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NUCLEAR SCIENCE AND TECHNIQUES

Nuclear Science and Techniques, Vol.17, No.1 (2006) 61–64

Non-dimensional analysis of static bifurcation in a natural circulation loop

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Abstract The steady-state characteristics of a two-phase natural circulation loop were investigated based on the homogenous model. Transcendental equations of non-dimensional loop mass flow rate under various conditions were also derived. The static bifurcation diagram of a two-phase natural circulation described with non-dimensional variables $N_{\rm pch}$-m^+ was obtained. In addition, various steady-state characteristics of a natural circulation loop were analyzed and discussed. These characteristics include the existence of multiple solutions under certain conditions, and the maximum mass flow rate. The authors also examined the effects of important parameters such as sub-cooling number, riser-to-heated-region length ratio, and riser-to-heated-region diameter ratio.

Key words Non-dimensional analysis, Two-phase natural circulation, Static bifurcation

CLC numbers TL33, TK12.

1 Introduction

Because of their simplicity, high heat transfer capability and passive nature, two-phase natural circulation loops have been adopted in many industrial applications. Since Wallis and Hessley [1] reported bouyancy bifurcation in 1961, extensive attention has been paid to these loops by academic researchers studying the complex interaction between heat transfer and fluid flows in the loop.

Unlike a forced circulation system, the loop mass flow rate in a natural circulation loop is not an independent variable, but is a function of heating power, inlet fluid sub-cooling, system pressure, and loop configuration. Natural circulation boiling loops present a more complicated relationship between loop mass flow rate and heating power. In general, the loop mass flow rate increases with increasing heating power in a low power region and decreases with increasing power in a high power domain; and thus a maximum loop mass flow rate also appears at an intermediate heating power. Ramos [2] and Knaoni [3] reported static bifurcation phenomenon in two-phase natural circulation. Also, Lee and Lee [4] and Yao [5] noted the existence of multiple steady state flow rates at a given power, and considered the range of heat flux with multiple solutions of flow rate to be statically unstable. Jeng [6] also studied two-phase flow characteristics in a natural circulation loop using the drift-flux model taking the flow pattern change into account.

In this study we have investigated the two-phase flow and heat transfer characteristics of a natural circulation loop based on a homogenous model with non-dimensional analysis.

2 Theoretic model

2.1 Assumption

A typical natural circulation loop is defined in Fig.1. Analysis of the current model is focused on the heated region, into which the fluid flows at a sub-cooled state. For simplicity, the following assumptions are made concerning the model:

Received date: 2005-06-30

(1) System pressure is constant, and the liquid and vapor properties may also be considered as constant and can be evaluated at the saturated state corresponding to the system pressure;

(2) The liquid density in the single-phase region is considered as a function of temperature;

(3) One-dimension flow;

(4) The heat flux applied at the heated section is uniformly distributed and the riser and down-comer are adiabatic.

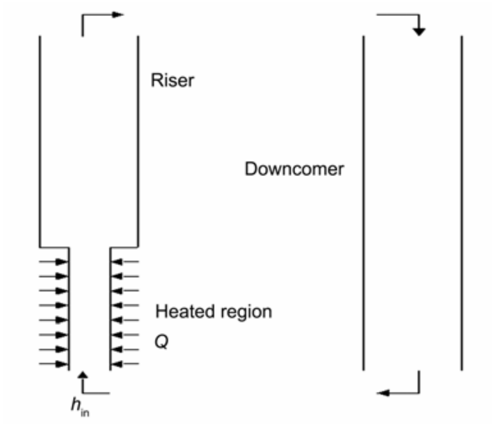

Fig.1 Schematic diagram of a simple two-phase natural circulation loop.

2.2 Non-dimensional analysis

It is well established that, at lower power, the gravitational pressure drop dominates the loop pressure drop, and the gravitational pressure drop decreases with an increasing void fraction in the heated region and riser. Hence, the loop mass flow rate increases with an increase in heating power. On the other hand, in the high power region, the frictional pressure drop becomes dominant and increases with an increase of void fraction. Consequently, the loop mass flow rate decreases with an increase of heating power. Hence, a maximum loop mass flow rate appears at an intermediate heating power. The maximum mass flow rate can be selected as the characteristic mass flow rate to nondimensionalize the transcendental equations.

$$u_{\mathrm{ref}} = \sqrt{\frac{gD_{\mathrm{h}}}{f_{2\Phi}}} \tag{1}$$

$$m_{\mathrm{ref}} = \rho_{\mathrm{f}} A_{\mathrm{h}} \sqrt{\frac{gD_{\mathrm{h}}}{f_{2\Phi}}} \tag{2}$$

where $f_{2\Phi} = 2 f_{1\Phi}$, and $f_{1\Phi} = 0.213 / Re^{0.241}$.

