

Two-parameter dynamical scaling analysis of single-phase natural circulation in a simple rectangular loop based on dilation transformation

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Abstract

Scaling analysis is widely used to design scaled-down experimental facilities through which the prototype phenomena can be effectively evaluated. As a new method, dynamic system scaling (DSS) must be verified as a rational and applicable method. A DSS method based on dilation transformation was evaluated using single-phase natural circulation in a simple rectangular loop. The scaled-down cases were constructed based on two parameters—length ratio and dilation number—and the corresponding transient processes were simulated using the Relap5 computational code. The results show that this DSS method can simulate the dynamic flow characteristics of scaled-down cases. The transient deviation of the temperature difference and mass flow rate of the scaled cases decrease with increases in the length ratio and dilation number. The distortion of the transient temperature difference is smaller than that of the mass flow; however, the overall deviation is within a reasonable range.

Keywords Dynamical system scaling analysis \cdot Single-phase natural circulation \cdot Transient scaling deviation \cdot Dilation transformation

1 Introduction

During the development and application of third-generation nuclear power technology, passive safety is an important factor to consider when designing safety facilities for nuclear power plants. In the AP1000 advanced passive Pressurized Water Reactor (PWR) system, designed by Westinghouse, the use of passive safety technology increases the safety margin of a nuclear power plant [1]. CAP1400 (China Advanced PWR 1400), a large-scale advanced nuclear power plant project in China, adopts passive safety technology similar to

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AP1000 [2]. This technology improves system safety over that of second-generation nuclear power plants, which are being widely built, while reducing cost and simplifying the structure. The HPR1000 (Hua-long Pressurized Reactor) nuclear power plant project independently developed by China also applies a reliable passive safety design concept [3].

Passive safety systems, such as passive core cooling and passive containment cooling, are widely used in thirdgeneration nuclear power plants and require rigorous safety performance assessments to verify the reliability and stability of the facilities before they are used. From an economic point of view, using full-scale prototype models to verify the reliability of passive safety features is unrealistic. Therefore, a validation experiment using a scaled-down experimental facility is a reasonable and widely used method. Many such scaled-down test facilities are already in use. Examples include the ACME (Advanced Core-cooling Mechanism Experiment) facility [2], APEX (Advanced Plant Experiment) facility [4], ATLAS (Advanced Thermal–hydraulic Test Loop for Accident Simulation) test loop [5], and ROSA (Rig of Safety Assessment) large-scale test facility [6].

In recent years, many scaling methods have been reported [7, 8]. Hsu et al. [9] analyzed the applicability of the linear scaling method based on the mass-energy balance equations and flow pressure drop relationship in the simulation of small-break LOCA accidents. Nahavandi et al. [10] derived the corresponding series of scaling criteria using three scaling methods: time-reducing scaling laws, time-preserving volumetric scaling laws, and time-preserving idealized model/prototype scaling laws. They discussed the rationality of these criteria for simulating and predicting the application of nuclear reactor systems. The power-volume method is suitable for establishing full-height test facilities [11]. Ishii et al. [12] deduced the corresponding scaling criteria for single-phase and two-phase natural circulation flows and indicated that it can be applied to experimentally simulate the natural circulation of pressurized water reactors in a LOFT (Loss Of Fluid Test) facility. Ishii developed a three-level scaling method to guide the design of an overall test facility [13]. This method obtains scaling criteria through both topdown and bottom-up scaling analysis. Zuber et al. [14, 15] developed Hierarchical Two-Tiered Scaling (H2TS) method suitable for more complex multiphase thermal experimental systems. By nondimensionalizing the conservation equations at multiple levels, a similar standard consisting of geometric and fluid parameter scaling factors was obtained. In addition, Zuber proposed the fractional scaling analysis method and applied it to analyze the loss of coolant accidents [16, 17].

