Study of algorithms of phase advance measurement between BPMs and its application in SSRF

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Abstract As a third generation light source, Shanghai Synchrotron Radiation Facility (SSRF) has up to 140 beam position monitors (BPM) installed to monitor the beam dynamics on its storage ring. Once the operation mode is chosen, the betatron functions are determined. Since the sinusoidal betatron oscillation is the dominant component of the transverse motion, these BPMs can be used to measure the motion to get the betatron functions. Three methods are compared to calculate the phase advance among the BPMs in this paper, aiming to find one or more feasible ways to check the beam optics in SSRF. Some experiments have also been made to verify the practicality of the phase information.

Key words Phase advance, Storage ring, Correlation function, Discrete Fourier transform, Singular value decomposition

1 Introduction

Stability is the most important factor for particle accelerators, especially for Shanghai Synchrotron Radiation Facility (SSRF), a third generation light source. One way to check the beam optics and find errors is to measure the betatron phase advances around the storage ring and compare them to the lattice model[1]. Here, the 140 synchronized BPMs on the storage ring here in SSRF are ready for the beam stability test[2], but it is still necessary to pick one major method to calculate the phases at those positions.

The betatron oscillation of a single particle can be expressed as[3]

$$u(z)=\sqrt{\varepsilon}\sqrt{\beta(z)}\cos\left(\varphi(z)+\varphi_0\right)$$

where $\beta(z)$ is the so-called β-function. ε and φ_0 are integration constants. φ_0 represents the initial phase and stays all the same around the ring. $\varphi(z)$, the phase at z, may vary in different initial states, thus only the phase advances between probes are concerned.

Once the proper operating conditions are established, the β-function is decided. The phase function can be written as the following[3]

$$\varphi(z)=\int_0^z\frac{\mathrm{d}\overline{z}}{\beta(\overline{z})}+\varphi_0$$

So the phase will be shifted by a fixed angle after each cycle despite the details of the lattice. The phase advance between two probes is unrelated to any parameters except for the β-function:

$$\Delta\varphi(z_1,z_2)=\int_{z_1}^{z_2}\frac{\mathrm{d}\overline{z}}{\beta(\overline{z})}\tag{1}$$

The data series of the i^{th} BPM have the following form:

$$u_i(n)=A_i\cos\left(\omega nT_c+\varphi_i\right)=A_i\cos\left(2\pi n\nu+\varphi_i\right)\tag{2}$$

This means that the series is a sinusoidal signal. Fig.1 is a typical waveform that a BPM can get after the beam is excited by white noise in the frequency domain. Plenty of classic methods are available to extract the phase information due to its narrowband characteristics. The commonly used tools which are suitable for the machine are discrete Fourier analysis[4],

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correlation analysis[5] and singular value decomposition (SVD). An experiment addressing the phase advance measurement algorithms has been performed for this reason.

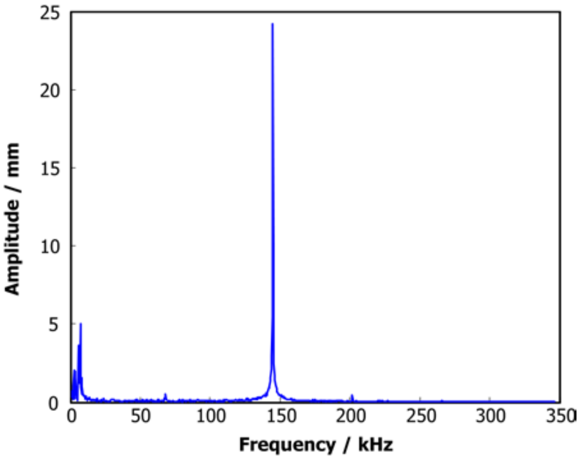

Fig.1 Typical turn-by-turn data in frequency domain of transverse motion excited by white noise.

2 Experimental

2.1 Discrete Fourier analysis

Fourier analysis is widely adopted to process monochrome data like Eq.(2). These data could be decomposed into its constituent frequencies. The output frequency spectrum contains the amplitudes as well as the phase information of the signal. The phase advance could be obtained by simply subtracting one phase from another.

The discrete Fourier transform (DFT) only approximates the continuous Fourier transform. Since there are some resonance conditions[3] to be avoided to prevent the closed orbit from divergence, it is better that the fractional tune usually does not match the revolution frequency. In that case, the waveforms fetched from the BPMs are band-limited periodic and are truncated to consist of other than an integer multiple of the characteristic period, so that leakage and distortion are inevitable. Weighting function is chosen to reduce leakage and phase adjustment is applied to calculate the "exact" phase[6].

