

Nusselt number correlation for turbulent heat transfer of helium– xenon gas mixtures

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Abstract A gas-cooled nuclear reactor combined with a Brayton cycle shows promise as a technology for highpower space nuclear power systems. Generally, a heliumxenon gas mixture with a molecular weight of 14.5–40.0 g/mol is adopted as the working fluid to reduce the mass and volume of the turbomachinery. The Prandtl number for helium-xenon mixtures with this recommended mixing ratio may be as low as 0.2. As the convective heat transfer is closely related to the Prandtl number, different heat transfer correlations are often needed for fluids with various Prandtl numbers. Previous studies have established heat transfer correlations for fluids with medium-high Prandtl numbers (such as air and water) and extremely low-Prandtl fluids (such as liquid metals); however, these correlations cannot be directly recommended for such heliumxenon mixtures without verification. This study initially assessed the applicability of existing Nusselt number correlations, finding that the selected correlations are unsuitable for helium-xenon mixtures. To establish a more general heat transfer correlation, a theoretical derivation was conducted using the turbulent boundary layer theory. Numerical simulations of turbulent heat transfer for helium-xenon mixtures were carried out using Ansys Fluent. Based on simulated results, the parameters in the

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derived heat transfer correlation are determined. It is found that calculations using the new correlation were in good agreement with the experimental data, verifying its applicability to the turbulent heat transfer for helium–xenon mixtures. The effect of variable gas properties on turbulent heat transfer was also analyzed, and a modified heat transfer correlation with the temperature ratio was established. Based on the working conditions adopted in this study, the numerical error of the property-variable heat transfer correlation was almost within 10%.

Keywords Gas-cooled nuclear reactor \cdot Space nuclear power \cdot Helium–xenon mixtures \cdot Convective heat transfer \cdot Nusselt number

Abbreviations

- *x* Axial distance from inlet
- *i* Inlet of tube
- y Radial distance from wall
- C Constant, 0.3
- T Temperature
- *u* Axial velocity (m/s)
- v Radial velocity (m/s)
- q Heat flux (W/m^2)
- M Molar weight (g/mol)
- G Mass flux (kg/m² s)
- f Darcy friction factor
- $g_{\rm c}$ Conversion factor
- $R_{\rm g}$ Gas constant for a gas
- u_{τ} Friction velocity (m/s)
- r_0 Radius of tube (m)
- D Diameter of tube (m)
- Pe Peclet number, Pr·Re
- $Pr_{\rm t}$ Turbulent Prandtl number, $\varepsilon_{\rm m}/\varepsilon_{\rm h}$
- y⁺ Non-dimensional wall distance

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| U^+ Non-dimensional velocity, normalized by <i>i</i> |
|--|
|--|

 Pe_{t} Turbulent Peclet number, $\varepsilon_{m}/v \cdot Pr$

Subscript

| b | Bulk |
|----------|----------------------------|
| c | Center of tube |
| W | Wall |
| ср | Constant property |
| vp | Variable property |
| ∞ | Turbulent core region |
| crit | Value of demarcation point |
| Т | Temperature |
| Cal | Calculation |
| Exp | Experimental data |
| local | Local radial value |

1 Introduction

Deep-space exploration, a landing on Mars, and the construction of planetary bases will become established activities in the near future. The development of advanced and reliable space power will become hugely important in order to achieve these ambitious milestones. Compared to typical power sources used in space, such as chemical fuel cells or solar photovoltaic arrays, the space nuclear reactor power is characterized by wider power coverage and longer operating duration; as such, it is regarded as a promising technology for human space exploration [1-3]. Among existing technology roadmaps of space nuclear reactors, the gas-cooled nuclear reactor combined with a closed Brayton cycle is perceived as the most rational scheme for the system power exceeding 200 kW [4-6]; this is because it can achieve a higher energy conversion efficiency and a smaller specific mass. Typically, pure helium is adopted as the working fluid for the terrestrial Brayton cycle because of its excellent thermal and transport properties [7]. However, its small molecular weight inevitably results in a larger aerodynamic loading of the turbomachinery, which in turn increases the mass and volume of the system [8]. To reduce the aerodynamic loads and obtain an acceptable heat transfer capacity, a helium-xenon mixture with a molecular weight of 14.5-40.0 g/mol is generally considered as the working fluid for Brayton cycles [9-13]. The Prandtl number (Pr) of helium-xenon mixtures in this molecular weight range is approximately 0.21-0.30, which is significantly lower than that of conventional air or water (Pr > 0.70). For such low-Pr fluids, the thermal boundary layer is thicker, and the similarity of velocity and temperature will be disrupted. Hence, the convective heat transfer characteristics of low-Pr helium-xenon mixtures differ from those of fluids with medium-high Prandtl numbers. Previous studies have also shown that the DittusBolter (DB) [14] and Colburn equations [15] obtained from experimental data of those conventional fluids overpredicted the Nusselt number of low-Pr gas mixtures [16, 17]. Therefore, to better describe the convective heat transfer of helium–xenon mixtures, it is necessary to establish a suitable heat transfer correlation.