Scale analysis based on the above scaling criteria often focuses on the static parameter deviation of the scaled model, whereas the actual situation it simulates is mainly a dynamic process. This type of simulation can lead to scaling errors if the actual parameters change over time throughout the dynamics. To address this defect, recent natural circulation simulation studies have sought improvement by focusing on transient processes. Kuran et al. [18] simulated the start-up transient of a boiling water reactor following a natural cycle at low pressure and flow using a PUMA (Purdue University Multi-Dimensional Integral Test Assembly facility). Lakshmanan et al. [19] confirmed that the Natural Circulation Test Facility (CNTF) model developed by Relap5 could predict transient cases of natural circulation under low pressure and power. Dixit simulated two-phase natural circulation instability in an integrated modular water reactor [20]. Basu used rectangular natural circulation to study the dynamic response to a change in the input power of a singlephase natural circulation system [21]. Reves developed a new dynamic system scaling (DSS) method to simulate the dynamic process and evaluate the dynamic distortion of the scaled model [22]. In this scale-up process, five types of similarity criteria are generated, for which different arrays of similar criteria can be obtained. The dynamic deviation generated during the scaling process can be effectively evaluated by comparing the original dynamic process with the scaling process. Li et al. [23] used the DSS method to evaluate the scaling distortion of gravity-driven drainage systems and explored the dynamic changes of key parameters in the gravity drainage process under different scaled-down cases. In addition, a similarity criterion array based on the DSS identity transformation law for single-phase natural circulation is derived. Using a simple loop model, dynamical scaling analysis theory is enriched by comparing the difference between the simulation results of the DSS method and that of the traditional H2TS method [24].

To further understand the DSS method and expand its application in the field of natural circulation, a scaling criterion based on dilation transformation was deduced using the two-parameter affine transformation concept. Based on single-phase natural circulation in a simple rectangular loop, the corresponding two-parameter dynamical scaling analysis similarity arrays under different combinations of length ratios and dilation numbers are obtained. The Relap5 code is used to establish a natural-circulation loop model. Different scaling cases are developed according to the calculated similar array, and the dynamical scaling characteristics are evaluated under different length ratios and dilation numbers.

2 Two-parameter scaling analysis based on dilation transformation

2.1 Physical model

For the convenience of analysis, a simple rectangular loop model was built. The simplification of the model involves several assumptions.

- (1) Flow is one-dimensional along the loop axis, and the fluid properties of the cross section remain uniform.
- (2) Boussinesq approximation is applied to loops.
- (3) Heat loss is negligible throughout the rest of the piping, except for the piping sections used to simulate heating and cooling.
- (4) Pipes have the same diameter and cross-sectional area throughout the circuit.

A simple rectangular loop is illustrated in Fig. 1. The circuit is mainly composed of heating, cooling, ascending and descending pipes, and horizontal pipe sections. The heating section is used to simulate the heating of the water in the core of the nuclear reactor, whereas the cooling section is used to simulate the heat exchange and cooling process between the primary coolant circuit and the secondary side of the steam generator. The density of the heated fluid decreases, and the density increases after cooling, thereby realizing circulation in a rectangular loop. To create a dynamic process for analysis, in the heating section,



Fig. 1 Sketch of the rectangular loop

Table 1 Prototype model parameters

Item	Value
System pressure (MPa)	1
Loop length (m)	24
Heating section length (m)	2
Cooling section length (m)	2
Ascent section length (m)	8
Drop section length (m)	8
Horizontal section length (m)	2
Pipe inner diameter (m)	0.08
Pipe wall thickness (m)	0.0005
Initial input power (W)	12,000
Initial water temperature (°C)	20

the thermal component provides the circuit with an initial power of 12,000 W. In this case, the natural circulation in the loop reaches a steady state. The power is then increased by 5% at 12,600 W, after which the cycle gradually reaches the second steady state. The dynamic process to be analyzed lies between these two states. The main parameters of the simple rectangular natural-circulation loop are listed in Table 1.

2.2 Similarity criterion

Using the above simple rectangular loop model, a similarity criterion array based on the dilation transformation can be derived.

The momentum conservation equation for single-phase natural circulation is given by:

$$\frac{L}{A}\frac{dW}{dt} = \beta_{\rm f}g\rho\Delta TH - \frac{W^2}{2\rho A^2} \left(\frac{fL}{d} + k\right). \tag{1}$$

Equation (1) can be transformed into the following form:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \left(\frac{A}{L}\right)\beta_{\mathrm{f}}g\rho\left(T_{\mathrm{h}} - T_{\mathrm{c}}\right)H - \frac{W^{2}}{2\rho AL}\left(\frac{fL}{d} + k\right). \tag{2}$$

Here, *L* is the total length of the circuit, *A* is the crosssectional area of the pipe, *W* is the mass flow rate, β_f is the thermal expansion coefficient, ρ is the fluid density, ΔT is the temperature difference at the outlet of the hot and cold sections, *H* is the height difference between the centers of the cold and heat sources, *f* is the friction loss coefficient, *d* is the pipe diameter, and *k* is the flow loss coefficient.