2.2 Correlation analysis

The autocorrelation functions and the cross-correlation function of two sinusoidal signals

$f(t) = A\cos(\omega t + j_0)$ and $g(t) = B\cos(\omega t + j_0 + j)$

converge at Eqs.(3−5).

$$R_{fg}(0) = \frac{1}{N}\sum_{t=1}^{N} f(t)\times g(t) = \frac{AB}{2N}\sum_{t=1}^{N}\cos\varphi + \cos(2\omega t + 2\varphi_0 + \varphi) \xrightarrow{N\to\infty} \frac{AB\cos\varphi}{2} \tag{3}$$

$$R_{ff}(0) = \frac{1}{N}\sum_{t=1}^{N} f(t)\cdot f(t) = \frac{A^2}{2N}\sum_{t=1}^{N} 1 + \cos(2\omega t + 2\varphi_0) \xrightarrow{N\to\infty} \frac{A^2}{2} \tag{4}$$

$$R_{gg}(0) = \frac{1}{N}\sum_{t=1}^{N} g(t)\cdot g(t) = \frac{B^2}{2N}\sum_{t=1}^{N} 1 + \cos(2\omega t + 2\varphi_0 + 2\varphi) \xrightarrow{N\to\infty} \frac{B^2}{2} \tag{5}$$

The following relationship can be easily obtained from the above equations as long as enough data points are available

$$\cos\varphi \cong \frac{R_{fg}}{\sqrt{R_{ff}R_{gg}}}$$

There is no information about whether the phase difference between these two signals is a phase advance or a phase retard without more calculations. The locations of the BPMs are well designed so fortunately the β-function between any adjacent BPMs will not cause the phase advance greater than π in Eq.(1) and no more autocorrelation functions and cross-correlation functions with time delay are needed in this case.

2.3 SVD

Model-Independent Analysis (MIA) is a tool that applies Principal Component Analysis (PCA) to the particle accelerator physics. The MIA relies on a statistical analysis of the BPM data matrix and studies the beam dynamics without knowing the machine model. The PCA uses SVD to convert the BPM data matrix B into a product of three matrices[7].

$$B_{P\times M}=U_{P\times P}S_{P\times M}V^{\dagger}_{M\times M}$$

where

$$S=\mathrm{diag}\left(\lambda_1,\lambda_2,\cdots,\lambda_k\right)$$

is the singular matrix.

When the beam is excited and the betatron oscillation is the dominant term of the transverse motion, the first two columns of *U*, *S* and *V* denote the two orthogonal bases of this betatron oscillation. The matrix therefore approximately equals to the sum of these two principal components[7]:

$$B\cong\lambda_1u_1v_1^{\dagger\dagger}+\lambda_2u_2v_2$$

Where

$$v_1=\frac{1}{\lambda_1}\sqrt{\langle J\rangle\beta_m}\cos\left(\varphi_0+\varphi_m\right)\qquad m=1,2,\cdots,M$$

$$v_2=\frac{1}{\lambda_2}\sqrt{\langle J\rangle\beta_m}\sin\left(\varphi_0+\varphi_m\right)\qquad m=1,2,\cdots,M$$

$$u_1=\sqrt{\frac{2J_p}{P\langle J\rangle}}\cos\left(\varphi_p-\varphi_0\right)\qquad p=1,2,\cdots,P$$

$$u_2=\sqrt{\frac{2J_p}{P\langle J\rangle}}\sin\left(\varphi_p-\varphi_0\right)\qquad p=1,2,\cdots,P$$

The following formula[7] could be derived directly from the above expressions of v_1 and v_2.

$$\varphi=\arctan\frac{\lambda_2v_2}{\lambda_1v_1}$$

And the sign of v_1 and v_2 will be used to determine which quadrant the phase falls in.

3 Beam experiment

An experiment was performed on September, 2011. The beam energy was 3.5 GeV, the current was about 150 mA, the fractional tune is around 0.22 and a normal model was chosen to be run during the experiment. The beam was excited by white noise and 2048 readings were immediately taken from each BPM. The excitation was repeated by 150 times so there were 150 groups of BPM matrices available for the calculation of the phase advances.

3.1 Goodness of coincidence

There are two dimensions of the transverse betatron oscillation: the horizontal and the vertical. Only the horizontal results will be listed in this article for the sake of brevity. The three methods mentioned above were used to process these data and the results of the phase advances are shown in Fig.2. The solid line with asterisks is the theoretical model. The dashed line with diamonds is the result of MIA. The dots are the result of the DFT and the five-pointed stars are the results of the correlation analysis. A rough glance gives the impression that the correlation analysis fits the model best.

