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Brilliance of novel X-ray source based on bremsstrahlung

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Abstract A novel X-ray source based on tiny target bremsstrahlung and a low energy tabletop synchrotron was developed in Japan. In this paper, its brilliance formula is deducted and compared with BEAMnrc (EGS4) simulation. The brilliance of a tiny target bremsstrahlung X-ray prototype is discussed.

Key words Brilliance, Bremsstrahlung, X-ray source

CLC number O57

1 Introduction

As a powerful tool, X-rays are used in medical imaging, material and life sciences. For many applications, a compact X-ray source is desired. A novel X-ray source, based on tiny target bremsstrahlung and a low energy tabletop synchrotron, has developed in Japan [1-3]. It has some advantages over conventional X-ray sources [2].

As an important characteristic for an X-ray source, brilliance should be calculated accurately. But the brilliance of this novel X-ray source shall be further investigated, because, for example, the brilliance calculated with the beam current formula given in *Ref.[2]* seemed unconvincingly large. In this work, initiated by developing a prototype tiny target X-ray source, two methods, i.e. analytical method and Monte Carlo simulation, were used for brilliance calculation of the X-ray source, and the results are compared.

2 Brilliance calculation methods

According to bremsstrahlung cross-section and brilliance definition, a formula to calculate brilliance of the prototype X-ray source was derived in Section 2.1 by analytical method. But the formula becomes less reliable when the atomic number increases, when the initial electron energy decreases, when the photons approach the high energy limit, and when θ_0, the angle between the emitting photon and the incident electron, is less than γ^{-2} (γ is the relativistic factor).

Specifications of the prototype X-ray source, such as the photon number, and their energy and angular distribution, are related to energy loss and multiple scattering of the electrons in the tiny target of certain material and thickness, and photon absorption by the target. However, these are not taken into account in the analytical method.

Besides, the results differ in orders of magnitude from different brilliance formulas in *Ref.[2]* and this paper. Therefore, simulation method should be used for the brilliance calculation. With simulation method, we can obtain not only energy distribution and angular distribution of the emitted photons, just as the analytical method does, but also the information related to energy spread of the electrons and the target thickness. In addition, simulation works at high energy region of the X-rays.

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2.1 Analytical method

A general classification of differential cross-section of bremsstrahlung is given by Koch and Motz[4], either from the Born-approximation or the extreme-relativistic formula, which contain the Coulomb correction (E_0 >50 MeV, where E_0 is the initial energy of the electron). The formulas that branch out from the starting point are divided into two groups, designated as screened or non-screened. Because of high electron energy and tiny target size, we use Schiff's formula [4, 5] in this paper.

The cross section expressed in photon energy and angular distributions is given by

$$\mathrm{d}\sigma_{k,y}=(\frac{4Z^2r_o^2}{137})(\frac{\mathrm{d}k}{k})y\mathrm{d}y\{\frac{16y^2E}{(y^2+1)^4E_0}-\frac{(E_0+E)^2}{(y^2+1)^2E_0^2}+ [\frac{E_0^2+E^2}{(y^2+1)^2E_0^2}-\frac{4y^2E}{(y^2+1)^4E_0}]\ln M(y)\} \tag{1}$$

$$\begin{cases} y=E_0\theta_0/\mu \\ M(y)=[(\frac{\mu k}{2EE_0})^2+(\frac{Z^{1/3}}{111(y^2+1)})^2]^{-1} \end{cases} \tag{2}$$

where E_0 and E are the initial and final energy of the electron, $k = E_0–E$ is the energy of the emitted photon, $\mu= mc^2$ is the rest energy of an electron, Z is the atomic number of the target, and r_0 represents the classical radius of electron.