Reyes provides a detailed description of the theoretical analysis of the DSS method [22]. Here, we use the DSS method to process the momentum balance equation above. Define

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \varphi_{\mathrm{T}}^{\mathrm{W}} + \varphi_{\mathrm{W}}^{\mathrm{W}}.$$
(3)

Combining Eqs. (3) with (2) yields

$$\varphi_{\rm T}^{\rm W} = \frac{\beta_{\rm f} g \rho A \left(T_{\rm h} - T_{\rm c} \right) H}{L},\tag{4}$$

$$\varphi_{\rm W}^{\rm W} = -\frac{W^2}{2\rho AL} \left(\frac{fL}{d} + k\right). \tag{5}$$

 ω is used to represent the dimensionless mass flow response variable, which can be expressed by the following two equations:

$$\omega_{\rm T}^{\rm W} = \frac{\varphi_{\rm T}^{\rm W}}{W_0} = \frac{\beta_{\rm f} g \rho A (T_{\rm h} - T_{\rm c}) H}{L W_0},\tag{6}$$

$$\omega_{\rm W}^{\rm W} = \frac{\varphi_{\rm W}^{\rm W}}{W_0} = -\frac{W^2}{2\rho A L W_0} \left(\frac{fL}{d} + k\right). \tag{7}$$

Defining $W^+ = \frac{W}{W_0}$, the dimensionless responses can be expressed as

$$\omega_{\rm W}^{\rm W} = \frac{\varphi_{\rm W}^{\rm W}}{W_0} = -\frac{W_0}{2\rho AL} \left(\frac{fL}{d} + k\right) (W^+)^2.$$
(8)

The natural circulation agents-of-change similarity criteria are

$$\lambda_{\rm A}^{\rm W} = \widetilde{\Omega}_{\rm W,R}^{\rm W} = \tau_{\rm s,R}^{\rm W} \omega_{\rm W,R}^{\rm W} \tag{9}$$

$$\lambda_{\rm B}^{\rm W} = \widetilde{\Omega}_{\rm W,R}^{\rm T} = \tau_{\rm s,R}^{\rm T} \omega_{\rm W,R}^{\rm T}.$$
 (10)

Assuming that the dimensionless mass flow response variables of the model and prototype are $\omega_{W,M}^W$ and $\omega_{W,P}^W$, respectively, we obtain

$$\omega_{\rm W,R}^{\rm W} = \frac{\omega_{\rm W,M}^{\rm W}}{\omega_{\rm W,P}^{\rm W}} = \frac{W_{0,\rm R}}{(AL)_{\rm R}} \left(\frac{fL}{d} + k\right)_{\rm R} \left[\frac{W_{\rm M}^+(t_{\rm M,W})}{W_{\rm P}^+(t_{\rm W})}\right]^2.$$
 (11)

In Eq. (11), $t_{M,W}$ represents the model reference time (more details can be found in [25] and [26]), the subscripts M and P represent the corresponding parameters of the model and prototype, and R represents the ratio of the parameters of the model and prototype,

$$t_{\rm R} = \tau_{\rm s,R}^{\rm W} = \frac{\lambda_{\rm A}^{\rm W}}{\lambda_{\rm B}^{\rm W}} = 1, \qquad (12)$$

$$t_{\mathrm{M,W}} = \tau_{\mathrm{s,R}}^{\mathrm{W}} t_{\mathrm{R}}.$$
 (13)

Combining Eqs. (9) and (11), we have

$$\tau_{\mathrm{s,R}}^{\mathrm{W}} = \frac{\lambda_{\mathrm{A}}^{\mathrm{W}}}{\omega_{\mathrm{W,R}}^{\mathrm{W}}} = \frac{\lambda_{\mathrm{A}}^{\mathrm{W}}}{\frac{W_{0,\mathrm{R}}}{(AL)_{\mathrm{R}}} \left(\frac{fL}{d} + k\right)_{\mathrm{R}} \left(\frac{W_{\mathrm{M}}^{+}(t_{\mathrm{M}})}{W_{\mathrm{P}}^{+}(t_{\mathrm{P}})}\right)^{2}}.$$
(14)

According to the DSS analysis method,

$$\lambda_{\rm A}^{\rm W} = \frac{W_{\rm M}^+(t_{\rm M})}{W_{\rm P}^+(t_{\rm P})},\tag{15}$$

$$\lambda_{\rm A}^{\rm T} = \frac{T_{\rm M}^+(t_{\rm M})}{T_{\rm P}^+(t_{\rm P})}.$$
(16)

