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Predominance of linear Q and μ systematics in random-interaction ensembles

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Abstract In random-interaction ensembles, the electric quadrupole moments (Q) and magnetic moments (μ) of the $I^{\pi} = 11/2^{-}$ isomers of the Cd isotopes predominantly exhibit a linear correlation with the neutron numbers, corresponding to the recently emphasized linear Q and μ systematics in realistic nuclear systems. Although the seniority scheme enhances such predominance (more essentially for μ), the configuration mixing due to quadrupole-like and δ-force-like proton–neutron (pn) interactions is responsible for the linear Q and μ systematics, respectively, in realistic nuclear system, as well as random-interaction ensembles.

Keywords Random interaction · Electromagnetic moment · Linear systematics · Proton–neutron interaction

1 Introduction

Nuclear low-lying spectra and electromagnetic properties are highly regulated with simple patterns. For example, doubly even nuclei always have $I^{\pi} = 0^{+}$ ground states and $I^{\pi} = 2^{+}$ first excited states, with few exceptions. Another example is that 2_1^{+} quadrupole moments of doubly even nuclei are almost proportional to the 2_1^{+} ones with a unified ratio of -1 across the whole nuclide chart [1]. These regularities are also predominant in an ensemble of the nuclear structural model, where all the two-body interaction matrix elements are random numbers [2, 3]. Such predominant patterns in random-interaction ensembles demonstrate how simple regularity emerges out of complex nuclei, even with interactions mostly deviating from reality [4–8]. It is also feasible to summarize the interaction property of certain robust nuclear patterns via an interaction sampling for such patterns within random-interaction ensembles.

As recently reported, the Q and μ values of the $I^{\pi} = 11/2^{-}$ isomers of the neutron-rich cadmium isotopes are also regulated by simple linear systematics [9], where the Q linear systematics is attributed to the seniority scheme beyond the neutron $h_{11/2}$ orbit. Several theoretical efforts to understand such linear systematics have been made, such as the BCS with quadrupole–quadrupole force [10], the density functional theory [11], and the schematic shell-model description [12]. In particular, Ref. [11] further attributed the simpler Q linear systematics to pairing. However, it is still a challenge to explain how such simplicity exists in complex Cd isotopes [13], and whether there are other possible explanations besides the seniority scheme or pairing. As mentioned above, a random-interaction study may help to answer such questions. Therefore,

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our work aims to probe the potential linear Q and μ systematics in random-interaction ensembles, and clarify its interaction properties, if any. Previous random-interaction studies normally focused on the simple regularities of an individual doubly even nucleus. Therefore, random-interaction calculation is an innovative approach for the study on the systematics of odd-mass isotopes.

2 Calculation framework

Our random-interaction calculations are based on the shell model and include all the odd-mass Cd isotopes with $A = 113$–129. The single-particle space is limited in the $Z = 40$–50 and $N = 64$–82 shells with the $\pi 0g_{9/2}$, $\nu 2s_{1/2}$, $\nu 1d_{3/2}$ and $\nu 0h_{11/2}$ single-particle orbits. No further truncation is introduced. Degenerated single-neutron energies are adopted as in Ref. [9]. The two-body interaction is randomized within the two-body random ensemble (TBRE) [14–16]. In other words, the two-body interaction elements (denoted by V^J_{ijkl}) are all Gaussian random numbers constrained by

$$\begin{aligned} &\left\langle V^J_{ijkl} \right\rangle = 0, \\ &\left\langle V^J_{ijkl} V^{J'}_{i'j'k'l'} \right\rangle = \delta_{ii'}\delta_{jj'}\delta_{kk'}\delta_{ll'}\delta_{JJ'}(1+\delta_{ik}\delta_{jl}), \end{aligned} \tag{1}$$

where the brackets of $\langle\rangle$ denote average, J labels the interaction rank, and the i, j, k, and l indexes can equal 1, 2, 3, and 4, corresponding to the $g_{9/2}$, $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ orbits, respectively. For example, V^J_{1414} represents the pn interaction between the $g_{9/2}$ and $h_{11/2}$ orbits with rank J.