The photon energy distribution is

$$\mathrm{d}\sigma_k=\frac{2Z^2r_o^2}{137}(\frac{\mathrm{d}k}{k})\{[1+(\frac{E}{E_o})^2-\frac{2E}{3E_o}]\times [\ln M(0)+1-\frac{2}{b}\arctan b]+ \frac{E}{E_o}[\frac{2}{b^2}\ln(1+b^2)+ \frac{4(2-b^2)}{3b^2}\arctan b-\frac{8}{3b^2}+\frac{2}{9}]\} \tag{3}$$

$$\begin{cases} b=\frac{2E_oEZ^{1/3}}{111k\mu} \\ M(0)=[(\frac{\mu k}{2EE_o})^2+(\frac{Z^{1/3}}{111})^2]^{-1} \end{cases} \tag{4}$$

When electrons collide with target, the total bremsstrahlung differential cross-section (σ_{total}) can be expressed by the bremsstrahlung differential cross-section ($\mathrm{d}\sigma_{\mathrm{k}}$), target particle density (n_{t}), thickness (χ_t) and cross-section (α_{t}) as

$$\sigma_{\mathrm{total}}=\mathrm{d}\sigma_{\mathrm{k}}n_{\mathrm{t}}\chi_{\mathrm{t}}\alpha_{\mathrm{t}} \tag{5}$$

where $n_{\mathrm{t}} = N_{\mathrm{a}}\rho/A$, N_{a} is the Avogadro constant, ρ is the material density and A is the atomic weight.

The number of electrons, which collide with the tiny target in Δt, can be expressed as

$$N_{\mathrm{e}}=(\frac{I\Delta t}{e})(\frac{\alpha_{\mathrm{t}}}{S_{\mathrm{b}}}) \tag{6}$$

where $I\Delta t/e$ is the total electron number, I is the beam current, $\alpha_{\mathrm{t}}/S_{\mathrm{b}}$ is the probability of collision, and S_{b} is the beam cross-section.

During Δt, the total emitted photon number (N_{br}) is given as

$$N_{\mathrm{br}}=(\frac{\mathrm{d}\sigma_{\mathrm{k}}n_{\mathrm{t}}\chi_{\mathrm{t}}\alpha_{\mathrm{t}}}{\alpha_{\mathrm{t}}})(\frac{I\Delta t}{e})(\frac{\alpha_{\mathrm{t}}}{S_{\mathrm{b}}}) \tag{7}$$

According to its definition, the brilliance can be formulated as

$$B=\frac{N_{\mathrm{br}}}{\Delta t\alpha_{\mathrm{t}}\Omega} \tag{8}$$

where $\Omega = \pi\theta_{\mathrm{br}}^2$ is the solid angle.

Substituting Eqs. (3) and (7) for Eq. (8), the brilliance can be rewritten as

$$B=(\frac{2Z^2r_o^2}{137})(\frac{n_{\mathrm{t}}\chi_{\mathrm{t}}I}{eS_{\mathrm{b}}\Omega})(\frac{1}{1000})\times \{[1+(\frac{E}{E_o})^2-\frac{2E}{3E_o}]\times [\ln M(0)+1-\frac{2}{b}\arctan b]+ \frac{E}{E_o}[\frac{2}{b^2}\ln(1+b^2)+\frac{4(2-b^2)}{3b^2}\arctan b- \frac{8}{3b^2}+\frac{2}{9}]\} \tag{9}$$

2.2 Simulation

Bremsstrahlung simulation of the novel X-ray source was done with BEAMnrc, a general purpose Monte Carlo code for modeling radiotherapy sources. It is based on the EGSnrc code for modeling coupled electron and photon transport.

The photons are tracked by (1) generating an initial electron beam at the target surface, (2) simulating the beam in the target, (3) transporting the electrons and photons through a drift to a recording screen, and (4) analyzing the output file and deriving the brilliance.

With the BEAMnrc code, one obtains the photon energy distribution, from which the brilliance can be calculated, with a custom written code, according to its definition in BEAMnrc source code.

The simulation was performed with 10^6 electrons of 25 MeV bombarding a 10-μm thick target, in which 48980 bremsstrahlung photons were generated. The error of the simulation is 1% for low energy photons, and about 4.5% for high energy photons. These can be improved with more incident electrons.

3 Calculation results

Typical parameters of the prototype tiny target X-ray source are as follows.

