

The measurement of electron beam transverse sizes by synchrotron radiation interferometry for HLS II

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Abstract In order to improve accuracy of electron beam transverse sizes measurement, the vertical and horizontal synchrotron radiation light interferometers have been designed to measure beam transverse sizes for the upgrade project of Hefei Light Source (HLS II). In this paper, the light intensity distributions of the interferogram are discussed, and parameters of the interferometers are optimized. The interferograms are simulated under MATLAB. Taking integrations of two-dimensional interferogram intensity, normalized one-dimensional interference intensities distribution of the vertical and horizontal interferogram are obtained. The two curves are fitted to compare the beam transverse sizes with the designed sizes, and this verifies the feasibility of this method. Finally, several factors that affect the beam sizes measurement are discussed.

Key words Beam transverse sizes, Synchrotron radiation interferometer, Interferogram, Simulation

1 Introduction

Transverse sizes of an electron beam, i.e. the vertical and horizontal sizes, are fundamental parameters on beam status diagnostics of an electron synchrotron. A conventional and direct method for measuring beam sizes is to image the beam spot on CCD camera and calculate the size from the image coordinates^[1], but measuring an electron beam of small size in this way is of low accuracy. Thus, a double-slit synchrotron radiation (SR) interferometer was used for small beam size measurement at KEK Photon Factory^[2], inspired by the interferometry measurement of angular diameter of a star by A.A. Michelson. The method is based on the Van Cittert–Zernike theorem: an electron beam of smaller size has a more distinct interference fringe, hence a suitable method to measure a small size light source^[2–7].

Currently at Hefei Light Source (HLS), the horizontal and vertical transverse sizes are about 0.5 mm and 0.25 mm, respectively. In the project to upgrade HLS, the horizontal and vertical transverse

sizes will be, respectively, 0.126 mm and 0.06 mm at 1% coupling. An SR interferometer must be developed for monitoring the SR beam spot. In this paper, the design of SR light interferometry measurement system of electron beam transverse sizes is described.

2 Interferometry measurement system

Figure 1 shows schematically the interferometry measurement system.

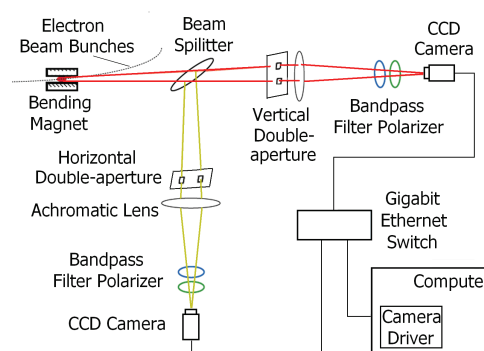


Fig.1 Layout of beam size measurement system.

Two interferometers based on wave-front division two-beam interferometer using polarized

quasi-monochromatic light are used. The SR light source is setup at a bending magnet of electron beam orbit. The SR beam f is half reflected by a beam splitter. The original SR beam is used to measure the vertical beam size, and the reflected SR beam is used to measure horizontal beam size. The SR beam passes through a double-aperture, places at L (in mm) from the light source. Each aperture is of a (horizontal) mm $\times$ b (vertical) mm, and the separation between two apertures is D_x (horizontal) and D_y (vertical). An achromatic lens to focus the SR beam, with a focal length of f_x (horizontal) and f_y (vertical), is placed a short distance behind the double-aperture. A polarizer and a bandpass filter at mean wavelength λ_0 are used to obtain polarized quasi-monochromatic light^[2]. The beams diffracted by double-aperture interfere at the focal plane of objective lens, and the interferogram is acquired by a digital CCD camera, which sends image data to the Gigabit Ethernet. A computer, on which the LabVIEW software development environment is installed, calculates the image data from LAN. Interferogram display and beam size calculation codes will be developed.

3 Basic principle

Taking the vertical SR interferometer as an example (Fig.2), a quasi-monochromatic SR beam with mean wavelength λ_0 is diffracted and divided by double-aperture. The intensity of one quasi-monochromatic beam diffracted from upper and low apertures are defined as I_1 , and I_2 , respectively. Suppose that I_1 and I_2 are superposed at some point $P(x_i, y_i)$ of the image plane (focal plane). The Δz in Fig.2 can be omitted.

