Nuclear Science and Techniques **20** (2009) 60–64

Multi-group pin power reconstruction method based on colorset form functions

HUANG Hao

School of Nuclear Science and Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

Abstract A multi-group pin power reconstruction method that fully exploits nodal information obtained from global coarse mesh solution has been developed. It expands the intra-nodal flux distributions into nonseparable semi-analytic basis functions, and a colorset based form function generating method is proposed, which can accurately model the spectral interaction occurring at assembly interface. To demonstrate its accuracy and applicability to realistic problems, the new method is tested against two benchmark problems, including a mixed-oxide fuel problem. The results show that the new method is comparable in accuracy to fine-mesh methods.

Key words Multi-group, Pin power reconstruction, Form function

1 Introduction

A common practice in current LWR core analysis is to reconstruct intra-assembly pin power distribution from the whole-core coarse mesh nodal solution. However, the pin power reconstruction methods[1-3] based on two-group assumption are not applicable to multi-group cases. While multi-group nodal reactor analysis tools are increasingly demanded for aggressive core designs nowadays, multi-group pin power reconstruction method should be developed.

Most pin power reconstruction methods assume that detailed pin-by-pin distributions (heterogeneous fluxes or powers) within an assembly can be approximated by the product of a global homogenized intra-nodal distribution and a local heterogeneous form function. Therefore, such a problem requests the approximation method for the intra-nodal flux distribution and the method to calculate the form functions.

The first successful method to obtain accurate intra-nodal flux distributions was introduced by Koebke *et al*[4], who expanded the intra-nodal flux shapes by nonseparable polynomial functions. Such polynomial expansion can accurately model fast flux shapes, but not the thermal flux shapes, especially when large localized gradients occur. In fact, the fast and thermal flux shapes could be approximated accurately by analytic basis function expansions[5]. The analytic basis functions are derived from decoupling the multi-group diffusion equations into "mode-group" partial differential equations. Unfortunately, the equation coefficients are not always real number in actual multi-group cases, in which a method to treat complex variables is needed.

To alleviate the problem, a semi-analytic function expansion method was adopted in this study. The flux shapes are expanded by combined polynomial functions and plane wave functions. The plane wave functions are derived from the homogeneous form of diffusion equations. To obtain an accurate result at large mesh size, eight terms of plane wave functions are used. Corner-point quantities are used as constraints to determine the coefficients in the flux expansion and node- and surface-averaged quantities. Corner-point quantities are usually determined by using additional flux, current continuity conditions and free source condition at each of corner point surrounded by four nodes. They can be calculated, inefficiently though, by low-order interpolation[4] or the iteratively sweeping for all corner-points[6]. In this paper, an efficient and simple method is adopted to

Partially supported by the National Natural Science Foundation of China via research project 10605016

* Corresponding author. *E-mail address:* nweapon@sjtu.org

Received date: 2008-11-18

determine corner-point quantities, and corner-point net currents are adopted as constraints and calculated from the parabolic distribution of surface-averaged net currents.

The form functions (FF) are usually calculated from single-assembly solution with zero net current boundary condition.

$$FF(x,y)=\frac{\Phi(x,y)^{het}}{\overline{\Phi}_g} \tag{1}$$

Eq.(1) is good for the flux inside an assembly, but not for large localized gradients introduced by spectral interactions occurring at the assembly interface. Against the difficulty, we use a form function derived from colorset calculations

$$FF(x,y)=\frac{\Phi(x,y)^{node,colorset}}{\Phi(x,y)^{node,\mathrm{hom}}} \tag{2}$$