Define the reference value of pressure $p_{\mathrm{ref}} = \rho_{\mathrm{f}}\, gL_{\mathrm{down}}$, and the reference length and diameter are the length and diameter of the heated region. The reference density is the density of saturated liquid.

With these characteristic values, a transcendental equation is obtained:

$$-m^{+2} \oint \frac{\Lambda}{A^{+^2}} \mathrm{d}\, z^{+} + \frac{1}{Fr} \oint \rho^{+} \mathrm{d}\, z^{+} = 0 \tag{3}$$

Let $b = \oint \rho^{+} \mathrm{d}\, z^{+}$ and $K = \oint \frac{\Lambda}{A^{+^2}} \mathrm{d}\, z^{+}$, respectively, represent non-dimensional buoyancy and resistance, the static equation is as follows:

$$\frac{1}{Fr} \times \frac{b}{K} - m^{+2} = 0 \tag{4}$$

3 Static bifurcation phenomenon and effect of parameters

3.1 Static bifurcation phenomenon

The non-dimensional mass flow rate is presented in Fig.2 as a function of non-dimensional heating power with the following parameters: system pressure is 0.2 MPa, inlet sub-cooling number is 48, heating region length is 1 m, diameter is 0.02 m, and without local friction.

It is shown that in the very low power region, single-phase flow prevails only in the loop and the loop flow rate increases as expected. Once the two-phase flow is initiated, the flow rate rises very quickly with heating power until it reaches its maximum value. Consequently, the flow rate decreases, increasing the heating power as well. Multiple steady-state solutions are received when non-dimensional heating power is between N_{pch1} and N_{pch2}, and points B and C are the bifurcation points. When $N_{\mathrm{pch}} < N_{\mathrm{pch1}}$ or $N_{\mathrm{pch}} > N_{\mathrm{pch2}}$, the system has only one static solution. When $N_{\mathrm{pch1}} < N_{\mathrm{pch}} < N_{\mathrm{pch2}}$, the system has three steady-state solutions. Boiling two-phase natural circulation loop revealed nonlinear characteristics. The homogeneous model opens out the static excursion characteristics of the two-phase natural circulation loop. The sudden reduction of the mass flow rate at some heating power can affect the safe operation of the two-phase natural circulation loop, to which special attention should be paid.

Fig.3 shows the relationship between mass vapor quality and void fraction at different system pressures (liquid-vapor density ratio). With the increase of pressure, $x-\alpha$ curve changes gradually, which is caused by decrease of liquid-vapor density difference. In the homogeneous model, the relationship between void fraction and mass vapor quality is affected mainly by the liquid-vapor density ratio.

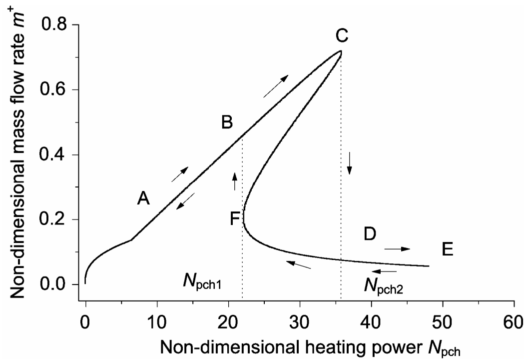

Fig.2 Static bifurcation diagram.

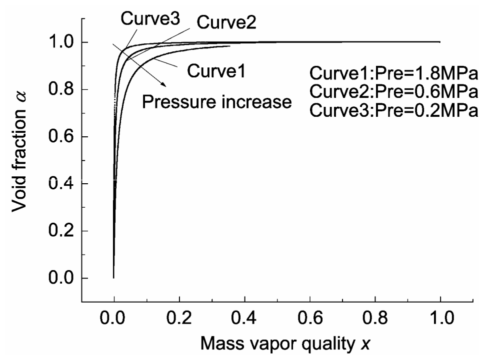

Fig.3 $x-\alpha$ relationship at different pressures.

3.2 Effects of some parameters

Study on the effects of changing important parameters of two-phase natural circulation loops is equally very important. Characteristics of two-phase natural circulation loops are varied due to the effects of changing important parameters, such as sub-cooling number, pipe size and local friction coefficient.

Fig.4 shows the effect of inlet sub-cooling number on the bifurcation diagram. Notably, the non-dimensional mass flow rate is of the order of unity, and the non-dimensional heating power is of the order of ten, revealing that the mass flow rate is a representative characteristic. Fig.4 also reveals the incipient power for two-phase flow increases with an increase of inlet sub-cooling number. The ascending slope in the beginning of the two-phase region decreases with an increase of the inlet sub-cooling number. The non-dimensional maximum loop mass flow rate is insensitive to the inlet sub-cooling number. The non-dimensional maximum power increases with an increase of the sub-cooling number. The region of multiple steady-state solutions exists only if the inlet sub-cooling of working fluid is high enough. Moreover, the power range of multiple solution region increases with the increase of the sub-cooling number.

