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Methodology for reliability allocation based on fault tree analysis and dualistic contrast

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Abstract Reliability allocation is a difficult multi-objective optimization problem. This paper presents a methodology for reliability allocation that can be applied to determine the reliability characteristics of reactor systems or subsystems. The dualistic contrast, known as one of the most powerful tools for optimization problems, is applied to the reliability allocation model of a typical system in this article. And the fault tree analysis, deemed to be one of the effective methods of reliability analysis, is also adopted. Thus a failure rate allocation model based on the fault tree analysis and dualistic contrast is achieved. An application on the emergency diesel generator in the nuclear power plant is given to illustrate the proposed method.

Key words Reliability allocation, Fault tree analysis, Dualistic contrast

CLC numbers TL364^{+}.2, TL38^{+}.7

1 Introduction

The safety goal of nuclear power plants (NPPs) is to reduce the likelihood of threatening situations to the lowest degree possible. To accomplish this, various approaches are adopted, such as overall safety analysis, and diversity and redundancy concepts in NPP design. However, it is impossible to reduce the risk to zero because of all kinds of technical and economical constraints, hence the development of various methods of reliability allocation[1,2], which can be applied to determine the reliability characteristics of systems, subsystems, major equipment in an NPP to improve its design, maintenance and safety[3].

Reliability allocation, as an important task in design or maintenance of NPPs, is an optimization process to minimize the total costs subject to overall plant safety goal constraints. The simplest method for allocating reliability is to distribute the reliabilities uniformly to all components. This simplifies the calculation, but the results deviate severely from the fact. Other methods for reliability allocation, such as uniform allocation and AGREE allocation, are not generally considered as the best way to allocate reliability for an NPP system, hence a need of optimizing reliability allocation for safety of NPPs.

On the other hand, it is a difficult task to take many factors into consideration, such as importance, technical level, cost, complexity, etc. Some of factors can be calculated or measured quantitatively, while others can just be assessed qualitatively[4], such as complexity, for which no calculation can be done with any applicable mathematic expression and no measurement can be done exactly, either.

In this paper a new method is developed based on fault tree analysis (FTA), dualistic contrast and fuzzy decision, so as to achieve reliability allocation for NPP systems. It can be used to deal with setting the reliability goals for individual subsystems so that a specified reliability can be met, with well-balanced factors.

2 Criteria of reliability allocation

Lots of basic reliability indexes are adopted such as failure rate, availability, and mean time between

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failures during the reliability allocation. In this paper, performance of an NPP system is represented by the λ parameters. For an exponential life distribution, failure rate of the system is constant. Then, λ_s represents failure rate of the overall system and λ_i is failure rate of the i^{th} subsystem (i = 1, 2, ….). For a given level of failure rate of the system, λ_s is allocated into the subsystems in such a way that the system failure rate does not exceed the specified value. The system failure rate can be obtained by solving the equation of $f(\lambda_1, \lambda_2, \ldots) = \lambda_s$.

A reliability allocation must have constraints. The following restriction factors, and their reliability, are chosen.

2.1 Structure importance

Structure importance is analyzed by minimizing cut sets of the fault tree. The whole system fails when structure importance of a subsystem is 1. A subsystem of higher component importance is allocated with a high reliability.

2.2 Mode importance

Mode importance to recognize weakness of the system design can be calculated by the FTA. A subsystem with higher mode importance has a weaker reliability. It is allocated with a higher reliability, too.

2.3 Complexity

Complexity is a complicated task. We have to count the number of essential parts of the subsystem to show the degree of complexity based on the structure of a subsystem. A subsystem with higher complexity fails more frequently. It is difficult to improve the reliability, and the complex subsystem is allocated with a low reliability.

2.4 Technical level

Technical level is a fuzzy factor. It includes standards or quality of the design, manufacturing, maintenance, and management. A subsystem that is designed, produced, repaired and managed with advanced technology available is relatively easy to assure a high reliability, and is allocated with a high reliability.

2.5 Working condition

Different working conditions (temperature, vibration, concussion, corrosion, radiation, and time of operation as well) have different effects on reliability. A low reliability is allocated to a subsystem in hazardous conditions.

2.6 Cost

A balance should be met between reliability and costs in the system's design, operation, maintenance, etc. A subsystem costs more to improve its reliability, hence a higher reliability.

Of course, based on practical needs, some other factors can be chosen as criterion of reliability allocation, such as maintenance and criticality of failure.