where $\Phi(x, y)^{node,colorset}$ is the flux distribution obtained from colorset fine mesh heterogeneous transport calculation (for a heterogeneous problem) or from colorset fine mesh homogenous calculation (for a homogeneous problem), and $\Phi(x, y)^{node,hom}$ is the homogenous flux distribution within the node. By defining one form function with non-reflective outer boundary conditions for each quad of a colorset, the $\Phi(x, y)^{node,hom}$ of Eq.(2) within a quad will no longer be flat. Therefore, one may use various methods to calculate the flux distribution. In this paper, the method to calculate $\Phi(x, y)^{node,hom}$ should be consistent with the one used to reconstruct the intra-nodal flux after the global coarse mesh calculation, and all the approximations to calculate $\Phi(x, y)^{node,hom}$ (and discontinuity factors) at the colorset level, such as the quadratic transverse leakage approximation, should be fully consistent with those for the downstream whole core calculation. In this way, when the consistent form function is eventually multiplied to the homogenized intra-nodal flux distribution reconstructed from the global solution, most part of the errors introduced by various approximations and spatial truncation will be canceled out. This is the basic idea of the proposed method based on colorset form function.

2 Pin power reconstruction methodology

The following equations apply to every energy group, hence no index for a group.

2.1 Intra-nodal Flux Distributions

The radial intra-nodal flux shape is approximated as a combination of nonseparable semi-analytic functions.

$$\begin{aligned}\Phi(\xi,\eta)=&\sum_{i=0}^{i=2}\sum_{j=0}^{j=2}p_{i.j}P_i(\xi)P_j(\eta)+\sum_{i=3}^{4}p_{i,0}P_i(\xi)+\sum_{j=3}^{4}p_{0,j}P_j(\eta)\\&+a_1\sinh\left(B_x\xi\right)+a_2\cosh\left(B_x\xi\right)+a_3\sinh\left(B_y\eta\right)+a_4\cosh\left(B_y\eta\right)\\&+a_5\sinh\left(\frac{B_x\xi}{\sqrt{2}}\right)\cosh\left(\frac{B_y\eta}{\sqrt{2}}\right)+a_6\sinh\left(\frac{B_x\xi}{\sqrt{2}}\right)\sinh\left(\frac{B_y\eta}{\sqrt{2}}\right)\\&+a_7\cosh\left(\frac{B_x\xi}{\sqrt{2}}\right)\sinh\left(\frac{B_y\eta}{\sqrt{2}}\right)+a_8\cosh\left(\frac{B_x\xi}{\sqrt{2}}\right)\cosh\left(\frac{B_y\eta}{\sqrt{2}}\right)\end{aligned} \tag{3}$$

where

$$\xi=x/\Delta x;\quad \eta=y/\Delta y$$

$$B_x=\Delta x\sqrt{\frac{\Sigma_r}{D}};\quad B_x=\Delta y\sqrt{\frac{\Sigma_r}{D}}$$

2.2 Constraints for intra-nodal flux distributions

Eq.(3) has 21 unknown expansion coefficients, hence the need of finding equal number of the constraints.

Like the common nodal methods, the semi-analytic nodal code NLSANM[8] used in this study provides for each node nine node-averaged quantities, i.e., one node-averaged flux, four surface-averaged fluxes, and four surface-averaged net currents. They are used as constraints on the flux expansion. If a form expansion in Eq.(3) is constrained with just these quantities, however, the flux expansion will contain no cross terms (xy, x^2y, etc.), and the flux expansion is not accurate[9]. Therefore, one must use additional constraints to determine the cross terms, which are the net currents at nodal corner-points in this study. The corner-point net currents are calculated from surface-averaged currents of nodal solutions. The currents on surfaces of a node and its neighboring nodes are assumed to have a quadratic shape

$$J(u)=\overline{J}+\sum_{i=1}^{2}a_i p_i\left(\frac{u}{\Delta u}\right) \tag{4}$$

where $p_i(t)$ is i^{th} order of Legendre polynomials.