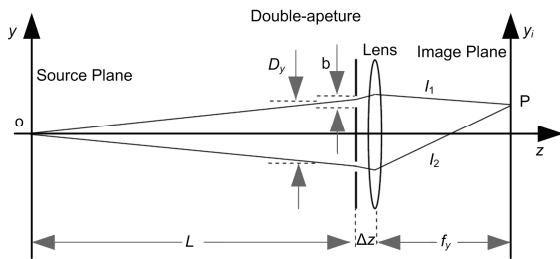


Fig.2 Schematic diagram of vertical interferometer.

Supposed $I_1 = I_2$, the total light field intensity $I_v(P)$ of vertical interferogram at P is given by^[4,8].

$$I_v(P) = 2I_0 a^2 b^2 \text{sinc}^2[\pi a x_i / (\lambda_0 f_y)] \text{sinc}^2[\pi b y_i / (\lambda_0 f_y)] \cdot \{1 + |\gamma_y| \cos[2\pi D_y y_i / (\lambda_0 f_y)]\} \quad (1)$$

where, γ_y is the vertical complex degree of coherence, I_0 is the central intensity of I_1 and I_2 .

Integrating Eq.(1) in x direction, one has the distribution of interference fringes in y direction as

$$I(y_i) = 2I_0 a^2 b^2 \text{sinc}^2[\pi b y_i / (\lambda_0 f_y)] \{1 + |\gamma_y| \cos[2\pi D_y y_i / (\lambda_0 f_y)]\} \quad (2)$$

According to the Van Cittert–Zernike theorem, the degree of coherence $|\gamma_y|$ is equal to the absolute value of the normalized Fourier transform of the light source intensity function^[8]. Approximately, the electron beam can be represented as a two-dimensional Gaussian distribution. So the degree of coherence $|\gamma_y|$ in vertical direction is given by

$$|\gamma_y| = \frac{\left| \int \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-y_0}{\sigma_y}\right)^2} e^{-i2\pi v y} dy \right|}{\left| \int \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-y_0}{\sigma_y}\right)^2} dy \right|} \quad (3)$$

where, v is spatial frequency at light source point,

$$v = D_y / (\lambda_0 L) \quad (4)$$

From Eq.(3), the coherence degree is.

$$|\gamma_y| = \exp\{-[2\pi D_y \sigma_y / (\lambda_0 L)]^2 / 2\} \quad (5)$$

As long as the value of $|\gamma_y|$ is known, the σ_y considered as vertical size of electron beam could be calculated. A curve fitting can be made according to Eq.(2), so as to match with the image of interference fringes acquired by CCD and obtain the coherence degree $|\gamma_y|$.

Similarly, the total light field intensity $I_h(P)$ of horizontal interferogram formed by the horizontal interferometer is given below.

$$I_h(P) = 2I_0 a^2 b^2 \text{sinc}^2[\pi a x_i / (\lambda_0 f_x)] \text{sinc}^2[\pi b y_i / (\lambda_0 f_x)] \cdot \{1 + |\gamma_x| \cos[2\pi D_x x_i / (\lambda_0 f_x)]\} \quad (6)$$

where, γ_x is horizontal complex degree of coherence.

Integrating Eq.(6) in y direction, distribution of interference fringes in x direction $I(x_i)$ is given by.

$$I(x_i) = 2I_0 a^2 b^2 \text{sinc}^2[\pi a x_i / (\lambda_0 f_x)] \{1 + |\gamma_x| \cos[2\pi D_x x_i / (\lambda_0 f_x)]\} \quad (7)$$

The horizontal degree of coherence $|\gamma_x|$ is

$$|\gamma_x| = \exp\{-[2\pi D_x \sigma_x / (\lambda_0 L)]^2 / 2\} \quad (8)$$

By the same method as the vertical direction, the horizontal electron beam size σ_x can be obtained.

4 Simulation of interference

4.1 Optimization of SR Interferometers

We plan to use Leutron PicSight G32M-GigE digital camera with 656×494 pixels to acquire the interference image, its image sensor size is $6.5241 \text{ mm} \times 4.8906 \text{ mm}$ ^[9]. This camera could be sensitized by the light range from 400 nm to 1000 nm. Considering the SR light intensity and beam size, we will use the bandpass filter centered at 550 nm with 40 nm bandwidth to extract 550nm quasi-monochromatic SR beam.