We generate 3,000,000 sets of random two-body interaction elements, and input them into the shell-model code [17] to calculate the Q and μ values of the $11/2^-_1$ states for all the $^{113-129}$Cd isotopes. For the Q calculation, the effective charges are set as $e_\pi = 1.5e$ and $e_\nu = 0.5e$. For the μ calculation, the single-particle g factors are set as $g_{\pi s} = 5.586 \times 0.7\mu_N$, $g_{\pi l} = 1\mu_N$, $g_{\nu s} = -3.826 \times 0.7\mu_N$, and $g_{\nu l} = 0$, where the spin g factors are conventionally quenched with a factor of 0.7.

To quantitatively describe the Q (μ) systematics, we introduce the Pearson correlation coefficient [18] [denoted by ρ_Q (ρ_μ)] as a measure of the linear correlation between the Q (μ) values and neutron numbers, which has a value between ± 1, where 1, 0, and -1 correspond to perfect positive linear correlation, no linear correlation, and perfect negative linear correlation, respectively. Its magnitude represents the degree of linearity. For instance, the experimental Q values of the Cd $I^\pi = 11/2^-$ isomers [9] give $\rho_Q = 0.997$, which is very close to 1, and thus reflects the positive linear Q systematics as observed. In the following random-interaction analysis, we focus on the positive linear systematics with $\rho_Q > 0.9$ or $\rho_\mu > 0.9$, because it agrees with the experimental reality [9].

For the random-interaction ensemble, the distribution of the calculated ρ_Q and ρ_μ is adopted to comprehensively describe the degree of Q and μ linearity, as denoted by $P(\rho_Q)$ and $P(\rho_\mu)$, respectively. Before any further investigation, it should be noted that random numbers can also have a considerable possibility to accidentally exhibit linear systematics, which introduces unexpected interference into our analysis. To eliminate this interference, the calculated $P(\rho_Q)$ and $P(\rho_\mu)$ should be normalized with the Pearson coefficient distribution of 9 random numbers.[1] Such a distribution can be taken as the background of the current analysis and is denoted by $P_{\rm bkg}$. The $P_{\rm bkg}$ is calculated with 25,000,000 sets of 9 random numbers from the normal distribution and is illustrated in Fig. 1a.

3 Linear Q systematics

The $P(\rho_Q)$ from the TBRE is shown in Fig. 1a and is compared with the $P_{\rm bkg}$. The TBRE $P(\rho_Q)$ is totally different from the $P_{\rm bkg}$ and maintains two major peaks around $\rho_Q = \pm 1$. Thus, the Q values from the TBRE potentially exhibit linear systematics with dominant possibility. We normalize the TBRE $P(\rho_Q)$ with the $P_{\rm bkg}$ as shown in Fig. 1b, where two peaks of $P(\rho_Q > 0.9)/P_{\rm bkg} \simeq 100$ and $P(\rho_Q < -0.9)/P_{\rm bkg} \simeq 50$ emerge, and thus quantitatively demonstrate the predominance of the linear Q systematics in the TBRE.

The linear Q systematics was previously attributed to the seniority scheme beyond the $h_{11/2}$ orbit [9]. This may also explain the predominance of the linear Q systematics in the TBRE, because a random-interaction ensemble favors seniority-like behavior [19, 20]. However, several arguments may challenge this explanation. Firstly, the predominance of the seniority scheme in random-interaction ensembles is based on the $J = 0$-ground-state situation of doubly even nuclei, which is inapplicable to the odd-mass Cd isotopes. Secondly, the Cd isotopes under investigation are all non-magic nuclei, whose pairing collectivity is supposedly depressed by pn interaction. Thirdly, the TBRE with degenerated single-particle energies, as adopted in this case, does not necessarily favor the seniority scheme [20]. In other words, the seniority-scheme interpretation of the linear Q systematics in the TBRE is still controversial.

To investigate the relationship between the seniority scheme and linear Q systematics in random-interaction

[1] The present study involves 9 Cd isotopes.