The particularity of flow and heat transfer for low-Pr fluids has been widely concerned [18–20]. However, most studies on this issue have mainly focused on liquid metals $(Pr \cong 0.01)$, where many heat transfer correlations for liquid metals were proposed [21–23]. Lyon et al. [24] first derived a semi-empirical heat transfer correlation for a boundary condition with constant heat flux. Subsequently, Azer et al. [25] and Sleicher et al. [26] proposed their heat transfer correlations for liquid metals using theoretical hypotheses. Based on experimental data for liquid metals, Stomquist et al. [27], Skupinski et al. [28], and Churchill [29] presented their empirical heat transfer correlations. The Pr of liquid metals was found to be one or two orders of magnitude lower than that of the helium-xenon mixture; importantly, the flow and heat transfer characteristics of these two fluids significantly differed. To better illustrate the difference in convective heat transfer, Fig. 1 presents the temperature profiles of the air, helium-xenon mixture, and liquid mercury in a heated tube [30]. The temperature profile of the liquid metal was flat; however, the temperature distribution of the helium-xenon mixture was similar to that of air and relatively steep near the wall. These results show that the thermal resistance of liquid metal is evenly distributed on the radial section, indicating that heat transfer was dominated by molecular heat conduction. The thermal resistance of the helium-xenon mixture was concentrated in the limited near-wall region, and turbulent heat



Fig. 1 (Color online) Temperature profiles of different fluids

diffusion continued to be dominant. Due to the difference in flow and heat transfer, the heat transfer correlations from liquid metals could not directly be used to calculate flow and heat transfer for helium–xenon mixtures without verification. As such, there is a need to conduct in-depth research on the heat transfer correlation for helium–xenon mixtures within the Pr range.

This study evaluates the applicability of existing Nusselt number correlations to helium-xenon mixtures. Subsequently, a theoretical derivation of the Nusselt number correlation was conducted to establish a more general heat transfer correlation. Numerical simulations of the turbulent heat transfer for helium-xenon mixtures were also carried out; the simulated results were used to determine related parameters in the derived correlation. Finally, the effects of varying gas properties on turbulent heat transfer were considered, and a new property-variable heat transfer correlation was presented.

2 Application of existing correlations

Table 1 shows that this study selected common correlations and explored their applicability to helium-xenon mixtures. Among these correlations, DB and Colburn's correlations are empirical correlations obtained from experimental data relating to medium-high Pr fluids. Churchill's correlation was determined by numerically fitting experimental data for different fluids with a wide range of Prandtl numbers. The uncertainty associated with the above three correlations is provided in Table 1. The Lyon's and Stomquist's equations were frequently used for comparison in many studies on the convective heat transfer of liquid metals; however, clear uncertainties have rarely been provided for these equations in published literature. Additionally, it should be noted that the applicable range of most correlations in Table 1 was intentionally extrapolated to evaluate their suitability for helium-xenon mixtures.

Figure 2 shows that calculations were compared with experimental data [17]; the correlations from DB and Colburn had clearly overpredicted the Nusselt number of helium–xenon mixtures, while other calculations were underestimates compared with experimental data. Although predictions by Churchill's correlation were relatively closer to the experimental data when the Reynolds number (*Re*) equaled 84 000, it still exceeded the error limit of 10% at *Re* = 34 000. This indicates that the selected correlations from liquid metals are unsuitable for helium–xenon mixtures.

3 Theoretical derivation

To propose a more universal Nusselt number correlation for helium-xenon mixtures, a theoretical derivation based on the turbulent boundary layer theory and eddy viscosity hypothesis was conducted. Obtaining the temperature profile was key to deriving the theoretical heat transfer correlation [31]. First, the derivation processes for the general logarithmic law of temperature was briefly reviewed. Ignoring the varying properties of gas mixtures, the wall shear stress and wall heat flux in the fully developed region may be expressed as follows [32]:

$$\tau_{\rm w} = \rho(\nu + \varepsilon_{\rm m}) \frac{\partial \overline{u}}{\partial y},\tag{1}$$

$$q_{\rm w} = \rho C_{\rm p} (\alpha + \varepsilon_{\rm h}) \frac{{\rm d}(T_{\rm w} - \overline{T})}{{\rm d}y}. \tag{2}$$

A two-layer model was used to simplify the boundary layer [33], in which the flow boundary layer was divided into linear and logarithmic regions. Similarly, the thermal boundary layer was divided into molecular conduction and turbulent conduction regions. The linear and molecular conduction regions were dominated by molecular viscosity (v) and molecular heat diffusivity (α) , respectively, neglecting the effect of turbulent momentum diffusivity

| Name | Correlations | Range of application | Max deviation |
|------------------------------|---|------------------------|------------------|
| Dittus-Bolter et al. [14] | $Nu = 0.023 Re^{0.8} Pr^{0.4}$ | $0.7 \le Pr \le 120$ | $\pm 20\%$ |
| Colburn [15] | $Nu = 0.023 Re^{0.8} Pr^{1/3}$ | $0.5 \le Pr \le 100$ | $\pm 25\%$ |
| Churchill [29] | $Nu = 6.3 + \frac{0.079 Re \sqrt{f} Pr}{\left(1 + Pr^{0.8}\right)^{5/6}}$ 1/ $\sqrt{f} = 2.21 \cdot \ln(Re/7)$ | $0.001 \le Pr \le 200$ | $\pm 30\%$ |
| Stomquist et al. [27] | $Nu = 3.6 + 0.025(PrRe)^{0.8}$ | <i>Pr</i> <0.1 | _ |
| Lyon [24] | $Nu = 7.0 + 0.025 (PrRe/2)^{0.8}$ | <i>Pr</i> < 0.1 | _ |