Electron energy / MeV	25
Energy spread / %	0
Emittance / mm mrad	0
Repetition rate	100
Pulse width / μs	3
Radiation angle / mrad	20
Beam current / mA	200
Beam size / mm	5
Target size / μm	10
Target material	Tungsten

When calculating the brilliance, the electron energy spread and emittance were assumed as zero, as the emittance increases solid angle of the emitted photon, hence decreased brilliance. The energy, size and current of the electron beam affect the brilliance, too. Higher energy electrons generate more photons in smaller solid angle, i.e. increased brilliance. The brilliance is proportional to the beam current density (mA·cm^{-2}).

3.1 Comparison of two methods

Fig. 1 is the peak brilliance of the prototype X-ray source obtained from the M-C simulation and Eq.(9).

No obvious difference can be seen between the two methods except at high energy region, where the photon energy approaches its high frequency limit, and Eq. (9) becomes less reliable.

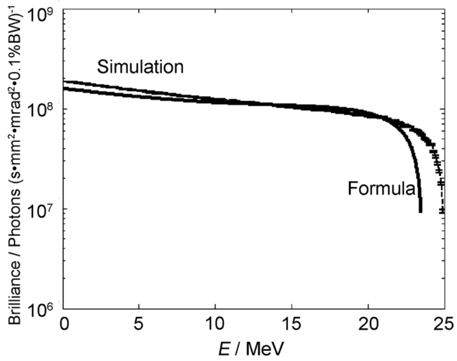
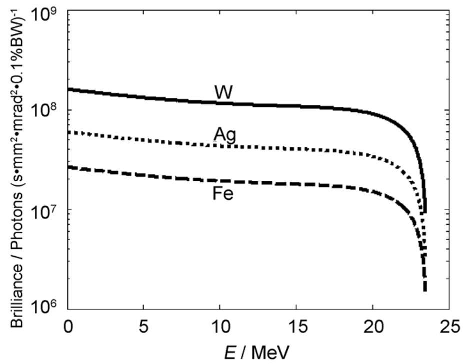


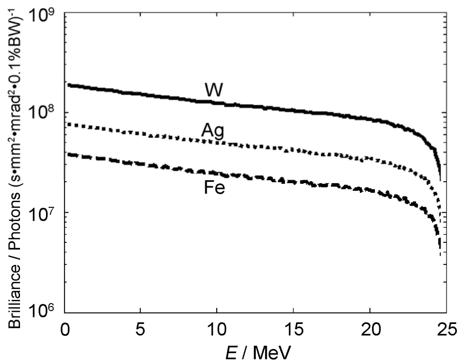
Fig.1 Peak brilliance with two methods.

3.2 Brilliances of different targets

Brilliances of three different target materials were calculated (Fig.2), with the same parameters as Fig.1. One finds that the brilliance increases with atomic number. Also, tungsten has the highest melting point and lowest vapor pressure of all metals, hence the choice for target material of the prototype X-ray source.



(a) Analytical result



(b) Simulation result

Fig.2 Peak brilliances with different targets.

3.3 Other characteristics

From Eq. (1), the angular distributions of photons for different energies of the prototype X-ray source were calculated. As shown in Fig.3, the photon flux decreases with increasing energy, where $\gamma^{\theta}=\gamma\theta_0$ is the reduced angle, and θ_0 is the angle between the initial electron and the emitted photon. The peak power point of bremsstrahlung is $1/(2\gamma)$[4,6], and no photon generates at $\theta_0 = 0$.

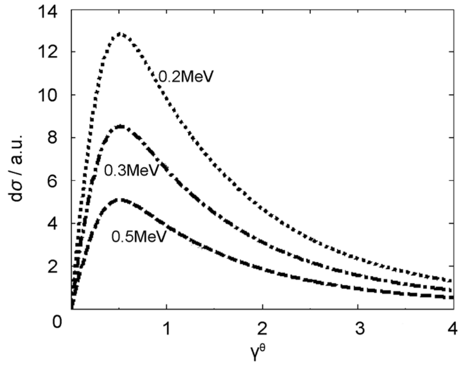


Fig.3 Angular distributions of photons for different energies from the prototype X-ray source.

4 Conclusion

The prototype tiny target X-ray source has its unique characteristics. Its brilliance can be calculated by analytical and simulation methods, and the results are consistent with each other.

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