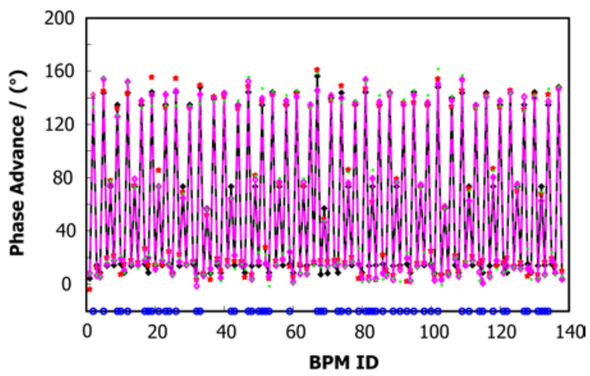

Fig.2 Resulting phase advances given by the three methods and a comparison between them and the machine model.

The real lattice may appear different from the designed one due to quadrupole gradient errors, beam energy offset and even wake-field, so the goodness of coincidence between the results and the model is not the criterion used to choose the most suitable algorithm in the machine. If the results of the phase advance between two adjacent BPMs are too far away from the model for all the three methods, one of the BPMs can be considered to be a problematic candidate. Fig.2 shows that we do not have high degree of trust and confidence in the BPMs that correspond to the circles on the horizontal axis which can be muted in the future but were not excluded in this experiment and will take part in the computations.

3.2 Repeatability

Figure 3 shows the repeatability performances of the three methods using all 150 groups with various data points in different line types as listed in the legend. Basically the performance of each method improves significantly when more samples are included. The correlation analysis is clearly more stable than the others while DFT gives the largest standard deviations.

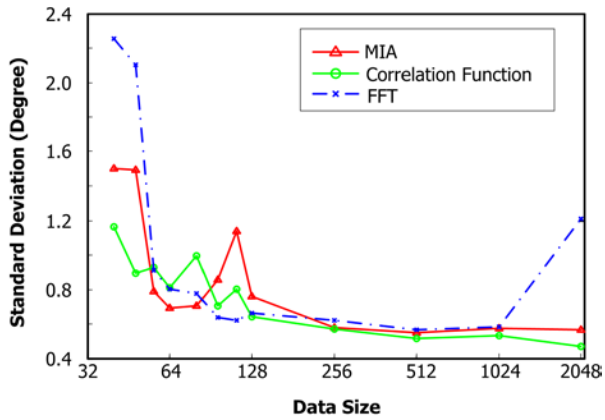

Fig.3 Standard deviations of different methods and different samples.

Distortion and leakage in DFT arise because of the requirement for sampling and truncation and the low frequency resolution makes it even worse. Any slight noise in the BPM readings can change the phase significantly. That is why the output has such large deviations when only a few points are available. The tune was drifting as the machine kept running, so the frequency band was widened as the data points grew. Since the single phase of the frequency with the largest amplitude is usually computed, it becomes unreliable when the number of points reaches some value and some other frequencies are mixed in the signal. This experiment shows that DFT could give results with the standard deviation close to 0.59° when 1024 samples were used and extra data would be redundant due to the signal to noise ratio.

The correlation analysis acts just perfectly. It gets better when more points are included because the more data are used, the smaller the oscillation term becomes, and thus the more stable it can be. The standard deviation reaches 0.47° when 2048 samples were used.

3.3 More study in MIA

Figure 4 lists the standard deviations of the phase advance between two specified BPMs when different numbers of BPMs are included in MIA. MIA extracts the shared modes of all the BPMs, therefore, not only the number of points of each BPM will affect its performance, but also the number of BPMs included in the SVD process of the data matrix may have some contribution.

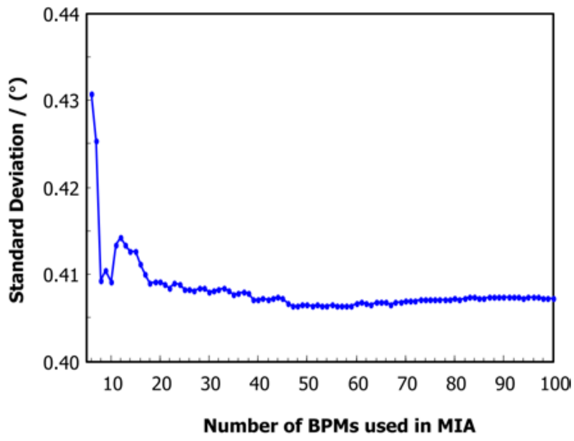

Fig.4 Standard deviations of different number of BPMs used in MIA.

