

# Initial fluctuation effect on elliptic flow in Au+Au collision at 1 GeV/A

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**Abstract** How the initial fluctuation affects on the elliptic flow is investigated by investigating the rapidity, transverse 4-velocity, centrality dependencies of elliptic flow for Au+Au at 1 GeV/A with the help of an Isospin Quantum Molecular Dynamics (IQMD). In addition, we compare the flow calculated with respect to participant plane created by the initial geometry in coordinate space with the flow reconstructed by the experimental event-plane method, and compare the flow with the experimental data of the FOPI collaboration. It shows that there exists some discrepancy between the flows reconstructed by the above two methods.

**Key words** Elliptic flow, Initial fluctuation, IQMD, Event plane method, Au+Au collision at 1 GeV/A

## 1 Introduction

The characterization of collective flow has proven to be one of the more powerful probes of the dynamics in heavy ion collisions. Elliptic flow is an excellent collective flow observable which has been studied extensively at Bevalac and SIS as well as AGS, SPS, and RHIC<sup>[1]</sup>. In intermediate energy, microscopic transport model calculations have stressed the importance of elliptic flow for extracting the equation of state (EOS) of nuclear matter.

The elliptic flow is mathematically defined as the second coefficient of the Fourier expansion of particle azimuthal distribution with respect to the reaction plane. The origin of the elliptic flow is the initial geometry anisotropy of the collision system which originates from the uneven distribution of the density in the early time of the collisions. Subsequent dynamical evolution of the system transforms the anisotropy from the coordinate space to the momentum space, which leads to the collective motion observed in the final state. Based on this physical picture, it can be imagined that the elliptic flow should be very sensitive to the initial state.

Recently, extensive studies of the initial fluctuation effect on anisotropic flow in relativistic nuclear collisions have been made. The initial fluctuation can be illustrated as the event-by-event fluctuation in the shape of the overlap region created in initial collisions. The initial eccentricity ( $\epsilon_2$ ), which is a parameter used for quantifying the initial spatial anisotropy, is affected by the initial spatial anisotropy.

In high energy collisions, it has already been predicted by model calculations that the anisotropic flow considering the initial fluctuation is larger<sup>[2]</sup>. Han *et al.* shows that the ratio of the elliptic flow to initial eccentricity ( $v_2/\epsilon_2$ ) is sensitive to the initial fluctuation<sup>[2]</sup>. The value of  $v_2/\epsilon_2$  stands for the conversion from the initial geometry anisotropy to the final momentum anisotropy. Many studies prove that the hydrodynamic calculations on flow are in good agreement with the experimental data, when the initial fluctuation is taken into account<sup>[3-10]</sup>. Furthermore, the initial fluctuation makes higher odd-order harmonic flow nontrivial. Previous studies have shown that the triangle flow is non-negligible, after the initial fluctuation is taken into account<sup>[11,12]</sup>.

However, the initial fluctuation effect on flow has only been investigated in high energy collisions so

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far. In the intermediate energy domain, its effect is not addressed yet.

In this article, the initial fluctuation effect on the collective flow will be demonstrated in the intermediate energy collisions. A transport model, named as an Isospin Quantum Molecular Dynamics (IQMD), which allows the generation of events with event-by-event fluctuating initial conditions, is used. We will also compare the flow results using two different methods. Finally, the centrality dependence of the eccentricity  $\varepsilon_2$  and the ratio of  $v_2/\varepsilon_2$  will be presented.

## 2 Analysis method

In the collisions, the particle azimuthal distribution relative to the reaction plane is not isotropic and is usually expanded in the Fourier series<sup>[13]</sup>:

$$E \frac{d^3 N}{d^3 p} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \psi_{RP})) \right) \quad (1)$$

where the  $v_n = \langle \cos[n(\phi - \psi_{RP})] \rangle$  coefficients are normally referred to  $n$ -th collective flow or anisotropic flow<sup>[14]</sup>, and the angle bracket means an average over all particles in all events.  $\psi_{RP}$  is the reaction plane angle. The reaction plane is spanned by the vector of the impact parameter and the beam direction. It can not be measured experimentally, but can be estimated in several ways.