Combining Eqs. (14) and (15) gives

$$\tau_{s,R}^{W} = \frac{1}{\lambda_{A}^{W} \frac{W_{0,R}}{(AL)_{R}} \left(\frac{fL}{d} + k\right)_{R}}.$$
(17)

According to the dilation transformation criteria,

$$\beta_{\rm R} = \lambda; \omega_{\rm R} = \lambda, \tag{18}$$

$$\widetilde{\Omega}_{\rm R} = \lambda; \tau_{\rm R} = t_{\rm R} = 1. \tag{19}$$

Equation (17) can be simplified to

$$\left(\frac{u_{\rm R}}{L_{\rm R}}\right) \left(\frac{fL}{d} + k\right)_{\rm R} = \frac{1}{\lambda},\tag{20}$$

$$\omega_{\mathrm{T,R}}^{\mathrm{W}} = \left(\frac{HT_0}{Lu}\right)_{\mathrm{R}} \lambda_{\mathrm{A}}^{\mathrm{T}}.$$
(21)

By simultaneously simplifying Eqs. (17), (20), and (21), λ_A^W can be expressed as

$$\lambda_{\rm A}^{\rm W} = \frac{1}{\lambda_{\rm A}^{\rm W}} \left[\frac{W_{0,\rm R}}{(AL)_{\rm R}} \left(\frac{fL}{d} + k \right)_{\rm R} \right]^{-1} \left(\frac{H\Delta T_0}{Lu} \right)_{\rm R} \lambda_{\rm A}^{\rm T}.$$
 (22)

The scaling of vertical and horizontal segment heights is assumed to follow the same guidelines, $\left(\frac{H}{L}\right)_{R} = 1$. By applying the dilation transformation criterion to Eq. (22), we obtain

$$\left(\frac{\Delta T_0}{u}\right)_{\rm R} \left(\frac{fL}{d} + k\right)_{\rm R}^{-1} \left(\frac{L}{u}\right)_{\rm R} = \lambda.$$
(23)

Substituting Eq. (20) into Eq. (23),

$$\Delta T_{0,\mathrm{R}} = u_{\mathrm{R}}.\tag{24}$$

The above shows the result derived from the momentum balance equation. The energy conservation equation for the natural circulation circuit is as follows:

$$\rho C_{\rm p} \left\{ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right\} = \frac{4q}{d}.$$
 (25)

Assuming that the circulating system is in a steady state, we ignore the correlation of the time differential in the equation

$$\frac{\partial T}{\partial z} = \frac{4q}{\rho C_{\rm p} du}.$$
(26)

Thus,

$$\frac{\partial (T - T_i)}{\partial z} = \frac{4q}{\rho C_{\rm p} d} (1 - \beta^{\rm w}), \tag{27}$$

where

$$\frac{\partial \beta^{\mathrm{T}}}{\partial t} = \omega^{\mathrm{T}}.$$
(28)

 β^{T} represents the dimensionless temperature difference between the hot and cold sections of the rectangular loop,

and $\omega^{\rm T}$ represents the corresponding change caused by the temperature change.

The constant scaling factor is given as

$$\lambda_{\rm A}^{\rm T} = \widetilde{\Omega}_{\rm W,R}^{\rm W} = \tau_{\rm s,R}^{\rm T} \omega_{\rm W,R}^{\rm T}.$$
(29)

Thus,

$$\tau_{\mathrm{s,R}}^{\mathrm{T}} = \lambda_{\mathrm{A}}^{\mathrm{T}} \frac{1}{\left(\omega_{\mathrm{W,R}}^{\mathrm{T}}\right)} = \lambda_{\mathrm{A}}^{\mathrm{T}} \left(\frac{T_{0,\mathrm{R}}d_{\mathrm{R}}}{q_{\mathrm{R}}}\right) \left[\frac{1 - W_{\mathrm{M}}^{+}(t_{\mathrm{M,T}})}{1 - W_{\mathrm{P}}^{+}(t_{\mathrm{T}})}\right]. \tag{30}$$

Assuming that the heat source is a point heat source,

$$\tau_{\mathrm{s,R}}^{\mathrm{T}} = \lambda_{\mathrm{A}}^{\mathrm{T}} \left(\frac{T_{0,\mathrm{R}} d_{\mathrm{R}}}{q_{\mathrm{R}}} \right). \tag{31}$$

The dilation similarity criterion gives

$$q_{\rm R} = \lambda T_{0,\rm R} d_{\rm R},\tag{32}$$

yielding

$$q_{\rm R} = \frac{Q_{\rm R}}{\pi d_{\rm R} L_{\rm R}} = \frac{\rho_{\rm R} u_{\rm R} A_{\rm R} \Delta T_{0,\rm R}}{\pi d_{\rm R} L_{\rm R}} = \frac{d u_{\rm R} \Delta T_{0,\rm R}}{L_{\rm R}}.$$
 (33)