 Table 1 Existing correlations

 of Nusselt number



Fig. 2 (Color online) Predictions of Nusselt number by various correlations

 (ε_m) and turbulent heat diffusivity (ε_h) . Conversely, turbulent diffusion predominates in the logarithmic and turbulent conduction regions, thus ignoring molecular viscosity and molecular heat diffusivity, respectively. Using the mixing-length theory [34], the profile of dimensionless velocity may be derived by integration as follows:

$$\begin{cases} U^{+} = y^{+}, & y^{+} \le y^{+}_{cirt} \\ U^{+} = \frac{\ln(y^{+}/y^{+}_{cirt})}{\kappa} + y^{+}_{cirt}, & y^{+} > y^{+}_{cirt} \end{cases}$$
(3)

where y_{cirt}^+ denotes the demarcation point between the linear and logarithmic regions, and κ denotes the Karman constant. Based on the definition of turbulent Prandtl number (Pr_t), Eq. (2) may be normalized, and the profile of dimensionless temperature can be acquired [35], as follows:

$$\begin{cases} T^{+} = Pry^{+}, & y^{+} \leq y^{+}_{cirt,T} \\ T^{+} = \frac{Pr_{t} \ln\left(y^{+} / y^{+}_{cirt,T}\right)}{\kappa} + Pry^{+}_{cirt,T}, & y^{+} > y^{+}_{cirt,T} \end{cases}$$
(4)

where $y_{cit,T}^+$ is the demarcation point between the molecular conduction and turbulent conduction regions, the above derivation is reported in several studies [36, 37]. Many empirical logarithmic laws of temperature have been proposed using regression analysis of experimental data. However, these empirical correlations lack a theoretical basis, and as such, their scope of application is limited. There is also an absence of relevant radial experimental data for helium–xenon gas mixtures in the current literature.

Therefore, in present investigation, the derivation of Eq. (4) was sequentially conducted to obtain the dimensionless mass-average temperature $(T_{\rm h}^+)$ and the theoretical

Nusselt number correlation. To facilitate this derivation, it was assumed that the thickness of the linear and molecular conduction regions was small and therefore negligible. One study reported that the demarcation point of the flow boundary layer was almost fixed [32]; that $is_{,y_{crit}}^+ = 11$, where y^+ was defined as per Eq. (5):

$$y^{+} = \frac{u_{\tau}y}{v} = \frac{\sqrt{\tau_{w}/\rho}}{v}y.$$
(5)

In which the wall shear (τ_w) may be expressed as follows:

$$\tau_{\rm w} = f \frac{\rho u_{\rm b}^2}{8}.\tag{6}$$

By adopting the Blasius formula to calculate the friction factor (*f*), the expression of y^+ can be simplified to Eq. (7):

$$y^{+} = \frac{u_{\rm b}\sqrt{f/8}}{v}y = \frac{0.2Re^{-0.125}u_{\rm b}}{v}y.$$
 (7)

In which $Re = \frac{u_b}{v_b}D$. The bulk viscosity (v_b) can be approximated to local viscosity (v), and as there is no significant difference in magnitude, then:

$$y^{+} = 0.2Re^{0.875} \frac{y}{D}.$$
 (8)

Using this, the ratio of the height of the demarcation point (*r*) from the wall to the diameter of the tube may be estimated. For $y_{\text{crit}}^+ = 11$, r/D = 0.035 when $Re = 10\ 000$ and r/D = 0.0086 when $Re = 50\ 000$; this indicates that the linear region in the tube is a very thin layer. As such, it is reasonable to dismiss the thickness of the linear region when carrying out integration derivation. Additionally, Zhao et al. [38] proposed a correlation to describe the relationship between $y_{\text{crit},T}^+$ and y_{crit}^+ as:

$$y_{\text{crit},\text{T}}^{+} = \frac{Pr_{\text{t}}}{Pr} y_{\text{crit}}^{+}.$$
(9)

Although this method may not be optimal for heliumxenon mixtures with $0.21 \le Pr \le 0.30$ (as per this study), it may be used as a rough estimate. When $Pr_t = 0.9$, then $y_{\text{crit,T}}^+$ is $33.0 < y_{\text{crit,T}}^+ < 47.1$; when combined with Eqs. (8) and (9), it is possible to calculate the ratio of the height of the demarcation point for the thermal boundary layer (r_t) from the wall to the diameter of the tube. Specifically, $r_t/D = 0.052 \sim 0.075$ when Re = 10000, and $r_t/D = 0.013 \sim 0.018$ when Re = 50000, which indicates that the molecular heat conduction region takes up a small proportion of area in the tube; as such, it is reasonable to ignore the thickness of the molecular heat conduction region.

At the wall surface (i.e., y = 0), the axial velocity is reduced to zero. At the center of tube (i.e., $y = r_0$), the axial velocity is the highest and can be described as follows:

$$U_{\rm max}^{+} = \frac{1}{\kappa} \ln r_0^{+} + y_{\rm crit}^{+} - \frac{1}{\kappa} \ln y_{\rm crit}^{+}.$$
 (10)

By integrating the area under Eq. (3), the expression of dimensionless area-average velocity, U_a^+ , can be obtained:

$$U_{\rm a}^{+} = \frac{\int_{0}^{r_0} U^{+} \cdot 2\pi r \mathrm{d}r}{\pi r_0^2}.$$
 (11)

By subtracting Eqs. (10) from (11), the following relationship can be obtained:

$$U_{\rm a}^{+} - U_{\rm max}^{+} = \frac{2}{\kappa r_0^2} \int_0^{r_0} \left[r \ln(r_0 - r) - r \ln r_0 \right] \mathrm{d}r = \frac{-3}{2\kappa}.$$
(12)