### 2.1 Initial fluctuation effect on the reaction plane

The original reaction plane is XZ plane in the model where  $x_{RP}$  represents for X-axis, however, the initial fluctuation has effects on the reaction plane as illustrated in Fig.1.

The effect of the initial fluctuation on the reaction plane is that the initial fluctuation makes the participant plane ( $x_{PP}$ ) deviate from the reaction plane ( $x_{RP}$ )<sup>[15]</sup>. The participant plane angle is defined as:

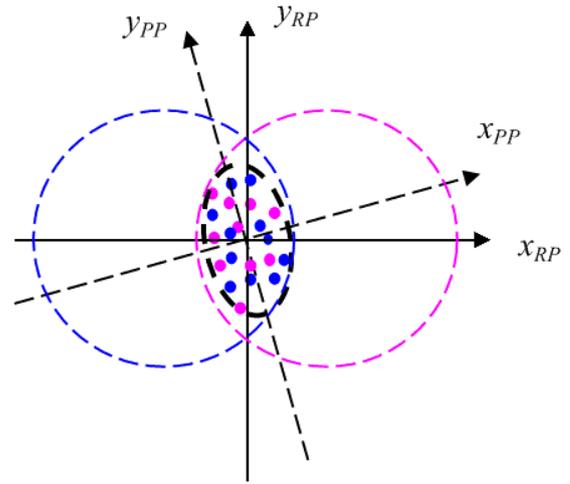
$$\psi_n \{PP\} = \frac{1}{n} \left[ \arctan \frac{\langle r^2 \sin(n\phi) \rangle}{\langle r^2 \cos(n\phi) \rangle} + \pi \right] \quad (2)$$

where the  $r$  and  $\phi$  are the coordinate position and azimuthal angle of each particle and  $\langle \dots \rangle$  is the participant density weighted in the initial state.

The  $n$ -th order eccentricity calculated with respect to this participant plane is defined as:

$$\varepsilon_n = \frac{\sqrt{\langle r^2 \cos(n\phi) \rangle^2 + \langle r^2 \sin(n\phi) \rangle^2}}{\langle r^2 \rangle} \quad (3)$$

When  $n=2$ , the above two variables  $v_2$  and  $\varepsilon_2$ , correspond to elliptic flow and eccentricity, respectively. In high energy heavy-ion collisions, it is found that the eccentricity taking the initial fluctuation into account is different from the eccentricity without the consideration of initial fluctuation<sup>[16]</sup>.



**Fig.1** Reaction plane and participant plane coordinate systems.

### 2.2 Experimental method

A common used method in experimental analysis is the event-plane method<sup>[17,18]</sup>. It uses the event-plane angle determined from the observed collective flow itself as an approximate reaction plane<sup>[14]</sup>.

The event plane angle is given as:

$$\begin{aligned} \psi_n \{EP\} &= \arctan 2(Q_{n,y}, Q_{n,x}) / n \\ Q_{n,x} &= \sum_i \omega_i \cos(n\phi_i) \\ Q_{n,y} &= \sum_i \omega_i \sin(n\phi_i) \end{aligned} \quad (4)$$

in which the sum runs over all the particles used in the reconstruction of the event plane. The  $\phi_i$  and  $\omega_i$  are the azimuthal angle and weight for particle  $i$ . We choose  $\omega_i = p_T$  for  $y_0 > 0.3$ ,  $\omega_i = -p_T$  if  $y_0 < -0.3$ <sup>[19]</sup>. The observed  $v_n$  with respect to the event plane is written by:

$$v_n^{obs} = \langle \cos[n(\phi_i - \psi_n \{EP\})] \rangle \quad (5)$$

The event plane differs in general from the original reaction plane for the finite multiplicity in

events. Therefore, the  $v_n$  has to be corrected by the event plane resolution  $R_n$ .

