Improvement of digital S-K filter and its application in nuclear signal processing

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Abstract On the basis of preliminary studies, a novel duo-parameter model consisting of amplitude filter factor and frequency filter factor for low-pass S-K filter is presented in this paper. The model is established by applying numerical differentiation method. Some simulation experiments and real data tests are carried out to verify the feasibility and superiority of the new algorithm. The results show that this duo-parameter model of low-pass S-K filter can be used to achieve high performance in signal processing and nuclear spectrum smoothing.

Key words Digital S-K Filter, Duo-parameter Model, Numerical Differential, Nuclear Spectrum Smoothing

1 Introduction

There are many practical forms of digital filter which can be roughly divided into finite impulse response and infinite impulse response. And both of the two types can be achieved by hardware and software. If achieved by hardware, the filter always consists of adders, multipliers and other units. It is different from the analog filter constituted with resistors, inductors and capacitors^[1,2]. General speaking, the digital signal processing system is easily achieved by digital IC which featuring as small size, high stability and programmability. The digital filter can also be achieved by software which is based on the theoretical algorithms executed by digital computer^[3–7].

S-K filter is a common active-filter circuit presented by R. P. Sallen and E. L. Key in $1955^{[1,2]}$. The analog S-K filter is widely used in the nuclear signal shaping. In the early data smoothing of nuclear spectrum, multi-points smoothing was always used. Recent years, there appear many new methods of data smoothing such as FFT converting, Kalman filter and the wavelet converting^[8–18]. In the preliminary study, a

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nuclear pulse signal was shaped digitally on real-time based on digital S-K filter^[19,20]. In this paper, some research work on nuclear spectrum data processing will be carried out based on S-K filter. Firstly, the digital S-K filter is established by adding the signal gain control unit. Secondly, both general signals and the spectrum data are simulated based on the new filter. Lastly, the feasibility of the new filter is verified by comparing the results of different smoothing methods.

2 S-K filter

Two representative circuits of S-K filter such as high-pass filter and low-pass filter exist. As a result of positive feedback control, it possesses a large quality factor. Applied to the shaping for nuclear pulse signal, Gaussian waveform is obtained by less order used^[20]. The schematic circuit of S-K filter is sketched in Fig.1.



Fig.1 Schematic circuit of S-K filter

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2.1 S-K filter numerical analysis and implementation

In the preliminary study, the signal amplitude will be amplified after digital shaping by basic S-K filter. Eq2.(1) and (2) are the algorithms.

The results of data processing are shown in Fig.2
$$\frac{1}{2}$$

$$y_n = \frac{(k+2k^2) \times y_{n-1} - k^2 \times y_{n-2} + 2x_n}{1+k+k^2} \quad n > 0 \quad (1)$$



Fig.2 Results of data processing of basic S-K filter (*k*=25)

In order to meet general digital signal processing, the S-K filter must be established by numerical analysis. And it is necessary to take amplitude factors into consideration. Four nodes are marked in Fig.1. Correspondingly, the voltage of each node is also marked as V_{f} , V_{p} , V_{n} and V_{o} . According to Kirchhoff's Current Law, four equations of voltage transmission can be described by the following ones.

$$\frac{(V_{in} - V_f)}{R_1} = \frac{(V_f - V_p)}{R_2} + C_2 \times \frac{d(V_f - V_o)}{dt}$$
(3)

$$\frac{(V_f - V_p)}{R_2} = C_1 \times \frac{\mathrm{d}V_p}{\mathrm{d}t} \tag{4}$$

$$V_n = V_0 \times \frac{R_3}{R_3 + R_4} \tag{5}$$

$$V_n = V_p \tag{6}$$

Assuming

$$a = \frac{R_3 + R_4}{R_4}$$

Eq.5 can be rewritten as Eq.(7).

$$a \times V_n = V_o \tag{7}$$

In Fig.1, the resistors R_1 and R_2 , the capacitors C_1 and C_2 are respectively designed to the same value. And taking Eq.(7) into Eq.(3), Eq.(4) and Eq.(6), we may condition, the three equations can be rewritten as Eq.(8).

$$aV_{\rm in} = (RC)^2 \times \frac{\mathrm{d}(\frac{\mathrm{d}V_{\rm o}}{\mathrm{d}t})}{\mathrm{d}t} + RC(3-a) \times \frac{\mathrm{d}V_{\rm o}}{\mathrm{d}t} + V_{\rm o} \qquad (8)$$

Eq.(8) can be written as Eq.(9) by form transformation.