3 The reliability allocation method

The fault tree technique, as a part of probabilistic safety assessment, has been widely applied for improving system reliability of NPPs. A fault tree consists of a top event, intermediate events, primary events and all kinds of gates. The top event is the most important subsystem failure that stops operation of the entire system. For example, reactor core melts is the top event for a reactor, while for an emergency diseal generator, failing to supply the power is the top event. Primary events and fault tree analysis are developed by logical functional relationship named as the top-level fault tree[5,6].

The equipments or components for reliability allocation are decided according to importance of the primary events, and are denoted by $v_1, v_2, \ldots, v_n$. Thus a subsystem vector is $V=(v_1, v_2, \ldots, v_n)$, where n represents the number of subsystems. In this paper, we have n = 8.

And the constraint vector is $U=(u_1, u_2, \ldots, u_m)$, where m is the number of allocation criteria. The six constraints of reliability allocation in § 2 are denoted by $u_1, u_2, \ldots, u_6$. The structure importance and mode importance are calculated by the FTA simulation, while the value of complexity is obtained by counting the number of essential parts of the subsystem based on structure of the system. Average rating scores of the technical level, working condition and cost were provided by relevant experts.

In order to set up a relation between different data of the same constraint, dualistic contrast method is adopted. The failure rate allocation ratio matrix $A^{(k)} = [\alpha_{ij}^{(k)}]_{n\times n}$, where $\alpha_{ij}^{(k)}$ is relative ratio for the k constraint between the i^{th} and the j^{th} subsystems, can be calculated by Eqs.(1) and (2):

$$\alpha_{ij}^{(k)} = \frac{IM_i}{IM_i + IM_j} \tag{1}$$

$$0 \le \alpha_{ij} \le 1 \quad , \quad \alpha_{ij} + \alpha_{ji} = 1 \tag{2}$$

For i and j= 1, 2,…n, k= 1, 2,…m, IM_i and IM_j are absolute weight of the i^{th} and the j^{th} subsystems, respectively. For criterion k, the matrix $A^{(k)}$ can be expressed as:

$$A^{(k)} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{bmatrix}, k = 1, 2 \dots m$$

Obviously $\alpha_{ii} = 0.5$.

The average value of the entries in row of the matrix $A^{(k)}$ represents the relative allocation factor for the i^{th} subsystem based on allocation criterion k. Define:

$$b_{ki} = \sqrt[n]{\prod_{j=1}^{n} \alpha_{ij}^{(k)}} \tag{3}$$

Finally, we can obtain a comprehensive fuzzy decision weight matrix B=$[b_{ki}]_{m\times n}$, where m and n are the number of constraints and subsystems respectively. It represents the relative allocation for all the subsystems.

The importance of every criterion is different in reliability allocation because of their different effects on reliability, so different weight vectors can be obtained by the expert rating method as follows:

$$W = (w_1, w_2, \dots, w_m) \tag{4}$$

where $\sum_{i=1}^{m} w_i = 1, w_i \ge 0, \ i = 1, 2, \dots, 6$.

After working out the comprehensive fuzzy decision matrix B and the weight vector W, fuzzy decision of reliability allocation can be made according to Eq.(5):

$$\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \tilde{v}_4, \tilde{v}_5, \tilde{v}_6) = W \bullet B \tag{5}$$

where $\tilde{v}_i$ represents relative failure rate allocation level of the i^{th} subsystem comparing with other subsystems, and the multiplier (•) represents a fuzzy operator. For every criterion's influence on reliability allocation can be considered, the fuzzy operator is named as M(•, +).

The failure rate λ_s of the whole system allocated to the failure rate λ_i of every subsystem can be plotted out according to the relative failure rate allocation level of the i^{th} subsystem comparing with other subsystems:

$$\lambda_i = \lambda_s \frac{\tilde{v}_i}{\sum_{j=1}^{n} \tilde{v}_j} \tag{6}$$

If failure rate of the subsystem submits to exponential distribution, reliability can be calculated by Eq.(7):

$$R_j = \mathrm{e}^{-\lambda_j T} \tag{7}$$

where T is operation time of the system.

4 Application of reliability allocation model

The emergency diesel generator is one of the important subsystems in the emergency power supply system. The top-level event is that it suddenly fails to supply the power during the operation. Therefore, possible causes were analyzed and a fault tree was developed (Fig.1). It consists of 17 primary events, six intermediate events and seven OR gates. The primary and intermediate events are listed in Table 1.

The mode importance and structure importance can be calculated by using CAFTA Code. Eight equipments or components, i.e. the injection pump (B13), fuel injector (B14), piston ring (B7), cylinder header gasket (B10), exhaust pipe (B6), electric system (B16), air cleaner (B5), and control system (B17) were chosen as the subsystems to be allocated.