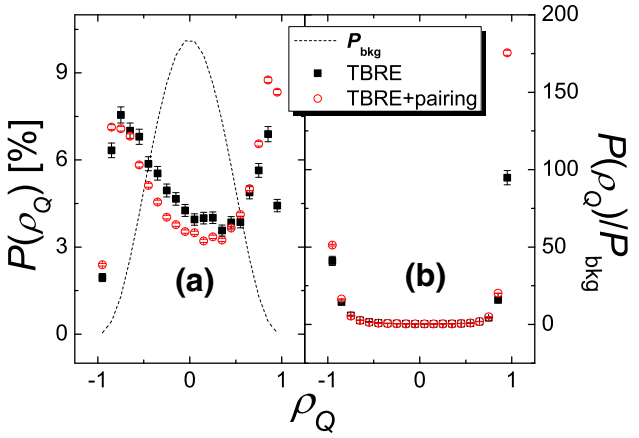

Fig. 1 $P(\rho_Q)$ in the TBRE and TBRE+pairing, compared with $P_{\rm bkg}$ (see text for definition). **a** provides a straightforward comparison. In **b**, the $P(\rho_Q)$ is normalized with the $P_{\rm bkg}$. The error bar represents the statistic error (Color online)

ensembles, we introduce an extra pairing force between like nucleons into the TBRE as

$$V^{J=0}_{j_1 j_2 j_1 j_2} = -V_0(2j_1+1)(2j_2+1), \quad (2)$$

where the pairing strength V_0 is equal to 0.01 to match the magnitude of the TBRE elements. Such a generated random-interaction ensemble is denoted by "TBRE+pairing", where the low-lying nuclear states are supposed to be predominately governed by the seniority scheme. In Fig. 1, the $P(\rho_Q)$ of the TBRE+pairing is presented. It can be seen that it has a shape similar to that of the TBRE; however, the TBRE+pairing $P(\rho_Q > 0.9)$ is twice of the TBRE one. Thus, the seniority scheme indeed reinforces the positive linear Q systematics as claimed in Ref. [9].

To demonstrate the interaction properties of the positive linear Q systematics, we calculate the average values of V^J_{ijkl}, which can reproduce $\rho_Q > 0.9$ in the TBRE and TBRE+pairing. These average values are presented as $\langle V^J_{ijkl}\rangle$ in Fig. 2 following the order defined in Table 1. It can be seen that $\rho_Q > 0.9$ does not require strong attractive $V^{J=0}_{iijj}$ elements, *i.e.,* the pairing in the TBRE, which means that the pairing may be inessential for the positive linear Q systematics. On the other hand, the TBRE $\langle V^J_{1414}\rangle$ values with $\rho_Q > 0.9$ obviously follow the parabolic rule [21] as analytically expressed by

$$V^J_{1414} = A[J(J+1)]^2 + BJ(J+1) + C, \quad (3)$$

where A, B, and C are constant factors independent of J. A schematic parabola, in blue, is illustrated in Fig. 2 to emphasize the regularity. The parabolic evolution of V^J_{1414} implies the quadrupole nature of the *pn* interaction between the $g_{9/2}$ and $h_{11/2}$ orbits [22]. Thus, even if the seniority scheme does not present, the positive linear Q systematics

Table 1 Order of $\langle V^J_{ijkl}\rangle$ presented in Figs. 2 and 4. The numbers, *i.e.*, 1, 2, 3 and 4, in the *ijkl* columns correspond to the $\pi 0g_{9/2}$, $\nu 2s_{1/2}$, $\nu 1d_{3/2}$ and $\nu 0h_{11/2}$ orbits, respectively, as mentioned in Sect. 2

Order	*ijkl*	*J*	Order	*ijkl*	*J*	Order	*ijkl*	*J*
1	1111	0	17	3333	0	33	1213	4
2	1111	2	18	3333	2	34	1213	5
3	1111	4	19	3344	0	35	1313	3
4	1111	6	20	3344	2	36	1313	4
5	1111	8	21	3434	4	37	1313	5
6	2222	0	22	3434	5	38	1313	6
7	2233	0	23	3434	6	39	1414	1
8	2244	0	24	3434	7	40	1414	2
9	2323	1	25	4444	0	41	1414	3
10	2323	2	26	4444	2	42	1414	4
11	2333	2	27	4444	4	43	1414	5
12	2344	2	28	4444	6	44	1414	6
13	2424	5	29	4444	8	45	1414	7
14	2424	6	30	4444	10	46	1414	8
15	2434	5	31	1212	4	47	1414	9
16	2434	6	32	1212	5	48	1414	10