$$(RC)^{2} \times y'' + RC(3-a) \times y' + y = ax$$
 (9)

Based on differential numerical method, Eq.(9) can be transformed as the follow one.

$$(RC)^{2} \times \frac{\left(\frac{y_{n+1} - y_{n}}{\Delta t} - \frac{y_{n} - y_{n-1}}{\Delta t}\right)}{\Delta t} + RC(3-a)$$

$$\times \frac{y_{n+1} - y_{n}}{\Delta t} + y_{n+1} = ax$$
(10)

And Eq.(10) can be written as Eq.(11).

$$(1 + \frac{RC(3-a)}{\Delta t} + (\frac{RC}{\Delta t})^2) \times y_{n+1}$$

= $(\frac{RC(3-a)}{\Delta t} + 2(\frac{RC}{\Delta t})^2) \times y_n - (\frac{RC}{\Delta t})^2 \times y_{n-1} + ax$ (11)

Assuming

$$k = \frac{RC}{\Delta t}$$

Eq.(11) can be written as Eq.(12).

$$(1+k(3-a)+k^{2}) \times y_{n+1} = (k(3-a)+2k^{2}) \times y_{n} - k^{2} \times y_{n-1} + ax$$
(12)

According to Eq.12, the output y_n can be described as Eqs.(13) and (14).

$$y_{n} = \frac{(k \times (3-a) + 2k^{2}) \times y_{n-1} - k^{2} \times y_{n-2} + ax_{n}}{1 + k(3-a) + k^{2}}, \quad n > 0$$

$$y_{n} = 0, \quad n \le 0$$
(13)
$$(13)$$

In summary, the output signal of digital Gaussian shaping can be achieved by recursive execution of Eqs.(13) and (14). And the factors k and a play an important role in regulation of the output signal. Correspondingly, factor k is defined as

frequency filter factor which can adjust the width of the output signal. And factor a is defined as amplitude filter factor which can adjust the amplitude of the output signal.



Fig.3 Digital S-K filtering waveforms for sinusoidal signal

3 The application study of digital S-K filter on signal processing

The digital S-K filter can be used in real-time signal processing. A sinusoidal signal (range 1000), whose period is set as 360 points in simulating experiments, is used as original signal. A noise signal with the amplitude ranging from -100 to 100, which is produced by random number generator in VBA (Visual Basic for Applications) platform, is added to the original signal to simulate input signal. The optimal output signal with phase-shifting can be obtained at filtering parameters k=25, a=1.15. Fig.3 shows the output signal with phase-shifting. The analysis result on correlation between original signal and output signal is presented in Fig.4. The analysis on correlation between original signal and output signal which is left-shifted for 45 points is also carried out and the result is given in Fig.5.

Under the condition of high noise level, which can be designed in experiments, numbers of simulating results presented in Table 1 indicate that the similarity coefficient between original signal and output signal can exceed 0.998 after phase-shifting processing. The signal-to-noise ratio of filtered signal can be improved from 6 db to about 30 db. When the signal-to-noise ratio of input signal is 10 db, which can be implemented by adjusting the amplitude of noise signal, the signal-to-noise ratio of output signal can exceed 40 db. Therefore, digital S-K filter can be used in real-time signal processing with favorable performance.



Fig.4 Analysis result on correlation between original signal and output signal.



Fig.5 Analysis result on correlation between original signal and output signal after phase-shifting processing.

Simulation times	1	2	3	4	5	6	7	8	9
Similarity coefficient	0.9987	0.9993	0.9987	0.9991	0.9996	0.9991	0.9997	0.9993	0.9990
Simulation times	10	11	12	13	14	15	16	17	18
similarity coefficient	0.9995	0.9992	0.9993	0.9992	0.9985	0.9992	0.9991	0.9993	0.9995

 Table 1
 Similarity coefficient between original signal and output signal

A numerical simulation experiment for detector output signal is done in this paper which improves the simulation results of Ref.[20]. In this experiment, a noise signal produced by using random number generator is added to a standard negative exponential signal to simulate the real nuclear signal. The negative exponential signal with noise makes it easier to analyze the smoothing effect of digital S-K filter for noise signal. The result is obtained in Fig.6. When the signal-to-noise ratio of input signal is 10db, the signal-to-noise ratio of output signal can exceed 30 db with digital S-K filter, at filtering parameters k = 5, a = 1.15.



Fig.6 Simulation of nuclear pulse signal of the digital filtering.

It can be seen from the simulating results of the two different signals that the signal quality can improved by digital S-K filter. This is the same with the analog S-K filter.