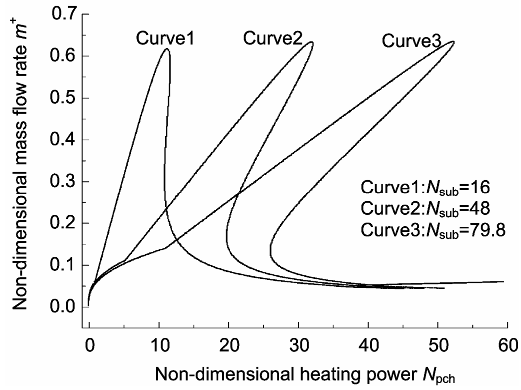

Fig.4 Effect of sub-cooling number on loop mass flow rate.

Fig.5 summarizes the effects of the riser-to-heated-region length ratio on the non-dimensional mass flow rate of a natural circulation loop. According to the figure, the maximum mass flow rate, the incipient power of two-phase flow and the slope of the flow increase all increase with increasing length ratio. Since increasing the riser length leads to the increase of the gravity head, the frictional pressure drop would not expand significantly.

The effect of riser-to-heated-region diameter ratio on the non-dimensional mass flow rate is displayed in Fig.6, resembling those of length ratio. Notably, the alternation of riser-to-heated-region diameter ratio does not affect the gravitational pressure head. Moreover, increasing the riser diameter ratio will reduce the frictional pressure drop in the riser, thus enhancing the loop mass flow rate.

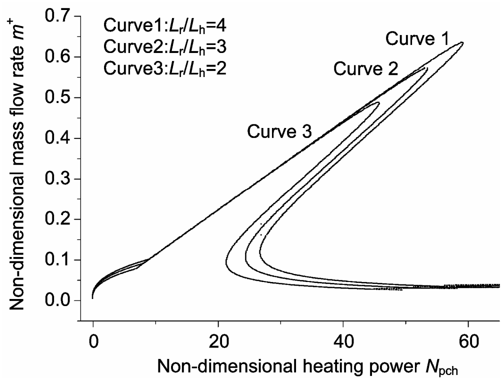

Fig.5 Effect of length ratio of rising to heating section on the bifurcation diagram.

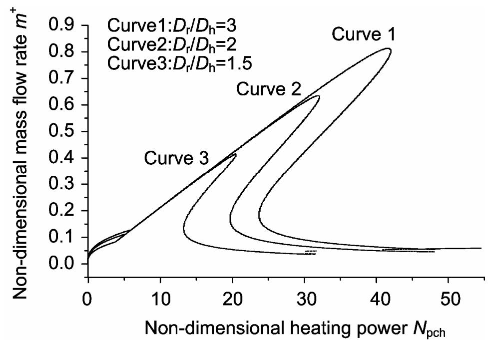

Fig.6 Effect of diameter ratio of rising heating section on the bifurcation diagram.

4 Conclusions

This study theoretically explores the steady-state characteristics of a two-phase natural circulation loop based on the homogeneous model with non-dimensional analysis. Based on the results of this study, we conclude that the homogeneous model with non-dimensional analysis can deduce the major static bifurcation of a natural circulation loop and the region of multiple steady-state solutions exists only if the inlet sub-cooling of working fluid is high enough. Moreover, the power range of the multiple solution region increases with an increase of sub-cooling number; and increasing the riser-to-heated-region length or diameter ratios can enhance the loop mass flow rate.

Nomenclature

A pipe cross-sectional area (m^2)
D pipe diameter (m)
L length (m)
Z axial coordinate
f friction factor
ρ density (kg/m^3)
v specific volume (m^3/kg)
Q heating power (kW)
q'' heat flux (kW/m^2)
h enthalpy (kJ/kg)
m mass flow rate (kg/s)
p pressure (MPa)
x mass vapor quality
u velocity (m/s)

N_{sub} sub-cooling number, $N_{sub} = \dfrac{(h_f - h_{in})v_{fg}}{h_{fg}v_f}$

$N_{pch} = \dfrac{Qv_{fg}}{m_{ref}h_{fg}v_f}$

Eu Euler number, $Eu = \dfrac{p_{ref}}{\rho_f u_{ref}^2}$

Fr Froude number, $Fr = \dfrac{u_{ref}^2}{gL_h}$

Λ friction number, $\Lambda = \dfrac{fL_h}{2D}$

Re Reynolds

Superscript

+ non-dimensional variables

Subscripts

1Φ single-phase
2Φ two-phase
f saturated liquid phase
h heating section
r riser section
down down-comer
ref reference variables

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