The coefficients in Eq.(4) are determined by the surface-averaged currents of the node and its two neighboring nodes. Therefore, the corner-point net currents can be expressed as

$$\begin{aligned} J_{u^-} &= \bar{J} - a_1 + a_2 \\ J_{u^+} &= \bar{J} + a_1 + a_2 \end{aligned} \tag{5}$$

In this manner, eight additional currents at four corner-points can be obtained. Nevertheless, the 17 constraints obtained from nodal solutions are not enough to determine 21 coefficients in Eq.(3). In this case, the transverse integration intranidal fluxes are required as close to corresponding one-dimensional intanodal flux obtained from nodal solution as possible. Therefore, a least square method is adopted,

$$\begin{aligned} &Min\left\{\int_{-\Delta x}^{\Delta x}\left((\int_{-\Delta y}^{\Delta y}\Phi(x,y)dy-\Phi_x(x)\right)^2 dx+\right. \\ &\left.\int_{-\Delta y}^{\Delta y}\left((\int_{-\Delta x}^{\Delta x}\Phi(x,y)dx-\Phi_y(y)\right)^2 dy\right\} \end{aligned} \tag{6}$$

This is a problem of conditional extremum. The Lagrange multiplier method is applied to convert it to a problem of non-conditional extremum.

$$\begin{aligned} &Min\left\{F=\left(\int_{-\Delta x}^{\Delta x}\left((\int_{-\Delta y}^{\Delta y}\Phi(x,y)dy-\Phi_x(x)\right)^2 dx+\right.\right. \\ &\left.\left.\int_{-\Delta y}^{\Delta y}\left((\int_{-\Delta x}^{\Delta x}\Phi(x,y)dx-\Phi_y(y)\right)^2 dy+\sum_{i=1}^{17}\lambda_k g_k\right)\right\} \end{aligned} \tag{7}$$

where λ_k is the undetermined coefficient and g_k is constraint described above.

A set of linear equations can be derived from Eq.(7).

$$\begin{cases} \dfrac{\partial F}{\partial p_{i,j}}=0, & \{i=0,1,2;\ j=0,1,2\}\cup \\ & \{i=0;\ j=3,4\}\cup \\ & \{j=0;\ i=3,4\} \\ \dfrac{\partial F}{\partial a_i}=0, & i=1,2,\ldots,8 \\ \dfrac{\partial F}{\partial \lambda_k}=0 & k=1,2,\ldots,17 \end{cases} \tag{8}$$

By solving the linear system of 38 equations in Eq.(8), the expansion coefficients $p_{i,j}$ and a_i in Eq.(3), and the homogeneous intra-nodal flux shapes, can be obtained. The groupwise pin flux distribution within an assembly can be calculated by timing a homogeneous intra-nodal flux shape with a local form function calculated in Eq.(2).

3 Numerical verification

The pin power reconstruction method was tested on the IAEA two-dimensional benchmark problem and the problem defined based on the OECD L-336 benchmark problem [10]. The main purpose of first-step numerical verification is to test multi-group pin power reconstruction method. Therefore, both the problems are benchmark problem with homogenized fuel assembly parameters, and the reference pin flux solutions are obtained by pin-by-pin fine mesh NLSANM calculation.

The method was performed in two steps: colorset calculation and full core calculation.

In the colorset calculation, the fine-mesh colorset diffusion calculations with reflective boundary condition for four quads of four adjacent fuel assemblies sharing a common corner point were preformed at each intersection point of the core, and the surface-averaged net current at the interface of two adjacent quads were obtained. These net currents were served as boundary conditions to define discontinuity factors (DFs) for all the quad surfaces, and were used as homogenized parameters for downstream coarse-mesh calculation and represents not only the heterogeneity within a quad, but also the effect introduced by the approximation in the nodal method for downstream global calculation. Generating the DFs, a very subtle process, will not be elaborated here, but all the approximations in the process are consistent with the downstream nodal calculation. The form functions defined in Eq.(2) for each quad could be obtained once the homogeneous intra-quad flux distributions were reconstructed from the coarse mesh calculation for each quad.