Similarly, expressions for dimensionless temperature at the center (T_{max}^+) can be derived as follows:

$$T_{\max}^{+} = \frac{Pr_{t}}{\kappa} \ln \frac{r_{0}^{+}}{y_{\text{crit, T}}^{+}} + Pr y_{\text{crit, T}}^{+}.$$
 (13)

Furthermore, the relationship between T_{max}^+ and the dimensionless area-average temperature, T_a^+ , can be obtained using Eq. (14):

$$T_{\rm a}^+ - T_{\rm max}^+ = \frac{-3Pr_{\rm t}}{2\kappa}.$$
 (14)

Combining Eqs. (12)–(14), the expression for T_a^+ can be derived using Eq. (15):

$$T_{a}^{+} = Pr_{t}U_{a}^{+} - \frac{Pr_{t}}{\kappa}\ln y_{\text{crit},\text{T}}^{+} + Pry_{\text{crit},\text{T}}^{+} + \frac{Pr_{t}}{\kappa}\ln y_{\text{crit}}^{+} - Pr_{t}\ln y_{\text{crit}}^{+}.$$
(15)

However, to derive the theoretical correlation of the Nusselt number, the T_b^+ still needs to be determined. Several previous studies [39] have assumed that T_{max}^+ as $T_{\rm b}^+$, because differences between the two parameters were < 10% for turbulent heat transfer as per Kays et al. [32]. In the present study, it was assumed that $T_{\rm b}^+ = T_{\rm a}^+$ to further reduce deviation. Specifically, a correlation to calculate $T_{\rm b}^+$ was proposed based on theoretical derivation [39], as:

$$T_{\rm b}^+ - T_{\rm max}^+ = \frac{3Pr_{\rm t}}{2\kappa^2} \sqrt{\frac{f}{8}} - \frac{3Pr_{\rm t}}{2\kappa}.$$
 (16)

Combine Eqs. (14) and (16), then,

$$\frac{T_{\rm a}^+}{T_{\rm b}^+} = 1 - \frac{3Pr_{\rm t}Nu}{2\kappa^2 RePr}.$$
(17)

Based on a working condition with $Re = 30\ 000$ and Pr = 0.30, where the Churchill correlation was used to calculate the Nusselt number, the second term on the right of Eq. (17) equals 0.030 and decreases with an increase in Re; as such, it was reasonable to assume $T_{\rm b}^+ = T_{\rm a}^+$.

Using the definition of Nusselt number and T^+ , the Nusselt number correlation may be transformed as follows:

$$\begin{cases} Nu = \frac{q_{\rm w}}{T_{\rm w} - T_{\rm b}} \frac{D}{\lambda} \\ T^+ = \frac{\rho C_{\rm p} u_{\rm r}(T_{\rm w} - T)}{q_{\rm w}} \Rightarrow Nu = \frac{RePr\sqrt{f/8}}{T_{\rm b}^+}, \\ f = 8\tau_{\rm w}/\rho u_{\rm b}^2 \end{cases}$$
(18)

where f is the friction factor. Combining Eqs. (15) and (18), the following theoretical expression for the Nusselt number was obtained:

$$Nu = \frac{RePr\sqrt{f/8}}{Pr_{\rm t}\sqrt{8/f} - \frac{Pr_{\rm t}}{\kappa}\ln y_{\rm crit,T}^+ + Pry_{\rm crit,T}^+ + \frac{Pr_{\rm t}}{\kappa}\ln y_{\rm crit}^+ - Pr_{\rm t}y_{\rm crit}^+}.$$
(19)

Equation (19) shows that the appropriate f, y_{crit}^+ , $y_{crit,\tau}^+$, and average Pr_t still need to be determined in order to obtain the new Nusselt number correlation. To do this, numerical simulations of turbulent heat transfer for helium-xenon mixtures were carried out, and these parameters were determined using the simulation results.

4 Numerical simulations

4.1 Thermophysical and transport properties

Accurate properties of helium–xenon mixtures are essential to numerical simulations. Typically, existing designs of space gas-cooled nuclear reactor systems have an operating pressure of 2 MPa and a temperature range of approximately 400–1300 K. Under these conditions, the theory of diluted gases based on the Chapman-Enskog approach was not found to be applicable. Tournier et al. [40] reviewed the experimental data for the properties of dense noble gas mixtures across a pressure range of 0.1–20 MPa and temperature up to 1400 K, summarizing semi-empirical property correlations. Based on the study of Tournier et al. [40], a property-calculation code for helium–xenon mixtures was developed, and its accuracy was validated in a previous study [30]. Figure 3 presents the calculation of variations in gas mixture properties with temperature.

These results indicate that variations in thermal conductivity (λ) and dynamic viscosity (μ) are proportional to temperature, while the specific heat capacity (C_p) and density (ρ) decrease as temperature increases. As the value of the specific heat capacity varies very slightly with temperature, it can be set as a constant for a particular gas mixture in numerical simulations. Other property correlations were implemented in Ansys Fluent using user-defined functions.

4.2 Numerical approach

The turbulent heat transfer of the helium-xenon mixtures in a tube was simulated using Ansys Fluent. The calculation model refers to the test section of the experiment in Taylor et al. [17], as shown in Fig. 4.

The experimental loop was a closed circuit driven by a gas booster pump; the flow and heat transfer of the gas mixture was investigated in the test section. Figure 4b shows the geometric parameters of the test section, which consisted of a straight tube with an inner diameter (D) of 5.87 mm. The total length (L) was 680.92 mm, of which the first part (L_1) was adiabatic, while the remainder was heated by a resistance wire to obtain uniform heat flux. A two-dimensional calculation domain was adopted, as the gravitational effect was neglected.