The relative allocation factors for subsystem were calculated with Eqs.(1)–(3). The structure importance criterion is β_{1i} =0.5, because the structure importance of every subsystem for this example is 1. The mode importance criterion, β_{2i}, was obtained by β_{2i}= $MO_j/(MO_i + MO_j)$, where MO_i and MO_j are the mode importance of the i^{th} and j^{th} subsystem, respectively. The mode importance and its value of relative allocation ratio of the eight primary events are listed in Table 2.

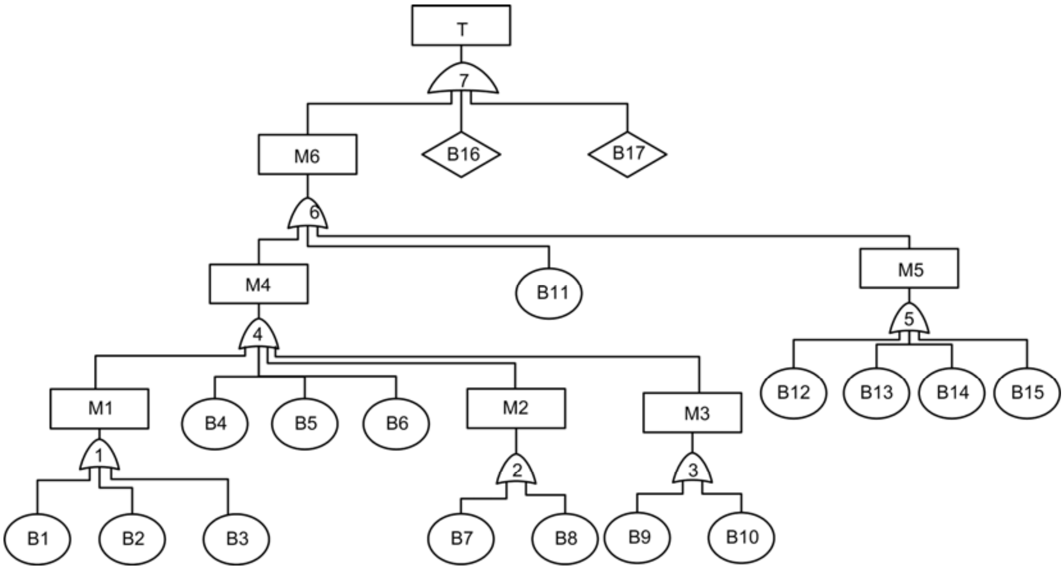

Fig.1 Fault tree: the emergency diesel generator has no power to supply.

Table 1 Six intermediate events and 17 primary events of the fault tree

Code	Event	Code	Event
M1	Failure of gas valve	B7	Lock of piston ring
M2	Lack of compressive stress	B8	Lock of gas valve rod
M3	Gas leaking between cylinder header and body	B9	Looseness of cylinder header nut
M4	Failure of diesel proper	B10	Failure of cylinder header gasket
M5	Failure of fuel supply	B11	Failure of cooling and lubrication system
M6	Failure of power system	B12	Fuel mixed with air in supply system
B1	Failure of gas valve spring	B13	Failure of injection pump
B2	Inaccuracy timing of gas valves	B14	Failure of fuel injector
B3	Error of valve gap	B15	Leak from the injector hole
B4	Too much carbon deposited	B16	Failure of electric system
B5	Jam of air cleaner	B17	Failure of control system
B6	Jam of exhaust pipe		

Table 2 The mode importance and relative allocation ratio of the eight primary events

Subsystem	Mode importance	Relative weight β_{2i}
B5	0.0166	0.76
B6	0.1358	0.33
B7	0.0270	0.67
B10	0.1864	0.27
B13	0.2904	0.20
B14	0.2121	0.24
B16	0.0737	0.45
B17	0.0280	0.67

The complexity criterion, β_{3i}, was derived by $\beta_{3i}=CM_i/(CM_i + CM_j)$, where CM_i and CM_j are the number of the essential parts of the i^{th} and j^{th} subsystem, respectively. The approximate numbers of the essential parts and β_{3i} of each subsystem are listed in Table 3.