Because of space limitation at HLS II, the distances from the light source to vertical and horizontal double-apertures are 7.2 m. With the vertical and horizontal transverse beam sizes of 0.06 mm and 0.126 mm, respectively, changes in the coherence degree, as a function of aperture separation, can be calculated with Eqs.(5) and (8) (Fig.3).

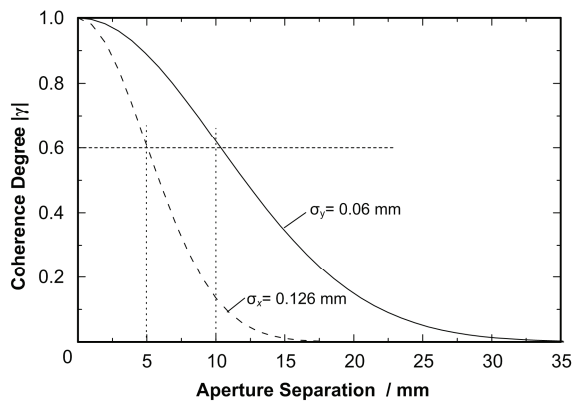


Fig.3 Coherence degree vs aperture separation.

It is advisable to measure beam size in condition of coherence degree range of 0.1–0.6^[5]. To reduce the background noise, the interferometer parameter setting should make the coherence degree range from 0.3 to 0.9^[6]. So we will use interference fringes with coherence degree 0.6 to measure beam size. From Fig.3, at vertical double-aperture separation $D_y=10 \text{ mm}$, the vertical coherence degree is 0.6356, while at horizontal double-aperture separation $D_x=5 \text{ mm}$, the horizontal coherence degree is 0.6068, which are suitable for beam sizes measurement on HLS II.

Optimized parameters of the beam spot size monitoring system are listed in Table 1. Taking SR light intensity, the size of CCD camera and wave front error into account, the apertures are of $1.0 \text{ mm} \times 1.0$

mm in size^[4]. Considering fringe spacing and CCD camera size, the focal length of vertical interferometer and horizontal lenses is 2 m.

Table 1 Optimized parameters of interferometers.

Light source-to-double-aperture distance L / m	7.2
Vertical aperture separation (D_y) / mm	10
Horizontal aperture separation (D_x) / mm	5
Aperture size ($a \times b$) / mm	1.0×1.0
Vertical focal length (f_y) / m	2
Horizontal focal length (f_x) / m	2

4.2 Simulation

Substituting the vertical interferometer parameters to Eq.(1), intensity distribution of vertical interferogram in the CCD sensor coordinate system can be simulated under MATLAB, as show in Fig.4. Summing the intensity distribution in x direction of Fig.4, we have the normalized intensity distribution curve in vertical direction as shown in Fig.5.

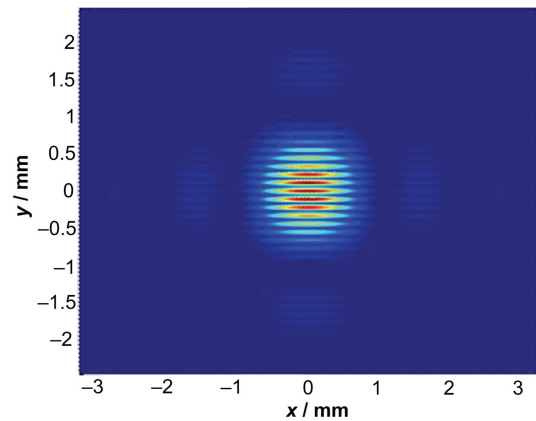


Fig.4 Simulation of the vertical interferogram.

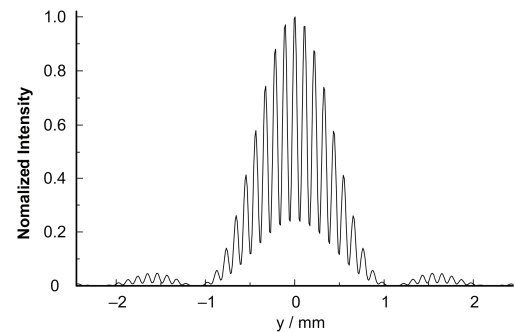


Fig.5 Normalized intensity distribution in vertical direction.

According to Eq.(6), intensity distribution of the horizontal interferogram, as shown in Fig.6.