Combining Eqs. (32) and (33) gives

$$u_{\rm R} = \lambda L_{\rm R}.\tag{34}$$

By reconsidering Eq. (20),

$$\left(\frac{fL}{d} + k\right)_{\rm R} = \frac{1}{\lambda^2},\tag{35}$$

the above dilation similarity criteria can be summarized as follows:

$$q_{\rm R} = \lambda^{\frac{17}{5}} L_{\rm R}^{\frac{8}{5}},\tag{36}$$

$$Q_{\rm R} = \lambda^{\frac{24}{5}} L_{\rm R}^{\frac{16}{5}},\tag{37}$$

 $\Delta T_{0,\mathrm{R}} = \lambda L_{\mathrm{R}},\tag{38}$

$$d_{\rm R} = \lambda^{\frac{7}{5}} L_{\rm R}^{\frac{3}{5}}.$$
 (39)

Clearly, the main parameter ratios of the scaled models are dilation number λ and length ratio $L_{\rm R}$. For $\lambda = 1$, a scaling criterion based on the identity transformation of the DSS method can be obtained.

By selecting the range of dilation numbers and length ratios, different scaling ratios can be obtained by changing the combination of the two variables. The two-parameter scaling curves based on the above criteria array are shown in Fig. 2.

By fixing the dilation number $\lambda = 0.5$, Fig. 2a shows the similarity criterion array formed by the parameters of the natural-circulation loop as the length scaling changes. For a fixed length ratio $L_{\rm R} = 0.5$, Fig. 2b shows the similarity criterion array formed by the loop parameters as the dilation number changes. The two graphs from this figure reflect the scaling law of the natural-circulation loop parameters from the two parameters, dilation number and length ratio. In general, the pipe diameter, heating power, and heat flux density of the scaled model increase with dilation number or length ratio. Considering the two cases, "fixed length ratio, dilation number gradually increases", in the latter case, the



Fig. 2 (Color online) Scaling number curve with fixed dilation number and length ratio

rate of increase of the parameters of the loop is significantly greater than in the former case. This phenomenon is particularly evident when the scaling ratio is large. However, the same conclusions hold for other dilation and length scales.

In addition, combining the two images reveals that the ratio of the heating power of the scaled-down model to that of the prototype case is smaller in the criterion array derived from the dilation transformation based on the DSS analysis (less than 0.15). This is conducive to the construction of a scale-model experimental bench, which would benefit from the smaller power. This is one of the advantages of DSS.

2.3 Numerical model

The simple rectangular-loop natural circulation model shown in Fig. 3 was established using the Relap5 computational code. Relap5 is a transient analysis program for lightwater reactors, developed by Idaho National Laboratory for the US Nuclear Regulatory Commission, and is widely used in thermal hydraulic analysis and calculation [27]. It should be noted that Relap5 uses a one-dimensional computational method for a simple natural loop.

The heating pipe section was simulated using Pipe160, whereas the cooling section was completed by heat exchange between the Pipe110 and Pipe120 components. The heat of the fluid in the main circulation loop was removed by two time-dependent control bodies: TDV200 and TDV220 components. The distance between Pipe110 and Pipe160 represents the height difference between the heating and cooling



Fig. 3 System nodalization

sections. This provides the driving force for natural circulation in the circuit. In addition, the Pipe180 and TDV182 components are set on the loop to simulate the pressurizer to ensure that the pressure in the loop is maintained at the set value during the cycle. Considering that only the natural circulation process is simulated, no pump components are set up in this simple circuit. Before the calculation, the node sensitivity of the simple rectangular loop model is analyzed, which ensures that the calculation is independent of the number of nodes.

3 Results and discussion

According to the array of dilation transformation and scaling criteria, the two-parameter scaling criteria, based on the length ratio and dilation number, are calculated, summarized in Table 2.

To ensure the rationality of the scaled-down case setting, after length ratio $L_{\rm R}$ is selected, dilation number λ should be chosen to ensure that the ratio of each scaled-down parameter is not greater than 1.

First, we analyzed the static errors of the flow velocity and temperature difference between the prototype and model cases.