is still evident with quadrupole-like *pn* interaction in the TBRE. On the other hand, the $\rho_Q > 0.9$ value in the TBRE+pairing presents a similar $\langle V^J_{1414}\rangle$ evolution to that in the TBRE. This means that the quadrupole nature of the *pn* interaction should be maintained for the positive linear Q systematics, irrespectively of whether the seniority scheme presents or not. Moreover, the theoretical reproduction of the linear Q systematics is always beyond a quadrupole perturbation [10, 12]. Therefore, it can be concluded that, in random-interaction ensembles, the quadrupole-like *pn* interaction is responsible for the predominance of the positive linear Q systematics, and the seniority scheme is not a determinant but a boost for this predominance, at least in the TBRE.

4 Linear μ systematics

In Fig. 3 the $P(\rho_\mu)$ in the TBRE and TBRE+pairing, normalized with $P_{\rm bkg}$, is presented. The TBRE $P(\rho_\mu)/P_{\rm bkg}$ is always close to 1, which demonstrates that it is not characterized by the linear μ systematics. In the TBRE+pairing ensemble, the pairing significantly increases the $P(|\rho_\mu| > 0.9)$, corresponding to the predominance of the linear μ systematics. Thus, the pairing is essential for the linear μ systematics. However, the $P(|\rho_\mu| > 0.9)/P_{\rm bkg}$ values in the TBRE+pairing are still smaller than the

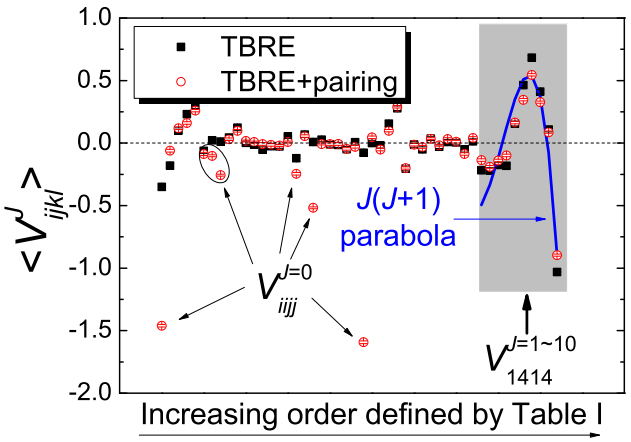

Fig. 2 $\langle V^J_{ijkl}\rangle$ for $\rho_Q > 0.9$ in the TBRE and TBRE+pairing. All the data are organized in the order defined in Table 1. The gray area highlights the *pn* interaction between the $g_{9/2}$ and $h_{11/2}$ orbits as labeled by V^J_{1414}, where a schematic parabolic curve of Eq. (3) is provided as a visual guide. The $V^{J=0}_{iijj}$ elements in the TBRE+pairing are most attractive, because of the additionally introduced pairing force defined in Eq. (2). The dashed line represents the ensemble average of the TBRE. The error bar represents the statistic error (Color online)

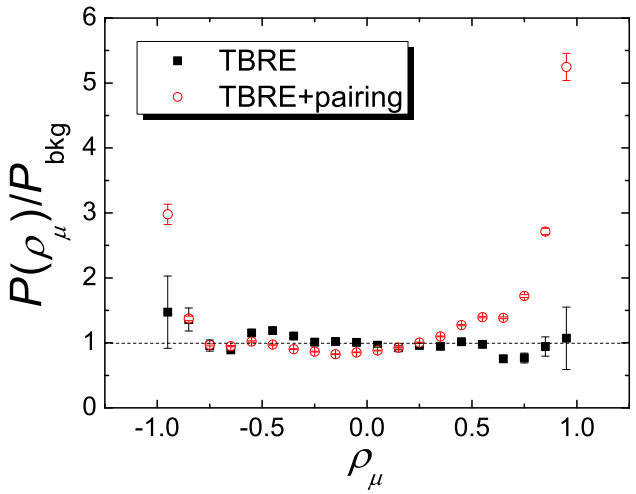

Fig. 3 $P(\rho_\mu)$ in the TBRE and TBRE+pairing, normalized with $P_{\rm bkg}$. The dashed line highlights the exact $P_{\rm bkg}$. The error bar represents the statistic error (Color online)

$P(|\rho_Q| > 0.9)/P_{\rm bkg}$ in both TBRE and TBRE+pairing (see Fig. 1b), which means that the linear μ systematics is always less evident than the Q systematics in random-interaction ensembles, similar to the realistic situation.