The MIA can detect the correlationship between all the BPMs involved in its computing, so the betatron motion will be found more easily if the number of BPMs increases and the mode can be expressed in a more precise way if the number of points increases. The MIA is considered to be more practical if sufficient BPMs and enough data are provided. Otherwise, there is a chance that wrong modes could be extracted by mistake which enlarges the standard deviations (see the strange bump at the front part of the curves in Figs.3 and 4). The resolution of the system reaches 0.55° when 512 samples were taken, and the phase advance accuracy could be less than 0.41° when 18 or more BPMs were used.

Nevertheless, it is worth mentioning that the actual readings of the BPMs are not perfect sampled sinusoidal functions so a lot of optimizations were applied during the DFT and the correlation analysis. There were neither pre-process nor post-process in MIA, even without a mechanism to prevent the wrong modes to be extracted. The phase accuracy of an experiment could achieve 0.3° in APS with up to 300 BPMs[8]. Besides, MIA is not just a phase advance calculator and some other parameters such as β-functions can be calculated at the same time (see Fig.5). Although it did not give the most stable results, it still has the potential to get enhanced after some deeper research.

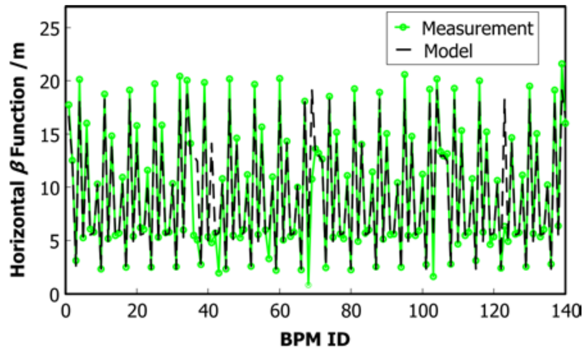

Fig.5 A comparison between horizontal β-function measured by MIA and that given as the model.

4 Applications

Another beam experiment was done at March, 2010 while a set of quadrupoles named Q1 was altered from 120% of its design strength to 150% of that with the beam current of 35 mA in a different lattice model. The initial phases of BPMs in cell 1 were recorded. Fig.6 shows a typical relation between the phase advance between 2 BPMs and the Q1 setting. Given that the standard deviation of our measurements is less than 0.6° when reasonable samples are taken. This system is capable of telling the difference of the lattice when the degree of variation of the quadrupoles has reached 5% if the positions where phase advances are to be monitored are carefully chosen based on the response matrix.

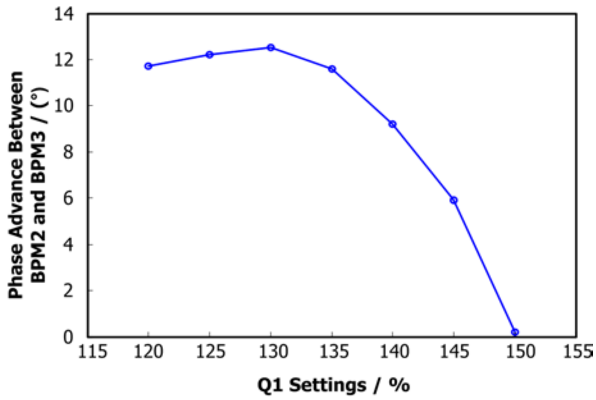

Fig.6 Relation between Q1 setting and the corresponding phase advance between BPM No.2 and BPM No.3.

5 Conclusion

Phase advance measurement is one of the new ways to monitor the stability of the beam optics in modern storage rings. Beam experiments in SSRF have shown the potential of calibrating quadrupoles' offsets and locating the positions of malfunctioned units by using the phase advance techniques, thus the precision of the phase measurement is mostly important.

Three popular methods: DFT, correlation analysis and MIA were used to calculate the phase advances between BPMs using the excited turn-by-turn beam data fetched by 140 BPMs on the storage ring in SSRF. It turns out that the correlation analysis could give the results with the least standard deviations so far, whether the number of points is sufficient or not, after some comparisons. Given enough data, the correlation function tool can generate the results with the standard deviation less than 0.47°.

It seems that the results from MIA are close to those of correlation analysis. MIA is worth further study and optimization to see if it can also be used in SSRF.

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