Table 3 The approximate number of the essential parts and the relative allocation ratios

Subsystem	Number of essential parts	Relative allocation ratio β_{3i}
B5	8	0.16
B6	7	0.14
B7	48	0.48
B10	24	0.34
B13	66	0.55
B14	240	0.79
B16	18	0.28
B17	169	0.73

For the criteria of the technical level, working condition and cost, no effective reliability data could be calculated, because the fuzzy information, β_{ki}, could just be calculated with $\beta_{ki}= CR_i/(CR_i + CR_j)$, where CR_i and CR_j are average rating scores of the i^{th} and j^{th} subsystem, respectively, provided by relevant experts, k = 4, 5, 6. Then the values of relative allocation ratio for each subsystem were calculated by Eq.(3). The average rating scores from experts and β_{4i}, β_{5i} and β_{6i} are listed in Tables 4 and 5.

Table 4 Average rating scores from the experts and relative allocation ratios

Code	Technical level	Relative allocation ratio β_{4i}	Working condition	Relative allocation ratio β_{5i}
B5	0.90	0.53	0.72	0.54
B6	0.95	0.55	0.90	0.60
B7	0.83	0.51	0.98	0.62
B10	0.98	0.55	0.72	0.54
B13	0.64	0.45	0.58	0.41
B14	0.52	0.39	0.85	0.58
B16	0.50	0.50	0.26	0.31
B17	0.75	0.49	0.10	0.15

Table 5 The average rating scores and relative allocation ratios

Code	Cost	Relative allocation ratio β_{6i}
B5	0.08	0.76
B6	0.02	0.08
B7	0.17	0.37
B10	0.06	0.19
B13	0.55	0.62
B14	0.48	0.59
B16	0.50	0.74
B17	0.82	0.70

The relative failure rate allocation ratios of each subsystem are 0~1. And the fuzzy decision matrix B is

$$B=\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.76 & 0.33 & 0.67 & 0.27 & 0.20 & 0.24 & 0.45 & 0.67 \\ 0.16 & 0.14 & 0.48 & 0.34 & 0.55 & 0.79 & 0.28 & 0.73 \\ 0.53 & 0.55 & 0.51 & 0.55 & 0.45 & 0.39 & 0.50 & 0.49 \\ 0.54 & 0.60 & 0.62 & 0.54 & 0.41 & 0.58 & 0.31 & 0.15 \\ 0.61 & 0.84 & 0.45 & 0.66 & 0.23 & 0.25 & 0.24 & 0.17 \end{bmatrix}$$

The weight vector W obtained by the expert rating method is W = (0.22, 0.15, 0.14, 0.09, 0.12, 0.18).

From Eq.(5), the fuzzy decision $\tilde{v} = W \bullet B$ = (0.5447, 0.4848, 0.5460, 0.4582, 0.3681, 0.4303, 0.3871, 0.4724).

The total reliability of the top-event of no power supply by the emergency diesel generator during 6h-operation is 0.982, calculated by FTA, and the failure rate is 30.273×10^{-4}.

Then, λ_i={4.463×10^{-4}, 3.975×10^{-4}, 4.477×10^{-4}, 3.757×10^{-4}, 3.018×10^{-4}, 3.528×10^{-4}, 3.174×10^{-4}, 3.873×10^{-4}}.

Correspondingly, R_i = {0.9973, 0.9976, 0.9973, 0.9977, 0.9982, 0.9979, 0.9981, 0.9976}.

The results of reliability allocation using the developed method and AGREE method for subsystems of the emergency diesel generator are listed in Table 6. It can be seen that reliability allocation using AGREE method is smaller than that from the developed method except B6. The reason is that much more constraint factors were taken into consideration in the developed method. This makes the result more rational. Reliability allocation of B17 with AGREE method is

0.8277, rather than 0.9976 with the developed method, because the AGREE method calculated the control system (B17, which has a number of components) with only two constraints that weighed 0.5 each.

Table 6 The result of reliability allocation with different methods

Code	Developed method	AGREE method
B5	0.9973	0.9850
B6	0.9976	0.9983
B7	0.9973	0.9458
B10	0.9977	0.9959
B13	0.9982	0.9929
B14	0.9979	0.9652
B16	0.9981	0.9924
B17	0.9976	0.8277

5 Conclusion

A reliability allocation model was developed based on fault tree analysis and fuzzy math. The fault tree analysis was implemented for an emergency diesel generator in a nuclear power plant. The results show that the combination of fault tree analysis and fuzzy math is a suitable way to handle the complex reliability allocation process.

Selecting subsystem by the top-level fault tree analysis is to avoid allocating reliability to all equipments or components including unnecessary parts. The importance measure of the subsystem was calculated with this model, which makes the allocation result more rational. Dualistic contrast was used to make fuzzy decision. The fuzzy characteristics of some factors worked out successfully.

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