Fig. 3 (Color online) Variations in the properties of helium-xenon mixtures with temperature at P = 2 MPa

With respect to the simulated boundary conditions, the inlet was set to 'mass-flow-inlet', the outlet was set to 'pressure-out', and the wall surface adopted a non-slip boundary. The simulation parameters were set according to experimental conditions, listed in Table 2. A previous paper [41] evaluated the applicability of different turbulence models to helium-xenon mixtures; based on the findings from this paper, the *SST* κ - ω turbulence model was selected for this study. To improve the accuracy of the numerical simulation for turbulent heat transfer of low-*Pr* helium-xenon mixtures, a local *Pr*t model (shown in Eqs. (20) and (21)), was proposed in Zhou et al. [30]. Based on comparison with more experimental data, in this study the first constant in the expression of *Pr*t, ∞ was adjusted slightly from the original 0.80 to 0.86.

$$Pr_{t} = 1 / \left\{ \frac{1}{2Pr_{t,\infty}} + CPe_{t} \sqrt{\frac{1}{Pr_{t,\infty}}} - (CPe_{t})^{2} \\ \times \left[1 - \exp\left(\frac{-1}{CPe_{t} \sqrt{Pr_{t,\infty}}}\right) \right] \right\}$$
(20)

$$Pr_{t\infty} = 0.86 + \frac{30}{Re_{\text{local}}^{0.888} \cdot Pr}, \quad Re_{\text{local}} = \frac{u \cdot D}{v}$$
(21)

Grid independence verification was investigated to ensure the validity of the numerical calculation. Four mesh files were tested: mesh1: 20×3000 , mesh2: 30×4000 , mesh3: 40×6000 , and mesh4: 60×8000 . The bulk temperature ($T_{\rm b}$) of helium–xenon mixtures and the wall temperature ($T_{\rm w}$) at x = 0.45, 0.55, 0.60, and 0.65 m were calculated. Figure 5 shows that when the grid number exceeded 120 000, the difference in the bulk and wall temperatures as calculated by various meshes was negligible, and the maximum error was < 0.8%. Based on the calculation accuracy and efficiency, mesh3 was adopted in this study.

Figure 6 presents the grid of mesh3; the grid refinement was applied in the near-wall region, where the spacing of the first grid was 0.004 mm, spacing ratio was 1.08, and maximal aspect ratio was 28.4. Using the Run696 as an example, the corresponding y^+ of the first grid was y^+ = 0.22. In fact, the wall-resolved method was enabled for the SST κ - ω model when the mesh in the near-wall region was sufficient. At that time, the y^+ of the first grid was generally required to be < 1.0; as such, mesh3 was able to meet the calculation requirements. In order to further verify the simulation models adopted in the present study, additional experimental data on turbulent heat transfer for helium-xenon gas mixtures were introduced for comparison. Figure 7 shows that the calculated deviation of the wall temperature under various working conditions was essentially 5%. Therefore, the modified turbulence model

Fig. 4 (Color online) Test section of experiment for helium-xenon mixtures by Taylor et al. a Schematic diagram of experimental device; b Heated tube of test section

| P | Cooler 1 Plenum 1 Control Valve |
|-----------------|---|
| Pressure trap 2 | |
| Thermocouples | V Image: Section |
| Gas flow | Transducers Cooler 2 Plenum 2 Pressure Regulator Pump |
| | |
| | (a) |
| inle | t ΔP $pt2$ |
| | |
| D |) \longrightarrow \longrightarrow |
| - | L = 56D uniform heating section |
| - | I = 116D |
| | (b) |
| | <- <i>/</i> |

| Table 2 Simulation parameters of helium–xenon mixtures | M (g/mol) | No | Pr | Rei | $q (W/m^2)$ | $T_{\rm w}/T_{\rm b}$ (max) | P _{out} (Pa) |
|---|-----------|--------|------|--------|-------------|-----------------------------|-----------------------|
| | 14.5 | Run711 | 0.30 | 40,869 | 45,712 | 1.15 | 993,150 |
| | | Run712 | 0.30 | 40,554 | 97,522 | 1.32 | 982,023 |
| | | Run713 | 0.30 | 35,374 | 175,055 | 1.61 | 828,120 |
| | | Run714 | 0.30 | 38,962 | 242,158 | 1.84 | 810,084 |
| | | Run715 | 0.30 | 34,042 | 296,622 | 2.04 | 806,581 |
| | 28.3 | Run707 | 0.23 | 53,390 | 157,094 | 1.67 | 928,019 |
| | | Run709 | 0.23 | 48,411 | 240,978 | 2.06 | 835,497 |
| | | Run710 | 0.23 | 49,001 | 33,809 | 1.16 | 779,391 |
| | 40.0 | Run696 | 0.21 | 36,183 | 136,770 | 1.99 | 563,474 |
| | | Run698 | 0.21 | 34,280 | 78,313 | 1.59 | 502,892 |
| | | Run700 | 0.21 | 34,865 | 18,151 | 1.15 | 465,129 |
| | | Run704 | 0.21 | 59,851 | 31,593 | 1.17 | 771,105 |
| | | Run705 | 0.21 | 61,772 | 159,072 | 1.77 | 990,763 |
| | 83.8 | Run687 | 0.25 | 85,802 | 43,637 | 1.42 | 471,502 |
| | | Run689 | 0.25 | 87,373 | 96,326 | 1.89 | 480,141 |
| | | Run693 | 0.25 | 33,513 | 37,260 | 1.62 | 262,358 |
| | | | | | | | |

may be used to study the turbulent heat transfer characteristics of helium-xenon mixtures.