The full core calculations are performed using homogenized parameters (cross sections and discontinuity factors) generated at colorset level. Once the global coarse mesh nodal solution is obtained, the pin flux within each node will be obtained following the methodology given in Section 2.

3.1 IAEA-2D benchmark problem

This is a two-group benchmark problem with three clusters of control rods inserted in one eighth core. Due to the insertion of control rods in the inner core, the fuel assembly power of the peripheral fuel assemblies are pushed high, hence a strong neutron leakage around core periphery, a severe test problem for pin power reconstruction.

Most of the colorset patterns for this problem consist of four identical quarter assemblies, and no colorset fine mesh calculation is needed for the patterns. Colorset fine mesh calculations are only required for colorsets having different type of fuel assemblies. The coarse mesh obtained by 2×2 nodes/assembly mesh division, with and without discontinuity factors, are given in Table 1, which demonstrates that for this benchmark problem, global the results of both calculations are fairly good, and the results with discontinuity factors.

Fig.1 shows the peak pin powers reconstructed from the global solution obtained by 2×2 nodes/assembly mesh division with discontinuity factors. It can be seen that the proposed pin power reconstruction method predicates accurately the peak power of each assembly and its location. The maximum error of the peak power is 1.23% and the maximum error in pin power is 2.72% (with the relative pin power as 0.58), the RMS pin power error is 0.29%.

Table 1 Results of homogenous diffusion calculation

Nodes/FA	k_{eff}	Maximum error of averaged assembly power (%)
15×15(reference)	1.02958	—
2×2 with DF	1.02958	–0.05
2×2 without DF	1.02961	–0.28

3.2 Modified OECD L-336 benchmark problem

The original OECD L-336 problem was issued to test the application of various modern core analysis methods to a core loaded with mixed-oxide (MOX) fuel assemblies. It has five difference core configurations. Among them the C5 configuration consists of UO_2 fuel, MOX fuel and water reflector. UO_2 and MOX fuels have a heterogeneous 17×17 configuration. Due to the strong spectrum interference among the two types of fuel and the strong neutron leakage, this problem is a severe problem for core analysis method.

0.999 (1,1) 1.001 (1,1) 0.21	1.459 (15,1) 1.456 (15,1) -0.20	1.477 (5,1) 1.477 (5,1) -0.04	1.399 (1,1) 1.397 (1,1) -0.19	0.872 (1,1) 0.869 (1,1) -0.34	0.996 (14,1) 0.996 (14,1) -0.02	0.990 (1,1) 0.991 (1,1) 0.01	0.936 (15,8) 0.937 (15,8) 0.04
	1.512 (15,1) 1.501 (15,1) -0.72	1.513 (2,1) 1.511 (2,1) -0.09	1.442 (1,1) 1.441 (1,1) -0.09	1.257 (1,1) 1.255 (1,1) -0.14	1.118 (1,1) 1.118 (1,1) -0.01	1.053 (1,1) 1.052 (1,1) -0.07	0.927 (15,15) 0.927 (15,15) 0.04
		1.513 (1,15) 1.512 (1,15) -0.06	1.442 (1,15) 1.441 (1,15) -0.07	1.269 (1,12) 1.269 (1,12) -0.05	1.129 (1,12) 1.129 (1,12) -0.01	1.081 (2,13) 1.081 (2,13) 0.04	1.236 (4,1) 1.221 (4,1) -1.23
			1.372 (1,16) 1.371 (1,16) -0.07	1.196 (1,15) 1.195 (1,15) -0.08	1.053 (1,15) 1.053 (1,15) 0.01	1.315 (15,12) 1.315 (15,12) -0.07	
Keff 1.02958 1.02959 0.0010				0.743 (1,15) 0.738 (1,15) -0.67	0.808 (8,15) 0.809 (8,15) 0.14	0.915 (15,15) 0.916 (15,15) 0.11	
					0.898 (15,13) 0.888 (15,13) -1.07	Reference peak power (location) Reconstructed peak power (location) Error (%)	

Fig.1 Peak powers of the IAEA-2D benchmark problem (Shaded area denotes a rodded assembly).