Summing the intensity distribution in y direction of Fig.6, we can get normalized intensity distribution curve in horizontal direction, as shown in Fig.7.

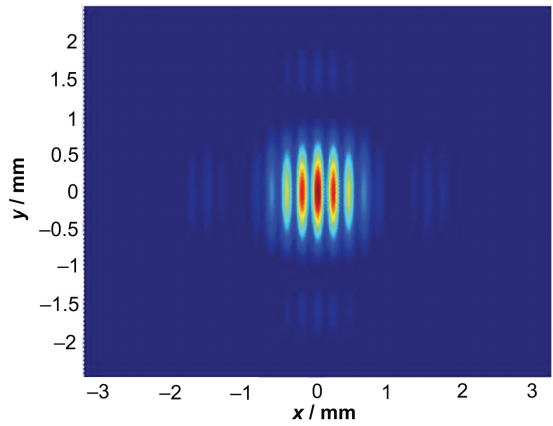


Fig.6 Simulation of horizontal interferogram.

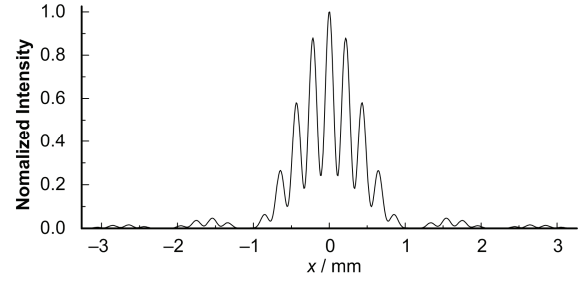


Fig.7 Normalized intensity distribution in horizontal direction.

Using Eqs.(2) and (7), we can match the intensity distribution curves, and make curve fittings in the vertical and horizontal direction. The fitting models used in the algorithm are given in Eq.(9)

$$\begin{cases} I(y_i) = a_1 + a_2 \text{sinc}^2 \left[\frac{\pi b(y_i - a_3)}{\lambda_0 f_y} \right] \left[1 + a_4 \cos \left(\frac{2\pi D_y y_i}{\lambda_0 f_y} + a_5 \right) \right] \\ I(x_i) = b_1 + b_2 \text{sinc}^2 \left[\frac{\pi a(x_i - b_3)}{\lambda_0 f_x} \right] \left[1 + b_4 \cos \left(\frac{2\pi D_x x_i}{\lambda_0 f_x} + b_5 \right) \right] \end{cases} \quad (9)$$

where, a_1 and b_1 are background values, a_2 and b_2 are central intensities, a_3 and b_3 are central positions of interference distributions, a_4 and b_4 are vertical and horizontal coherence degree, a_5 and b_5 are phase offsets of interference fringes, respectively.

With the fitting results of $a_4 = 0.6340$ and $b_4 = 0.6050$, we have $\sigma_y = 0.06$ mm, $\sigma_x = 0.126$ mm, which equal with designed transverse sizes of electron beam.

5 Analysis of effect factors

5.1 Bandwidth of bandpass filter

Because the SR light is not monochromatic, bandpass filter centered at 550 nm with 40 nm bandwidth is

used to extract quasi-monochromatic SR light. Its impact on the beam sizes measurement should be analyzed. The normalized wavelength transfer function $g(\lambda)$ of the bandpass filter is assumed as a Gaussian distribution function.

$$g(\lambda) = \exp \left\{ -[(\lambda - \lambda_0)/\sigma_\lambda]^2 / 2 \right\} \quad (10)$$

where, λ_0 is mean wavelength of the bandpass filter, σ_λ^2 is the variance of $g(\lambda)$, and σ_λ is defined below^[7].

$$\sigma_\lambda = \lambda_{\text{FWHM}} / (8 \ln 2)^{0.5} = 40 \text{ nm} / 2.355 = 17.0 \text{ nm} \quad (11)$$

Integrating over the bandpass filter, Eqs.(2) and (7) should be expressed by Eq.(12)^[7].