4.3 Determination of parameters

There is a need to analyze the momentum diffusion characteristics of helium-xenon mixtures to determine the

 y_{crit}^+ of the flow boundary layer. At the position of y_{crit}^+ , the turbulent momentum diffusion should be equal to the molecular momentum diffusion. Figure 8 shows that the molecular viscosity (v) and turbulent momentum diffusivity (ε_m) of helium–xenon mixtures with two different mixing ratios in the fully developed region (x = 600 mm)



Fig. 5 (Color online) Simulated bulk temperature (T_b) and wall temperature (T_w) with various meshes



Fig. 6 (Color online) Grid of mesh3



Fig. 7 (Color online) Comparison of simulated wall temperature (T_w) by the modified turbulent model and experimental data

were calculated. The results show that the intersections of the curves ($v = \varepsilon_m$) were always close to $y^+ = 11$; as such, the demarcation points of the flow boundary layer for the helium–xenon mixtures were determined to be $y_{crit}^+ = 11$.

Notably, the demarcation points of the flow boundary layer for water and air were also at approximately $y^+ = 11$ in Schlichting et al. [33]. These results indicate that Pr has little effect on the turbulent momentum diffusion of fluids.

To investigate the appropriate equation for the friction factor (*f*), four typical equations from conventional fluids [42-44] (see Table 3), were selected for testing. One study [16] suggested that the experimental friction factor of noble binary gases may be calculated using Eqs. (22)–(24). The term *f* is the overall average friction factor, calculated using the distance from the start of the heated section to the second pressure tap ('pt2' in Fig. 4):

$$\Delta P_{\rm f} = \Delta P - \Delta P_{\rm m},\tag{22}$$

$$\Delta P_{\rm m} = \frac{G^2 R_{\rm g}}{g_{\rm c}} \left(\frac{T_{\rm b2}}{P_2} - \frac{T_{\rm b1}}{P_1} \right),\tag{23}$$

$$f = \frac{2\Delta P_{\rm f}\overline{\rho}D}{lG^2},\tag{24}$$

where ΔP denotes the total pressure drop; $\Delta P_{\rm m}$ denotes the acceleration pressure drop; and $\Delta P_{\rm f}$ denotes the friction pressure drop. Figure 9 shows that the calculated values were relatively close to each other and calculation errors were generally within 5%. Based on the good agreement between the experimental data and the concise expression, the Blasius equation was used to calculate the friction factor for helium–xenon mixtures in Eq. (19).

Notably, the Karman constant (κ) exists in the velocity distribution (see Eq. (3)); as such, the simulated velocity distribution may be numerically fitted to determine κ for helium–xenon mixtures. Figure 10a shows that the fitting logarithmic law of velocity for Pr = 0.30 was initially obtained by setting $\kappa = 0.42$. Then, a comparison between the calculated logarithmic law with $\kappa = 0.42$ and the simulation results of Pr = 0.21 was conducted as shown in Fig. 10b. The calculated logarithmic law with $\kappa = 0.38$ was still in good agreement with the simulation results; as such, $\kappa = 0.42$ was adopted in Eq. (19).

To obtain the appropriate demarcation point of the thermal boundary layer for helium–xenon mixtures, variations in the molecular heat diffusivity (α) and the turbulent heat diffusivity (ε_h) were calculated. In this study, the criterion for defining $y_{crit,T}^+$ is that $\alpha = \varepsilon_h$. Figure 11 shows that it is evident that the demarcation points were not fixed for different *Prs*; the smaller the *Pr*, the farther the demarcation point from the wall. Therefore, it was necessary to establish a model of turbulent thermal diffusivity for helium–xenon mixtures to fix the value of $y_{crit,T}^+$.

The Reynolds-averaged Navier–Stokes equations (RANS) model was developed based on the Boussinesq eddy viscosity hypothesis [45], in which eddy viscosity was calculated using mixing-length theory; as such, ε_m/v



Fig. 8 (Color online) Variations in molecular viscosity (ν) and turbulent momentum diffusivity (ε_m) with y^+ for helium-xenon mixtures

Table 3 Four equations used todetermine the friction factor

| Name | Formula | Range of application |
|------------------------|--------------------------------------|--|
| Blasius [42] | $f = 0.316/Re^{0.25}$ | $5.0 \times 10^3 \le Re \le 10^5$ |
| Colebrook [43] | $1/\sqrt{f} = 1.8 \lg(Re/6.9)$ | $5.0 \times 10^3 \le Re \le 5.0 \times 10^7$ |
| Drewr [32] | $f = 0.0056 + 0.5 \times Re^{-0.32}$ | $3.0 \times 10^3 \le Re \le 3.0 \times 10^6$ |
| Taitel and Dukler [44] | $f = 0.184/Re^{0.2}$ | $Re \ge 3.0 \times 10^3$ |



Fig. 9 (Color online) Comparison of calculations using different friction factor equations with experimental data

was a function related only to y^+ . By analyzing the Pr_t model (Eqs. (20) and (21)), the Pr_t was also found to be a function only related to y^+ for a specific helium–xenon mixture, as Pe_t and Re_{local} were expressed as functions of y^+ . Therefore, the following relationship can be obtained:

$$\frac{\varepsilon_{\rm h}}{v} = \frac{\varepsilon_{\rm m}}{v} \frac{1}{Pr_{\rm t}} = f(y^+). \tag{25}$$