4 The application study of digital S-K filter on nuclear spectrum

The original spectrum from the nuclear instruments is not smooth because of the statistical fluctuation. Even there is some noise superposing on the original spectrum. Therefore it is necessary to smooth the spectrum data^[21–28]. Multi-points filter is widely used in spectrum smooth early such as 5-points filter, 7-points filter, 9-points filter and 11-points filter. Recently, some new smoothing methods are coming into people's attention, such as digital S-K filter, FFT **Table 2** Comparison of different smooth methods filter, Kalman filter and wavelet filter^[29–33]. Taking the digital S-K filter as an example, the filter factors k and a can be optimized by computer simulation continuously. Finally a favorable value will be obtained. The result of an original spectrum smoothed by the digital S-K filter is shown in Fig.7.

The smoothing effect is decided by the smoothing factor and the resolution of spectrum. A resolution experiment is carried out based on an EDXRF software system with digital S-K filter. In the experiment, Fe-55 nuclide is used as the radioactive source. The spectrum is shown in Fig.8.



Fig.7 Filtering result of digital S-K filter with k=1, a=1.1.



Fig.8 Measuring result of 55 Fe with digital S-K filtering technique.

Smooth methods	Comparing Factor				
	Resolution Fe (5.89 KeV)	Computation	Smooth factor R		
Without smooth filter	160.3 eV	No	0.7503		
5-points smooth filter	163.3 eV	Small	0.8402		
11-points smooth filter	164.5 eV	Bigger	0.8889		
FFT smooth filter	172.6 ev	Big	0.9348		
Digital S-K filter	166.7 eV	Small	0.9218		

It is difficult for Kalman filter and wavelet filter to be implemented based on hardware platform. And they are usually carried out based on software tools such as Matlab and VBA platform. Therefore, the resolution is usually analyzed by comparing the data from digital S-K filter, FFT filter, 11-points filter and 5-points filter with the original data from the detectors. At the same time, the computation of different types of data processing methods is compared as well. A study on the smooth factor of spectrum is carried out based on a mixed sample (SiSCaTiFe sample) by using an EDXRF analyzer with W target, 20 kV, 50 μ A and

Si-PIN detector. Corresponding to the experiment results respectively from the four types filters and without filter, quartic-polynomial-fitting curves are shown in Fig.9 to Fig.12. The smoothing factor of

different methods is evaluated between the fitting data and measuring data. And the smoothing factor of spectrum is presented as R^2 . The comparison analysis results are presented in Table 2.

 Table 3
 Comparison of peak position with different smooth methods

Smooth methods	Peak position						
	Ca(3.69 kev)	Ti(4.51 kev)	Fe(6.40 kev)	W(8.40 kev)			
Without smooth filter	247	321	493	673			
5-points smooth filter	247	321	493	673			
FFT smooth filter	247	321	493	673			
Digital S-K filter	249	323	495	675			

In summary, smoothing factor of the spectrum will be improved after digital filtering process while the resolution is deteriorated. That is to say, the better the smoothing factor is, the worse the resolution is. And multi-points filters are widely used in the spectrum processing software in order to acquire favorable resolution. However, digital S-K filter can both have the advantages of improving smoothing factor and keeping spectrum resolution. And it can also be applied in some occasions where the spectrum resolution is not the most prior factor.



Fig.9 Spectrum from 1000 to 2030 channel without smooth filter.

All the data from digital S-K filter which can be seen from Table 3 has been deviated two channels backwards. And it is different from the other smooth methods. The data deviation is caused by the characteristics of the iterative algorithm of digital S-K filter which is processing spectrum data from left to right. Improving the algorithm is the main solution to rectify spectrum deviation caused by digital S-K filter. And three steps should also be taken such as two channels left-shifting of the data from the filter, discarding the first two data and filling zero into the last two channels.



Fig.10 Spectrum from 1000 to 2030 channel with 11-points smooth filter



Fig.11 Spectrum from 1000 to 2030 channel with FFT filter



Fig.12 Spectrum from 1000 to 2030 channel with digital S-K smooth filter.

5 Conclusion

A new digital S-K filter with two shaping factors is established by simulation of the analog S-K filter and its numerical analysis. After some tests of signal processing on real-time, it verifies that the new filter can be widely used to data smoothing and signal processing on real-time. In addition, comparing with some other methods, it is a successful attempt that the digital S-K filter is introduced to data smoothing in nuclear spectrum. In summary, featuring as simple algorithm, easy factor selection and excellent results, the filter has some advantages in data processing of nuclear spectrum on real-time.

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