the calculated results for PWR and cluster problems are approximately 0.13% and 0.3%, respectively. The performance of the proposed procedure demonstrates that the approach is quite promising and can effectively tackle the complicated geometry. The proposed approach has more confidence and reliability for obtaining the reference Dancoff factor.

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