The original heterogeneous problem was modified to a problem with homogenized fuel assembly parameters, which were generated from whole core heterogeneous fine-mesh transport results.

As shown in Tables 2 and Fig.2, the results obtained by the proposed method agree well with the reference fine-mesh nodal solution. The maximum error of the peak power is 0.61% and the maximum error in pin power is 2.74% (with the relative pin power as 0.76), the RMS pin power error is 0.48%.

Table 2 Results of homogenous diffusion calculation

Nodes/FA	k_{eff}	Maximum error of averaged assembly power / %
17×17 (reference)	1.18423	—
2×2 with DF	1.18425	–0.06
2×2 without DF	1.18467	–0.58

2.282 (1,17)	1.887 (1,17)	
2.282 (1,17)	1.876 (1,17)	
0.010	–0.610	
1.887 (1,17)	0.822 (3,16)	Reference peak power (location)
1.876 (1,17)	0.821 (3,16)	Reconstructed peak power(location)
–0.610	0.100	Error (%)

Fig.2 Peak powers of the OECD L-336 benchmark problem.

4 Conclusion

A new multi-group pin power reconstruction method that fully exploits nodal information obtained from global coarse mesh solution is developed. The intro-nodal flux distribution is expanded as the combination of eight planar wave functions and thirteen Legendre polynomials. All the information that can be obtained from coarse mesh nodal solution, not only these node-averaged parameters, but also the transverse integrated 1D flux distribution, the parabolic transverse leakage profile, are exploited as the constraints to determine all these expansion coefficients. A form function generating method based on color set model and a consistent way to cancel out most of these numerical errors and the error in the method are also proposed. Numerical results for both IAEA 2D and modified OECD L-336 benchmark problems demonstrate that the proposed multi-group pin power reconstruction method is accurate and applicable to the multi-group core analysis. Comparison of results also indicates that even for a homogeneous problem, the introduction of the proposed "discontinuity factor" is necessary, it can noticeably improve the accuracy of these global parameters, needless to say the accuracy of pin power.

Acknowledgement

The author would thank Dr. Chao Yung'an of Westinghouse and Dr. Zhang Shaohong of Shanghai Jiaotong University for helpful discussions.

References

1 Rempe K R, Smith K S, Henry A F. Nucl Sci Eng, 1989, **103:** 334–342.
2 Zhu S, Xie Z. Jour. XJTU, 2000, **34:** 32–36.
3 Hu Y, Gan D. Jour. NWPU, 1995, **13:** 1–6.
4 Koebke K, Hetzelt L. Nucl Sci Eng, 1985, **91:** 123–131.
5 Noh J M, Cho N Z. Nucl Sci Eng, 1994, **116:** 165–180.
6 Mecir V, Chao Y A. Reactor Analysis, 1995, **72:** 346–347.
7 Yang W S, Finck P J, Khalil H. Nucl Sci Eng, 1992, **111:** 21–33.
8 Liao C K. NLSANM User's Manual, 2002, Department of Nuclear Science and Engineering, Xi'an Jiao Tong University.
9 Rempe K R. Pin Power Reconstruction of Nodal Reactor Analysis, Ph. D Thesis, 1988, Department of Nuclear Engineering, M. I. T., Cambridge, Mass.
10 Smith M A, Lewis E E, Byung-Chan Na. Prog Nucl Energy. 2003, **45:** 107–118.
11 Joon H G, Joon Y. Personal Communication, 2007.
12 Huang H, Tang C T, Zhang S H. Chin JourNucl Sci Eng., 2008, **28:** 97–101.

Errata

2008, Vol.19, No.4, page196, right column, line 6: delete "(minus)"