As mentioned in Ref. [9], the $h_{11/2}$ seniority scheme can only provide a constant μ value independent of the neutron number. Configuration mixing with proton excitation has to be introduced to reproduce the linear μ systematics [12]. To demonstrate the property of such configuration mixing in the TBRE+pairing, we also calculate the average values

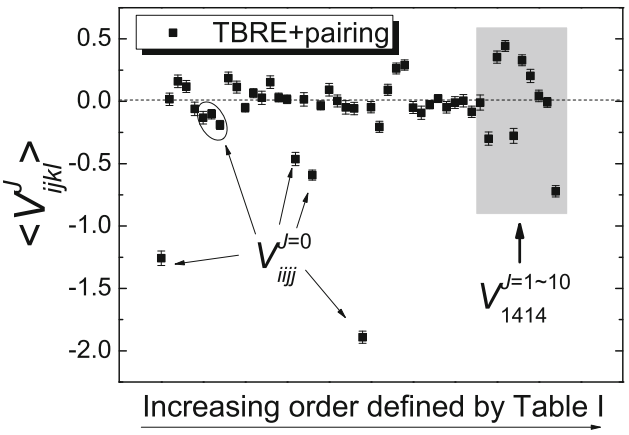

Fig. 4 As in Fig. 2, but for $\rho_\mu > 0.9$ in the TBRE+pairing

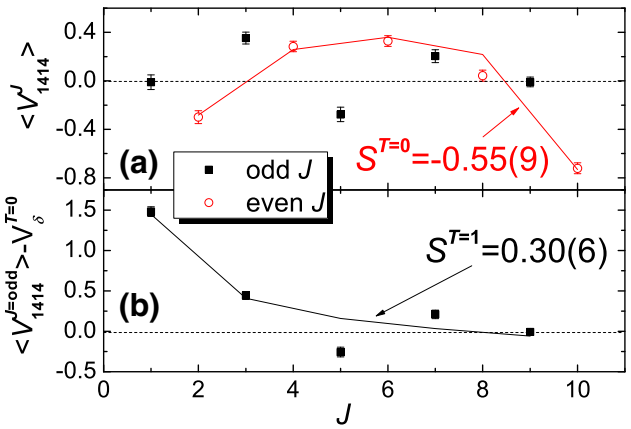

Fig. 5 $\langle V^J_{1414}\rangle$ for $\rho_\mu > 0.9$ in the TBRE+pairing. The solid lines represent the fitting to the δ force (see text for detail). Panel (a) provides an overall description, where the $\langle V^{J=\rm even}_{1414}\rangle$ and $\langle V^{J=\rm odd}_{1414}\rangle$ follow different regularities, and thus, are presented separately. Panel (b) presents the $\langle V^{J=\rm odd}_{1414}\rangle$ excluding the $T = 0$ component of the δ force with the same $S^{T=0}$ as for $\langle V^{J=\rm even}_{1414}\rangle$. The error bar represents the statistic error (Color online)

of V^J_{ijkl}, which can reproduce $\rho_\mu > 0.9$ in the TBRE+pairing, and present them as $\langle V^J_{ijkl}\rangle$ in Fig. 4, following the order defined in Table 1. Similar to the positive linear Q systematics, the positive linear μ systematics also requires some specific structure of $\langle V^J_{1414}\rangle$, which is responsible for the configuration mixing with proton excitation.

Let's take a close look at the $\langle V^J_{1414}\rangle$ with $\rho_\mu > 0.9$ in the TBRE+pairing, as shown in Fig. 5a. The $\langle V^{J=\rm even}_{1414}\rangle$ evolution presents an obvious parabolic feature against J, while the odd-J evolution seems less regulated. Such an odd–even difference also characterizes the δ force between the $g_{9/2}$ proton and $h_{11/2}$ neutron. More specifically, the even-J interaction element of the δ force has only the $T = 0$ contribution, according to Eq. (4.15) in Ref. [22], which also follows a parabolic evolution due to the short-range

property of the δ force. The odd-J interaction element, on the other hand, includes both the $T = 0$ and $T = 1$ components. The $T = 1$ component diminishes as $J \to J_{\max}$ because of the Pauli's principle, which is totally different from the $T = 0$ evolution. Thus, the odd-J interaction elements present a more complicated evolution than the even-J ones.