It is feasible to obtain Eq. (25) by numerical fitting. Since the thermal boundary layer of low-*Pr* helium–xenon mixture was thicker than the flow boundary layer, the fitting range of y^+ should ensure that $y^+ > 11$. First, the expression was initially determined by fitting the simulation results of Pr = 0.30, as shown in Fig. 12(a); this expression could be described by Eq. (26). Thereafter, the cases with the Prandtl number to 0.23 and 0.21 were simulated. The results are shown in Fig. 12b and c, where it is shown that the calculation of Eq. (26) was in good agreement with simulated data points. Therefore, for $0.21 \le Pr \le 0.30$ and $y^+ < 50$, Eq. (26) was able to reasonably describe the turbulent heat diffusivity of helium-xenon mixtures.

$$\frac{\varepsilon_{\rm h}}{\nu} = 0.0112(y^+)^{1.82} \tag{26}$$

As $\alpha = v/Pr$, the $y_{\text{crit,T}}^+$ may be determined as follows:

$$v_{\rm crit,T}^+ = 11.83 Pr^{-0.55}.$$
 (27)

The influence of the *Pr* on turbulent heat transfer may be quantitatively analyzed using the numerical model of the turbulent heat diffusivity. For air and water, the relationship between turbulent heat diffusivity and y^+ was $\varepsilon_h/v \propto$ $(y^+)^3$ [36, 37]. Compared with Eq. (26), it can be seen that the lower Prandtl number will lead to a smaller turbulent thermal diffusivity at the same radial position, resulting in weak convective heat transfer intensity. This conclusion may be used to understand the results in Fig. 2. The Prandtl number of liquid metals was lower than that of the binary



Fig. 10 (Color online) Fitted velocity profiles of helium-xenon mixtures



Fig. 11 (Color online)Variations in molecular heat diffusivity (α) and turbulent heat diffusivity (ϵ_h) with the y^+ of helium–xenon mixtures

gas mixtures. As the turbulent heat diffusion was weaker for the same Re, the Nusselt number correlations obtained from liquid metals underestimated the experimental data of binary gas mixtures. Conversely, the Pr of conventional fluids was higher than that of helium–xenon mixtures, resulting in more intense turbulent heat diffusion. Therefore, predictions from the DB and Colburn correlations represented overestimations of experimental data.

For working fluids with $Pr \gg 1.0$, the Pr_t tends to be constant of 0.85. Differently, for low-Pr fluids it tends to be larger [46]. However, it is difficult to obtain the section average Pr_t directly from the local function expressions. In this study, a constant value was used to roughly represent the section average Pr_t . To determine this parameter, the radial Pr_t distribution of Run711 was first calculated using



Fig. 12 (Color online) Numerical fitting of the expression for $\varepsilon_{\rm h}/v$



Fig. 13 Radial predictions of Pr_t at x = 0.60 m for Run711

the Zhou Pr_t model (Eqs. (20) and (21)). As shown in Fig. 13, by comparing with the prediction of the Zhou model, a Pr_t of 0.9 may be a candidate value; as such, this value was initially adopted as the section average Pr_t in Eq. (19) for the helium–xenon mixtures.

Once all parameters were determined, the semi-theoretical Nusselt number correlation was quantified using Eq. (28):

$$Nu = \frac{0.20PrRe^{0.875}}{4.53Re^{0.125} + 11.83Pr^{0.45} + 1.18\ln Pr - 10.05}.$$
(28)

5 Verification and analysis

Combining Eqs. (4) and (27), the radial temperature distribution of the corresponding helium–xenon mixtures with $0.21 \le Pr \le 0.30$, was determined as shown in Eq. (29):

$$\begin{cases} T^{+} = Pry^{+}, & y^{+} \le 11.83Pr^{-0.55} \\ T^{+} = 3.02(\ln y^{+} + 0.55\ln Pr - 2.47) + 11.83Pr^{0.45}, & y^{+} > 11.83Pr^{-0.55} \end{cases}$$
(29)

To verify the correctness of the derivation, the results using Eq. (29) where the *Pr* was 0.20, were compared with predictions of other temperature distribution correlations (shown in Table 4). Additionally, the direct numerical simulation results of Kawamura et al. [47] were introduced as benchmarks. As shown in Fig. 14, the calculations using Eq. (29) showed better agreement with the DNS results than those of other correlations, preliminarily proving the validity of Eq. (29) and the correctness of this derivation.

To test the accuracy of the new Nusselt number correlation, predictions using Eq. (28) were compared with experimental data, as shown in Fig. 15. The results show that the calculations are in good agreement with the experimental data of gas mixtures under both working conditions. This further verifies the accuracy of the derivation and its applicability to the turbulent heat transfer for helium-xenon mixtures where $0.21 \le Pr \le 0.30$.

A sensitivity analysis of each parameter was carried out to evaluate the influence of the parameters that were determined on the derived Nusselt number correlation. As the empirical range of the Karman constant was $0.43 \ge$ $\kappa > 0.35$, five different constants were tested. Figure 16 shows that even when the value of κ fluctuates in the entire empirical range, there is only a very small difference between the calculated Nusselt numbers. This proves that the calculation of the derived Nusselt number correlation is insensitive to κ . Thereafter, three additional constant Pr_t numbers were used for sensitivity analysis. Figure 17 demonstrates that the derived Nusselt number correlations vary with different Pr_t numbers. The predictions where Pr_t was 0.9 were in better agreement with experimental data, in which a fluctuation of 0.9 ± 0.1 appeared to be valid. Additionally, an error propagation analysis of the demarcation point of the flow boundary layer (y_{crit}^+) was also conducted. Figure 18 shows that the calculations where $y_{\text{crit}}^+ = 11$ were closer to experimental data, in which a fluctuation of 11 ± 1.0 may also be considered reasonable.