As shown in Figs. 4 and 5, compared with the prototype cases of natural circulation under steady-state cases, all cases based on the scaling of the two parameters can achieve good agreement regarding the flow parameters. The calculation

 Table 2
 Two-parameter scaling number group based on DSS method

L _R	λ	d_{R}	$Q_{\rm R}$	<i>u</i> _R	$\Delta T_{\rm R}$
0.25	0.25	0.0625	0.000015	0.0625	0.0625
	0.5	0.1649	0.0004	0.125	0.1250
	0.75	0.2910	0.0030	0.1875	0.1875
	1.0	0.4353	0.0118	0.25	0.25
	1.25	0.5949	0.0346	0.3125	0.3125
	1.5	0.7679	0.0829	0.375	0.375
	1.75	0.9528	0.1738	0.4375	0.4375
0.5	0.25	0.0947	0.0001	0.125	0.125
	0.5	0.2500	0.0039	0.25	0.25
	0.75	0.4410	0.0274	0.375	0.375
	1.0	0.6598	0.1088	0.5	0.5
	1.25	0.9017	0.3176	0.625	0.625
0.75	0.25	0.1208	0.0005	0.1875	0.1875
	0.5	0.3189	0.0143	0.375	0.375
	0.75	0.5625	0.1001	0.5625	0.5625
	1.0	0.8415	0.3983	0.75	0.75
1.0	0.25	0.1436	0.0013	0.25	0.25
	0.5	0.3789	0.0359	0.5	0.5
	0.75	0.6685	0.2514	0.75	0.75



errors of the steady-state flow velocity and temperature difference between the hot and cold sections can be controlled within a level of no greater than 7%. This shows that the setting of the length ratio-dilation number two-parameter scaling case is reasonable, based on the above criteria.

Figures 6 and 7 show the transient mass flow curves of the prototype case and each scaled-down model case.

Based on the two-parameter analysis criteria, a comparison of transient mass flow rate is carried out from the two parameters. In Fig. 6, the length ratio is given, and the transient change in the mass flow rate of the model operating cases corresponding to different dilation numbers is compared. Figure 7 shows a comparison of the transient trend of the mass flow rate under different length ratios





1.0

0.8

Fig.6 (Color online) Transient mass flow rate comparison (length scaling)

Fig. 7 (Color online) Transient mass flow rate comparison (dilation number scaling)

for a given dilation number. Notably, based on the DSS method, the processing time will be distorted for different scaled models, which is reflected in the time difference required for the fluid to circulate in a rectangular loop. To consider the influence of this process distortion on the analysis, the horizontal ordinate in the figure is defined as t^* , which has the following relationship,

$$t^* = \frac{t}{t_0},\tag{40}$$

where t_0 denotes the time required for the fluid to complete a cycle. The ordinate β_W represents the non-dimensional mass flow rate, and β_W is defined as $(W - W_{1,0})/(W_{2,0} - W_{1,0})$. $W_{1,0}$ and $W_{2,0}$ are the mass flow rates at the initial and final stages, respectively.

The same method is used to obtain the transient change curve of the normalized temperature difference between the hot and cold sections for each case, shown in Figs. 8 and 9. $\beta_{\rm T}$ is defined as $(\Delta T - \Delta T_{1,0})/(\Delta T_{2,0} - \Delta T_{1,0})$, where $\Delta T_{1,0}$ and $\Delta T_{2,0}$ are the temperature differences at the initial and final stages. Overall, the temperature difference shows a significantly different profile from that of the mass flow rate. Briefly, the transient temperature difference trends significantly in the early stages of the power step; that is, the normalized temperature difference jumps greatly to between 1.1 and 1.4 with the increase in power and then rapidly decreases to approximately 0.7 to 1.0. This oscillation process is completed in the first cycle of the loop after the power jump. After this time, the transient normalized temperature difference rises slowly, eventually reaching a steady state. Figures 8 and 9 show the evolution of the change in the normalized temperature difference as a function of the two parameters. The main variable in Fig. 8 is the dilation number, whereas it is the length ratio in Fig. 9. The conclusion is similar to that for the mass flow rate, in that increasing the dilation number and length scale produces simulation results of the scaled-down cases that match more closely with those of the prototype case. When the dilation number is small, the effect of the length scale on the results is less significant.

By comparison, the normalized temperature difference for the scaled-down cases containing a small length ratio and large dilation number (Fig. 8a–c) can be very close to that of the prototype case. However, for the normalized mass flow rate (Fig. 6a–c), even if a larger dilation number is selected, the time to reach the second steady state in the scaled-down cases is still significantly less than in the prototype case. This may be due to a change in the Reynolds number. A small length ratio corresponds to a small Reynolds number, which may cause errors in the calculation.