$$\begin{cases} I(y_i) = 2I_0 a^2 b^2 \text{sinc}^2 \left(\frac{\pi b y_i}{\lambda_0 f_y} \right) \cdot \left(1 + \exp \left[-2 \left(\frac{\pi \sigma_y D_y}{\lambda_0 L} \right)^2 - 2 \left(\frac{\pi \sigma_\lambda D_y y_i}{\lambda_0^2 f_y} \right)^2 \right] \cos \left(\frac{2\pi D_y y_i}{\lambda_0 f_y} \right) \right) \\ I(x_i) = 2I_0 a^2 b^2 \text{sinc}^2 \left(\frac{\pi a x_i}{\lambda_0 f_x} \right) \cdot \left(1 + \exp \left[-2 \left(\frac{\pi \sigma_x D_x}{\lambda_0 L} \right)^2 - 2 \left(\frac{\pi \sigma_\lambda D_x x_i}{\lambda_0^2 f_x} \right)^2 \right] \cos \left(\frac{2\pi D_x x_i}{\lambda_0 f_x} \right) \right) \end{cases} \quad (12)$$

With Eq.(12), the one dimensional interference intensity distributions can be obtained; and using Eq.(9), the curve fitting results are $\sigma_y=0.0670$ mm, and $\sigma_x=0.1328$ mm. They are comparable with the designed sizes, and they increases with the bandwidth.

5.2 Photons intensity imbalance at the apertures

There is an imbalance between two beams pass through the double-aperture^[4], with the imbalance ratio β being defined as

$$I_{01}=I_{02}/\beta=I_0 \quad (13)$$

The total light field intensity distributions in vertical and horizontal direction are given by

$$\left\{ \begin{array}{l} I_v(P) = I_0 a^2 b^2 \text{sinc}^2\left(\frac{\pi a x_i}{\lambda_0 f_y}\right) \text{sinc}^2\left(\frac{\pi b y_i}{\lambda_0 f_y}\right) \\ \quad \cdot \left(1 + \beta_y + 2\sqrt{\beta_y} |\gamma_y| \cos\left(\frac{2\pi D_y y_i}{\lambda_0 f_y}\right)\right) \\ I_h(P) = I_0 a^2 b^2 \text{sinc}^2\left(\frac{\pi a x_i}{\lambda_0 f_x}\right) \text{sinc}^2\left(\frac{\pi b y_i}{\lambda_0 f_x}\right) \\ \quad \cdot \left(1 + \beta_x + 2\sqrt{\beta_x} |\gamma_x| \cos\left(\frac{2\pi D_x x_i}{\lambda_0 f_x}\right)\right) \end{array} \right. \quad (14)$$

where, β_y and β_x are vertical imbalance ratio and horizontal imbalance ratio, respectively.

The simulated measurement error as a function of the imbalance ratio is shown in Fig.8. The errors caused by imbalance of two modes of photons passing through the double-aperture are not serious under small imbalance ratio (β). Therefore, the two apertures should be symmetrical to the SR axis, so as to minimize the imbalance effect on HLS II.

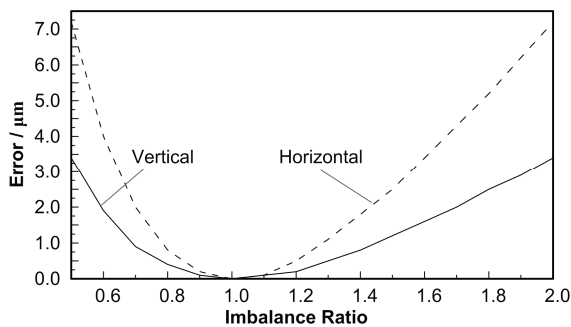


Fig.8 Simulated measurement error vs imbalance ratio.

5.3 The other factors

The optical components between the source and interferogram may introduce wave-front errors. To

minimize the wave-front errors, small double-aperture opening (1.0 mm × 1.0 mm) is used^[4].

The SR light is not monochromatic, so there is chromatic aberration when the SR lights pass the lens, so achromatic lenses are used to decrease chromatic aberration, and beam transverse sizes of HLS II are far larger than chromatic aberration. Therefore, we could neglect the effect of chromatic aberration.

Dark noise of image sensor may affect quality of interference image, which can be modeled by a Gaussian distribution with mean μ_d and variance σ_d^2 ^[10]. Other stray lights will impact on the interferogram as well. The noise in the image must be suppressed by image smoothing operations before curve fitting.

6 Conclusions

The vertical and horizontal interferometers had been designed at HLS II. We simulated the measurement of electron beam size by interferometry. Next, we will make some improvements on the designed interferometer and manufacture optical components. The SR interferometers will be realized before long.

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