The event plane resolution for each harmonic is given by:

$$\mathcal{R}_n = \langle \cos[n(\psi_n - \psi_{RP})] \rangle \quad (6)$$

where the angle bracket means an average over a large event sample. The event plane resolution depends on the multiplicity of particles which are used to define the flow vector as well as the average flow of these particles via the resolution parameter<sup>[18,20,21]</sup>:

$$\chi = v_n \sqrt{M}$$

$$\mathcal{R}_n(\chi) = \sqrt{\pi/2} \chi \exp(-\chi^2/2) (I_0(\chi^2/2) + I_1(\chi^2/2)) \quad (7)$$

where  $I$  is the modified Bessel function. To calculate the resolution we divide the full events up into two independent sub-events with an equal multiplicity<sup>[22,23]</sup>. Each sub-event resolution can be defined as:

$$\mathcal{R}_{n,sub} = \sqrt{\langle \cos[n(\psi_n^A - \psi_n^B)] \rangle} \quad (8)$$

where A and B denote the two subgroups of particles. For the given  $R_n$  the solution for  $\chi$  in Eq.(7) is done by iteration. The full event plane resolution is obtained by:

$$\mathcal{R}_{full} = \mathcal{R}(\sqrt{2}\chi_{sub}) \quad (9)$$

The final collective flow measured with respect to the event plane can be written as:

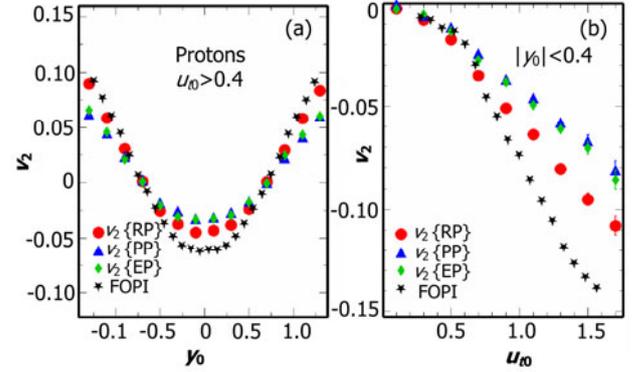
$$v_n = \frac{v_n^{obs}}{\mathcal{R}_n} \quad (10)$$

In this method, the event plane is calculated by the final momentum space. In this case, the flows reconstructed by the event plane method can be affected by the evolution of the dynamics. In Ref.[1] it shows that in Au+Au collisions at  $\sqrt{s_{NN}}=200$  GeV from a multiphase transport model the elliptic flow  $v_2$  and the triangle flow  $v_3$  relative to the event plane are larger than those relative to the participant plane. Therefore, it is reasonable to assume that the evolution of the dynamics has influence on the collective flow.

### 3 Results and discussion

Figure 2 shows the  $y_0$  and  $u_{t0}$  dependencies of the elliptic flow measured with respect to the reaction

plane ( $\{RP\}$ ), participant plane ( $\{PP\}$ ) and event plane ( $\{EP\}$ ), where  $y_0=y/y_p$  is the scaled rapidity and  $u_{t0}=u_t/u_p$  is the scaled transverse 4-velocity. Please note that in the  $v_2$  calculation, we have applied the detector geometry cut of the FOPI detector so that a quantitative comparison can be made<sup>[24]</sup>.



**Fig.2** Comparison for  $v_2\{RP\}$  (red circle),  $v_2\{PP\}$  (blue triangle) and  $v_2\{EP\}$  (green diamond) in Au+Au collisions at  $E=1$  GeV/A and centrality  $0.25 < b_0 < 0.45$  by the IQMD model with a parameter set of Soft and Momentum-dependent equation of state (SM). The black stars are the experimental data from the FOPI Collaboration. The panel (a) plots  $y_0$  dependence where  $v_2$  is integrated over  $u_{t0}$ , but constrained to  $u_{t0} > 0.4$ . The panel (b) shows the  $u_{t0}$  dependence in the indicated  $y_0$  bin.