Figures 10 and 11 show the comparison of the transient deviation of the normalized mass flow rate. The dynamic



Fig. 8 (Color online) Transient temperature comparison (length scaling)



Fig.9 (Color online) Transient temperature comparison (dilation number scaling)



Fig. 10 (Color online) Relative error of the mass flow rate (length scaling)



Fig. 11 (Color online) Relative error of the mass flow rate (dilation number scaling)

deviation of the normalized mass flow rate is defined as $(W_{\rm M}^* - W_{\rm P}^*)/W_{\rm M}^* \times 100\%$. Figure 10 shows the results with a fixed length ratio, comparing the scaled-down cases corresponding to different dilation numbers. During the first cycle, the normalized mass flow rate of all scaled-down cases oscillate significantly. For the same length ratio, a reasonably larger dilation number can reduce the error. At $L_{\rm R} = 0.25$, if the dilation number is greater than 1.5, the maximum error during the oscillation can be less than 20%. At $L_{\rm R} = 0.5$, the dilation number must be greater than 1.0 to achieve the same effect. This rule can be seen in Fig. 10a-c. For all cases, when natural circulation reaches 4.5 cycles, the transient error of the mass flow rate decreases to less than 10%, gradually reaching 0. Figure 11 shows the comparison of the scaled-down facility with different length ratios and a fixed dilation number. As shown in Fig. 11a, when the dilation number is selected to be small, the length ratio has little effect on the transient deviation of the mass flow rate. This is consistent with the conclusion drawn in Fig. 7a. For a fixed dilation number, selecting a larger length ratio can also reduce the transient error of the normalized mass flow rate, which can be seen in Fig. 11b-d. Of course, the length ratio should be selected in accordance with the cases of the scaled-down models.

The normalized deviations in the temperature difference between the hot and cold sections are similar to those of the mass flow rate (Figs. 12 and 13). Correspondingly, under a fixed length ratio, a larger dilation number yields a smaller error than that of the mass flow rate under the same parameters.

In addition, a comparison of the error between the normalized mass flow rate and the normalized temperature difference between the hot and cold sections shows an evident phenomenon: if a small dilation number is selected, the difference between the dynamic error curves corresponding to different length ratios is very small (Figs. 11a and 13a). The same conclusion is obtained for the normalized mass flow rate and normalized temperature difference between the hot and cold sections. When the dilation number is gradually increased, this phenomenon disappears (Figs. 11b–d and 13b–d). These results are consistent with the previous analysis of Fig. 11.

In the H2TS scaling analysis method, a series of criteria arrays was obtained by the dimensionless transformation of relevant control equations. In single-phase flow, this mainly includes the Richardson number, heat source number, and Stanton number. According to the scaling method, the theoretical ratios are all 1, and the specific scaling criteria are derived from these criteria. Therefore, in the H2TS method, the Richardson number, heat source number, and Stanton number are decisive criterion numbers. However, in the DSS method, the control equation



Fig. 12 (Color online) Relative error of the temperature difference (length scaling)



Fig. 13 (Color online) Relative error of the temperature difference (dilation number scaling)

of the system is treated using the entire method, by which the similarity criteria are obtained. This differs from the H2TS method.

 Table 3
 Similarity criterion number ratio

L _R	λ	Ri _R	Qs_{R}	St _R
0.25	0.25	4.000	1	1024.000
	0.5	2.000	1	73.517
	0.75	1.333	1	15.748
	1	1.000	1	5.278
	1.25	0.800	1	2.261
	1.5	0.667	1	1.131
	1.75	0.571	1	0.629
0.5	0.25	4.000	1	445.722
	0.5	2.000	1	32.000
	0.75	1.333	1	6.855
	1	1.000	1	2.297
	1.25	0.800	1	0.984
0.75	0.25	4.000	1	274.002
	0.5	2.000	1	19.672
	0.75	1.333	1	4.214
	1	1.000	1	1.412
1.0	0.25	4.000	1	194.012
	0.5	2.000	1	13.929
	0.75	1.333	1	2.984

Fig. 14 (Color online) Steadystate error of the Richardson number

To facilitate comparison with the traditional scaling analysis method, the relevant criterion numbers are defined as follows:

$$\prod_{Ri} = \frac{\beta_{fg} H \Delta T_0}{u^2},\tag{41}$$

$$\prod_{Qs} = \frac{4ql}{\rho C_{\rm P} du \Delta T_0},\tag{42}$$

$$\prod_{S \tan} = -\frac{4k_{\rm s}l}{\rho C_{\rm P} du\delta}.$$
(43)

Equations (41)–(43) represent the Richardson, heat source, and Stanton numbers, respectively. Notably, for the Richardson and Stanton numbers, their theoretical values are not 1 but will change with the length ratio and dilation number. The final scaling criteria were not determined based on their proportions. Therefore, in the DSS method, the Richardson number and Stanton number are considered non-decisive criterion numbers. The theoretical ratios of the criterion numbers for the different scaled-down cases are listed in Table 3. Here, the theoretical ratio of the heat source number is 1 for all scaled-down cases, whereas the Richardson number and Stanton number follow their respective ratios.