Importantly, helium-xenon mixtures flow through a strongly heated tube in space gas-cooled reactors; under these conditions, the wall temperature is generally higher than the bulk temperature. The physical properties of helium-xenon mixtures at different radial positions will be significantly different. Therefore, the Nusselt number

| Table 4 | Existing correlations | |
|----------|-----------------------|--|
| for temp | erature distribution | |

| Authors | Correlations | Scope of application |
|------------------------|---|----------------------|
| Duponcheel et al. [19] | $T^{+} = 5\ln(1 + 0.2Pry^{+})$ | 0.01 < Pr < 0.025 |
| Kader [31] | $T^{+} = 2.12 \ln y^{+} + (3.85Pr^{1/3} - 1.3)^{2} - 2.12 \ln Pr$ | 0.025 < Pr < 170 |
| Gowen and Smith [35] | $T^{+} = 2.5 \ln y^{+} + 5 \ln[(5Pr + 1)/30] + 8.55 + 5Pr$ | 0.7 < Pr < 14.3 |
| Kays et al. [32] | $T^+ = 2.2 \ln y^+ + 13.39 Pr^{2/3} - 5.66$ | 0.7 < Pr < 5.9 |



Fig. 14 (Color online) Comparison of prediction by Eq. (29) with other temperature correlations

correlation deduced from constant physical properties may need to be modified. To examine the influence of variable properties on turbulent heat transfer, the Nusselt number calculated using Eq. (28) (Nu_{cp}) was compared with measured experimental data (Ref. [13]) (Nu_{Exp}) at various temperature ratios, as shown in Fig. 19.

The ranges of the tested conditions are 18,000 < Re < 60,000, $0.21 \le Pr \le 0.30$, and $T_w/T_b < 2$. Figure 19 shows that the calculation deviation becomes obvious as T_w/T_b increases. When $T_w/T_b = 1.8$, the numerical error exceeded 30%. A seemingly exponential relationship between $Nu_{\rm Exp}/Nu_{\rm cp}$ and T_w/T_b was observed when $Nu_{\rm Exp}/Nu_{\rm cp}$ were presented in logarithmic coordinates. Moreover, as T_w/T_b approaches 1.0, the constant-property correlation would become valid, and $Nu_{\rm Exp}/Nu_{\rm cp}$ would approach 1.0. Therefore, by numerically fitting data points, the modified,



Fig. 16 (Color online) Sensitivity analysis on the Karman constant for the derived Nusselt number correlation

property-variable Nusselt number correlation for the helium-xenon mixture was obtained as follows:

$$Nu_{\rm vp} = \frac{0.20PrRe^{0.8/5}}{4.53Re^{0.125} + 11.83Pr^{0.45} + 1.18\ln Pr - 10.05} \cdot \left(\frac{T_{\rm w}}{T_{\rm b}}\right)^{-0.63}.$$
(30)

The results in Fig. 19 show that the error in predictions using Eq. (30) was almost within 5% and was consistently within 10%. Notably, the modified Nusselt correlation (Eq. (30)) was only determined using existing and limited experimental data; to fully verify its applicability, additional experimental data are required.



Fig. 15 (Color online) Comparison of predictions using Eq. (28) with experimental data



Fig. 17 (Color online) Sensitivity analysis on the Pr_t number for derived Nusselt number correlation



Fig. 18 (Color online) Sensitivity analysis on the y^+_{crit} for derived Nusselt number correlation

6 Conclusion

This study evaluates existing Nusselt number correlations and derives a theoretical Nusselt number correlation for helium–xenon mixtures using turbulent boundary layer theory. The turbulent heat transfer in a tube for helium– xenon mixtures was numerically simulated using a modified Pr_t model. Based on numerical simulations, a new turbulent heat diffusivity model in the thermal boundary layer was developed, and related parameters were determined in the theoretical Nusselt number expression. A preliminary modified Nusselt number correlation was



Fig. 19 (Color online) Comparison of calculations using Eq. (28) with measured experimental data

presented based on variations in gas properties. The main conclusions from this study were:

- (1) Compared with the extrapolated experimental data of gas mixtures, the selected Nusselt number correlations from conventional fluids and liquid metals were unsuitable for helium-xenon mixtures with $0.21 \le Pr \le 0.30$;
- (2) The calculation error of the friction factor for helium-xenon mixtures using the Blasius equation was within 5%, and the demarcation point of U⁺ for helium-xenon mixtures and conventional fluids was approximately equal;
- (3) For $0.21 \le Pr \le 0.30$ and $y^+ < 40$, $\varepsilon_{\rm h}/v = 0.0112(y^+)^{1.82}$ can reasonably describe the turbulent heat diffusivity of helium–xenon mixtures. A lower Pr led to a reduced turbulent thermal diffusivity, resulting in weak convective heat transfer intensity;
- (4) Predictions by the derived semi-theoretical Nusselt number correlation were in good agreement with experimental data, verifying the accuracy of the derivation and its applicability to turbulent heat transfer for helium–xenon mixtures.

Future research should focus on obtaining additional experimental research on low-*Pr* helium–xenon mixtures. This additional experimental data may be used to verify correlations proposed in this study. Furthermore, the new Nusselt number correlation should be used to develop a thermal–hydraulic transient analysis code for helium–xenon cooled reactor systems.

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Author contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Biao Zhou, Yu Ji, Jun Sun and Yu-Liang Sun. The first draft of the manuscript was written by Biao Zhou and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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