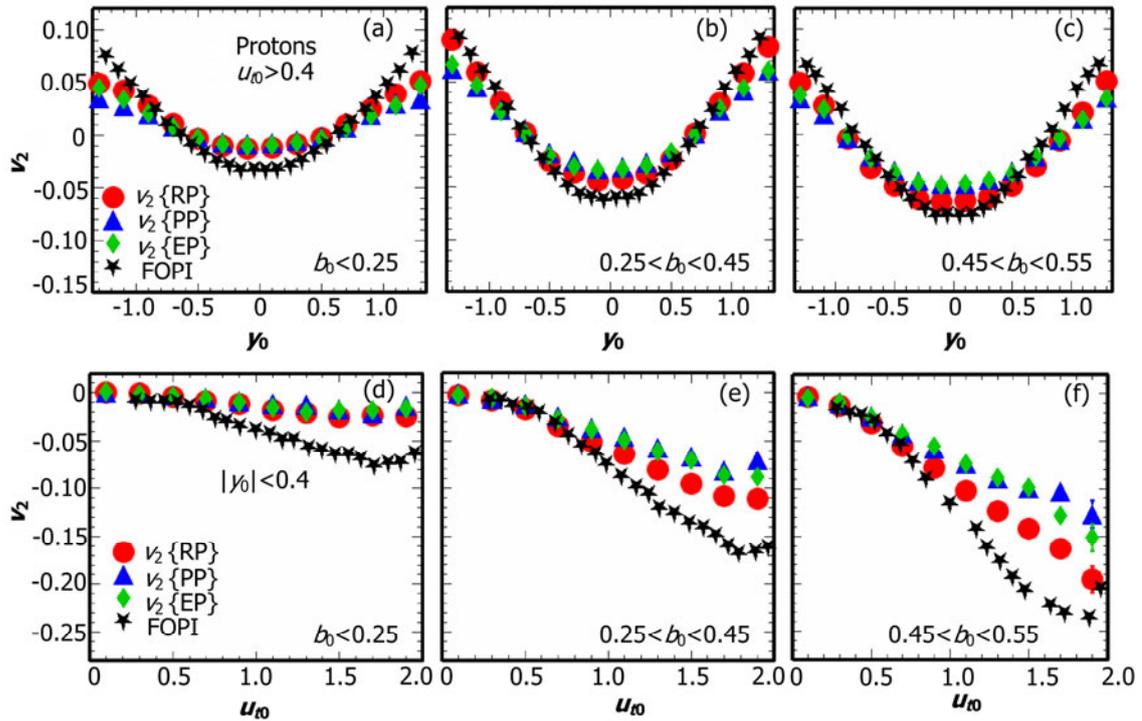
The rapidity dependence of Fig.2(a) shows a V-shape. The proton shows an in-plane emission ( $v_2 > 0$ ) in project-like and target-like regions (larger rapidity value in absolute value) while it shows a squeeze-out emission in the overlapping region (mid-rapidity). From the mid-rapidity to the projectile-like/target-like rapidities, the proton favors the squeeze-out emission to the in-plane emission. This is consistent with the shadowing effect.

The transverse velocity dependence (Fig.2(b)) shows that with the increase of  $u_{t0}$ , the squeeze-out emission is stronger, i.e. the proton with higher velocity can be more easily escaped from overlapping zone.

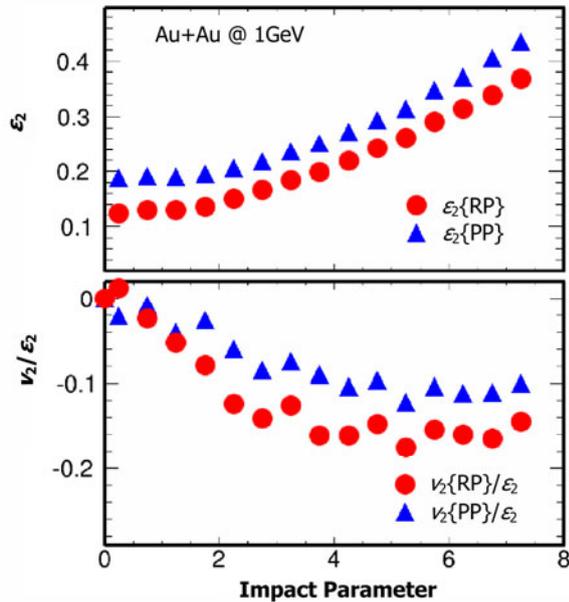
From Fig.2, we can see that the value of  $v_2\{RP\}$  is a little larger than  $v_2\{PP\}$ . However, there is almost no discrepancy between  $v_2\{EP\}$  and  $v_2\{PP\}$ . That means  $v_2$  becomes weak by the initial fluctuation but is not sensitive to the dynamical evolution. This phenomenon is different from what is known at high energies, where  $v_2$  is both enlarged by the initial fluctuation and the dynamical evolution. In the figure, the FOPI data is also plotted in order to check our