Fig. 16 (Color online) Steadystate error of the heat source number

Figures 14, 15 and 16 show the steady-state errors of the criterion numbers for each scaled-down case when the loop reached the second steady state. The error in the Richardson number is shown in Fig. 14. In all the cases, the static

deviation is within $\pm 15\%$. The deviation in the Stanton number is smaller, as shown in Fig. 15. All scaled-down cases are controlled within $\pm 7\%$. It is easy to explain from Eqs. (41) and (43) that the error sources of the Richardson

number and Stanton number are the deviation of the flow velocity and temperature difference between the hot and cold sections. An error analysis of multiple groups of independent variables reveals that the error of the Richardson number should be slightly larger. As shown in Fig. 16, the variation in the heat source number is very small compared to the first two criterion numbers, within $\pm 0.5\%$. This is because the number of heat sources, as a decisive parameter, is obtained during the derivation of the scaling criterion of the DSS method, and its theoretical ratio is always 1. The Richardson and Stanton numbers are composed of other criterion numbers. The error in the criterion number varies with the error of the independent variable. In addition, for each length scale that has been determined, there is a corresponding dilation number, making the steady-state heat source number of the scaled-down cases the same as that in the prototype case.

4 Conclusion

Based on the dilation transformation in the DSS method, the corresponding scaling criteria are deduced, and the twoparameter dilation scaling criteria array is obtained by considering the two parameters of the length scaling ratio and dilation number as the independent variables. Two-parameter scaled-down cases were constructed, and the natural circulation transient process was calculated using the Relap5 code. The static and dynamic deviations associated with the natural circulation flow parameters were evaluated, and the following conclusions were drawn.

- (1) The scaling criterion obtained using the dilation transformation method can be used to effectively simulate single-phase natural circulation in a rectangular loop. The DSS method considers the distortion of the process time in the dynamic process simulation of the scaled-down model and can evaluate the dynamic process. In addition, the error in the static parameters of natural circulation is small.
- (2) All scaled-down models based on two parameters, length scaling and dilation number, can simulate the variation trends of the mass flow rate and temperature difference under the prototype case. When the length ratio is fixed, the results obtained with the scaled-down cases are closer to those of the prototype case with an increase in the dilation number. When the dilation number is fixed, the scaled-down model case conform to the prototype case with an increase in the length ratio. Considering the factors of the two parameters, selecting a moderate length ratio and slightly larger dilation number can make the scaled-down model fit the prototype more closely. In the case of the same combination of dilation number and length ratio, the deviation of the

temperature difference is smaller than that of the mass flow rate.

(3) Overall, the static deviations of the Richardson number, Stanton number, and heat source number can be maintained within a reasonable range. Because the Richardson and Stanton numbers are non-decisive criterion numbers, the corresponding errors are significantly greater than that obtained when the heat source number is used as the decisive criterion number.

The above conclusions were obtained by analyzing the natural circulation of a simple rectangular loop using the dilation criterion of the DSS dynamical scaling method. To facilitate modeling, calculation, and analysis, ideal simplifications were made when the model was built using Relap5. Non-ideal cases such as pipe heat loss, wall thickness error, and two-parameter flow of fluid in the pipes need to be considered during the construction of the scaled model test bench. Additionally, there are other scaling criteria for the DSS method. It is especially noted that the β strain transformation and ω strain transformation are essentially singlephase dilation transformations. Further comparative analyses can be performed for an in-depth evaluation of this approach. The generalization of the conclusions from the simple natural-circulation loop to the reactor system is also worthy of further analysis. Furthermore, two-phase natural circulation occurs frequently. In addition to single-phase circulation, the applicability of the DSS method to two-phase natural circulation should be further considered, such that this method can be better applied in actual scaling experiments.

Author contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Jia-Ning Xu and Xiang-Bin Li. The first draft of the manuscript was written by Jia-Ning Xu, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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