To quantitatively demonstrate the *pn*-interaction property of the positive linear μ systematics in the TBRE+pairing, the $\langle V^{J}_{1414}\rangle$ in Fig. 5 are further fitted to a unified δ force. Analytically, the even-J interaction element of the δ force between the $g_{9/2}$ proton and $h_{11/2}$ neutron is written as

$$V_{\delta}^{J=\text{even}} = S^{T=0}\left\{1+\frac{121}{J(J+1)}\right\} \times \begin{pmatrix} 9/2 & 11/2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2, \tag{4}$$

and the odd-J interaction element is written as

$$V_{\delta}^{J=\text{odd}} = V_{\delta}^{T=0} + V_{\delta}^{T=1} \tag{5}$$

with

$$V_{\delta}^{T=0} = \frac{S^{T=0}}{J(J+1)} \begin{pmatrix} 9/2 & 11/2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \tag{6}$$

and

$$V_{\delta}^{T=1} = S^{T=1} \begin{pmatrix} 9/2 & 11/2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2, \tag{7}$$

where $S^{T=0}$ and $S^{T=1}$ are the interaction strengths for $T = 0$ and $T = 1$ components, respectively. With $S^{T=0}$ and a unified interaction offset as the fitting variables, the $\langle V^{J=\text{even}}_{1414}\rangle$ is fitted to Eq. (4) as shown in Fig. 5a. It can be seen that Eq. (4) describes the $\langle V^{J=\text{even}}_{1414}\rangle$ evolution suitably, and the best-fit $S^{T=0} = -0.55(9)$. If the $\langle V^{J=\text{odd}}_{1414}\rangle$ is characterized by the same δ force as the $\langle V^{J=\text{even}}_{1414}\rangle$, the $T = 0$ component in the $\langle V^{J=\text{odd}}_{1414}\rangle$ should have a strength of $S^{T=0} = -0.55$. Such a $T = 0$ component is excluded in the $\langle V^{J=\text{odd}}_{1414}\rangle$ according to Eq. (6), and the residual $\langle V^{J=\text{odd}}_{1414}\rangle - V_{\delta}^{T=0}$ is presented in Fig. 5b, which should correspond to the $T = 1$ component of a δ force governed by Eq. (7). Thus, the residues are fitted to Eq. (7) with $S^{T=1}$ and a unified interaction offset as the fitting variables. The best-fit $S^{T=1} = 0.30(6)$, and the corresponding fitting curve appropriately describes the plots in Fig. 5b. Therefore, it is concluded that a unified δ force with $S^{T=0} = -0.55$ and $S^{T=1} = 0.3$ characterizes the *pn* interaction for the positive linear μ systematics in the TBRE+pairing.

According to the best-fit $S^{T=0} < 0$ and $S^{T=1} > 0$, the positive linear μ systematics of the TBRE+pairing favors an attractive $T = 0$ *pn* interaction and a repulsive $T = 1$ one, and the $T = 0$ component should be stronger than the $T = 1$ one. On the other hand, the realistic *pn* interaction is also constructed with attractive $T = 0$ and repulsive $T = 1$ components (the $T = 1$ pairing between protons and neutrons is still attractive, but does not apply here), and the $T = 0$ strength is normally stronger [22]. This structural similarity between the *pn* interaction with $\rho_{\mu} > 0.9$ in the TBRE+pairing and the realistic *pn* interaction suggests that the linear μ systematics may share a same microscopic mechanism in both TBRE+pairing ensemble and realistic nucleus.

5 Examination with Sn isotopes

According to Sects. 3 and 4, the *pn* interaction is more crucial than the $h_{11/2}$ seniority scheme for positive linear Q and μ systematics, in random-interaction ensembles. It would be useful to investigate the impact of *pn* interaction on Q and μ systematics in a realistic nuclear system. Tin isotopes are chosen as the standard, because they are typical semi-magic nuclei with no *pn* interaction between the valence nucleons, and thus provide the best platform to study the effect of the absence of *pn*-interaction on the linearity of the $11/2^-_1$ Q and μ systematics in a realistic nuclear system.