model calculations. Even though quantitative agreements of various calculations are not reached, the

trend of  $v_2$  as functions of  $y_0$  and  $u_{t0}$  is consistent with the data.



**Fig.3**  $v_2$  of protons in Au+Au collisions for different centrality ranges. In the upper three panels,  $v_2$  is calculated for transverse 4-velocities  $u_{t0} > 0.4$ . In the lower three panels,  $v_2$  is calculated for the rapidity  $|y_0| < 0.4$ .



**Fig. 4**  $\epsilon_2$  and  $v_2/\epsilon_2$  as a function of impact parameter for Au+Au collisions at  $E=1\text{GeV}/A$ .

Figure 3 displays that  $v_2$  versus  $y_0$  and  $u_{t0}$  in three different centralities. While the shapes look similar in all centralities, the elliptic flow of protons in mid-rapidity (upper panels) or high velocity (lower panels) regions shows stronger squeeze-out effect at

some intermediate centralities ( $0.45 < b_0 < 0.55$ ). With the increase of  $b_0$ , the initial fluctuation and dynamical evolution also play an increasing role as seen from the behavior of increasing difference between  $v_2\{\text{PP}\}$ ,  $v_2\{\text{RP}\}$  and  $v_2\{\text{EP}\}$ . Again, the similarity between  $v_2\{\text{PP}\}$  and  $v_2\{\text{EP}\}$  indicates that there is almost no effect by the evolution of dynamics.

The upper panel of Fig.4 shows the impact parameter ( $b$ ) dependence of eccentricity  $\epsilon_2$ . It shows that  $\epsilon_2$  increases with  $b$  and the  $\epsilon_2\{\text{PP}\}$  is larger than  $\epsilon_2\{\text{EP}\}$ . The lower panel shows the ratio  $v_2/\epsilon_2$  as a function of  $b$ . We can see the absolute values of the ratio also increase with  $b$  and  $|v_2\{\text{PP}\}/\epsilon_2|$  is smaller than  $|v_2\{\text{RP}\}/\epsilon_2|$ . The smaller the absolute ratio is, the less efficient the conversion from the initial geometry anisotropy to the final momentum anisotropy is.

## 4 Conclusion

The elliptic flow of Au+Au collision at 1 GeV/A by three different methods for the determination of reaction plane was studied. And it was found that the initial event-by-event geometry fluctuation had effects on the final-state elliptic flow. In the present

calculation, the initial fluctuation weakens the elliptic flow, in other words the initial fluctuation makes the conversion from initial spatial eccentricity to final momentum smaller. In this paper, we studied the dependence of  $\varepsilon_2$  on impact parameter with the consideration of the initial fluctuation and compared the ratio  $v_2/\varepsilon_2$  with or without the consideration of the initial fluctuation. It shows that the initial fluctuation enhances the eccentricity  $\varepsilon_2$  but it decreases the ratio of  $|v_2/\varepsilon_2|$ . Moreover, we also found that the dynamical evolution has little influence on the elliptic flow in our studied cases. Our simulation gives a similar trend for rapidity and velocity dependencies, but the results are quantitatively smaller than the experimental data for mid-rapidity and high velocity protons, which may be caused by two reasons. One is the variation of physical parameters (such as the ground state densities, interaction ranges) whose precise values are not known; the other is the consequence of the impossibility to build a ground state nucleus with all its detailed structure in a semi-classical molecular dynamics approach<sup>[25]</sup>.

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