A plot of the Q and μ systematics of the $11/2^-_1$ states of the $^{115-131}$Sn isotopes is shown in Fig. 6. In panel (a), the Q systematics seems disorganized, especially near the $N =$

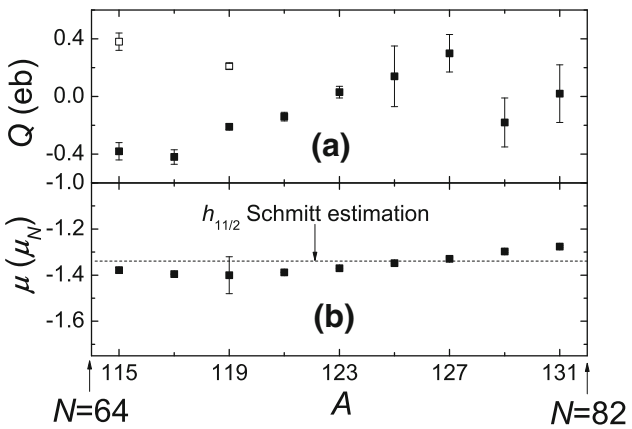

Fig. 6 Experimental Q and μ of the $11/2^-_1$ states of the Sn isotopes versus the mass numbers. All the data are taken from Ref. [23], except the Q value of ^{127}Sn, which is taken from Ref. [24], and the μ value of ^{117}Sn, which is taken from Ref. [25]. The uncertainty in the sign of the Q values of ^{119,115}Sn is represented by open squares in panel (a). The Schmitt estimation of the neutron $h_{11/2}$ orbit is represented by a dashed line to demonstrate the constant μ value of pure $h_{11/2}$ seniority scheme. The $N = 64$ and $N = 82$ shell closures are highlighted

64 and $N = 82$ shell closures, even though the $h_{11/2}$ seniority scheme should be mostly robust near the shell closure. Figure 6b presents the near constant (instead of linear) μ systematics of the Sn isotopes around the Schmitt estimation of the neutron $h_{11/2}$ orbit, which also supports the $h_{11/2}$ seniority scheme for Sn isotopes. Therefore, the absence of linear Q systematics in the Sn isotope chain indicates that the linear Q and μ systematics in Cd isotopes cannot be totally explained by the seniority scheme or pairing. The *pn* interaction may be also essential for both linear Q and μ systematics even in realistic nuclear system.

6 Summary

To summarize, random interactions predominantly reproduce the linear Q and μ systematics of the $I^{\pi} = 11/2^{-}$ isomers of the Cd isotopes as already reported in previous experimental work [9]. The importance of the *pn* interaction in such linear systematics is emphasized in such a Monte Carlo study.

The predominance of the linear Q systematics can be enhanced by the seniority scheme as expected. However, even without introducing seniority scheme, such predominance can still be perceived with a quadrupole-like *pn* interaction. We also note that the quadrupole collectivity is already emphasized in previous studies on the linear Q systematics [10, 12], which could further support this observation. For the linear μ systematics, the δ-force-like *pn* interaction and seniority scheme are both essential, and the *pn* interaction is further required to exhibit a similar structure to realistic nuclear interaction. Therefore, if the δ force plus quadrupole *pn* interaction is adopted in the shell-model calculation, the Q and μ linear systematics are both expected. A schematic calculation with $S^{T=0} = 0.55$, $S^{T=1} = -0.3$ and $\chi = -0.1$ for *pn* interaction and $S^{T=1} = -1.0$ for *pp* and *nn* interaction was performed, where χ is the strength of the quadrupole interaction. The resultant $\rho_Q = 0.947$ and $\rho_\mu = 0.833$ agree with expectation.

The effect of the *pn* correlation on the Q and μ systematics in a realistic nuclear system was also discussed. The experimental data of the $11/2_1^-$ states along the Sn isotope chain also suggests that the *pn* correlation is the major driving force of both the linear Q and μ systematics even in a realistic nuclear system.

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