# Gravitational waves from the axial oscillation of neutron star considering non-Newtonian gravity

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**Abstract** The eigen-frequencies of the axial *w*-modes of neutron star described by a super-soft equation of state (EOS) are investigated, by considering the non-Newtonian gravity. The results show that at the same stellar mass, the frequencies of $w_I$ and $w_{I2}$ for our model are lower than that of the typical EOSs (such as APR); and the frequencies increase with the stellar masses, which is contrary to that of the typical EOSs. These characters may provide a probe to testify the super soft symmetry energy and the non-Newtonian gravity in the future. Moreover, our model also has the universal behavior of the mass-scaled eigen-frequencies as a function of the compactness.

**Key words** Neutron stars, Oscillation, Non-Newtonian gravity

## 1 Introduction

Neutron stars (NSs) provide a perfect model for studying the gravitational waves and the equation of state (EOS) of the dense matters[1,2], which are two important topics in modern physics. For both the topics, it is believed that there is a very useful type of oscillation mode, the *w*-mode, being an important mechanism for producing gravitational waves[3-6], which will contain abundant information for the EOS of the super dense matters[7-10]. The *w*-mode only exists in general relativity, which is associated with the space-time and for which the motion of the fluid is negligible[1,3,4]. The *w*-mode is related to the space-time curvature and exists for all relativistic stars, including black holes. The standard axial *w*-mode is categorized as $w_I$.

At present time, however, the EOS of matter under extreme conditions is still rather uncertain. One of the main sources of uncertainties in the EOS of neutron-rich matter is the poorly known density dependence of the nuclear symmetry energy, $E_{\rm sym}(\rho)$[11]. Many of the theories predict that the symmetry energy increases continuously at all densities[12-16]. But many other models predict that the symmetry energy increases first but decreases above certain supra-saturation densities[17-31]. The latter kind of symmetry energy functions is generally regarded as being soft. Among the soft models, the original Gogny-Hartree-Fock (GHF) model[26] and group III[23] are super soft, in which the pressure drops quickly to zero around three times of the saturation density $\rho_0$. The super-soft models either cannot keep the NSs stable or predict maximum NS masses significantly below $1.4M_\odot$ ($M_\odot$ is the solar mass). On the other hand, the recent experimental results support the super-soft prediction by analyzing the FOPI/GSI experimental data on the $\pi^-/\pi^+$ ratio in relativistic heavy-ion collisions[32] within a transport model by using the MDI (Momentum-Dependent-Interaction) EOS[33]. In order to let the super-soft EOS support the mass observation of neutron star, the non-Newtonian gravity was introduced[34]. In this work, the *w*-mode is investigated by using a super-soft EOS, where the non-Newtonian gravity is considered.

In Section 2, we describe the EOS and the non-Newtonian gravity. In Section 3, we review the formalism used to calculate *w*-modes and present the results before concluding in Section 4.

Supported by National Natural Science Foundation of China (Nos.10947023, 11275073 and 11205061), and Fundamental Research Funds for the Central University, China (No. 2012ZZ0079). The project is sponsored by SRF for ROCS, SEM.

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Received date: 2013-06-27

## 2 Equation of state and non-Newtonian gravity

The MDI is a phenomenological effective interaction based on a modified finite-range Gogny interaction[26,33,35-37]. By using the MDI EOS, the circumstantial evidence for a super-soft symmetry energy was found recently from analyzing the FOPI/GSI experimental data[32] on the $\pi^-/\pi^+$ ratio in relativistic heavy-ion collisions[33]. For a neutron star consisting of neutrons, protons, and electrons (*npe*), the internal pressure of neutron stars is given by Eq.(1)[8,34,37], with the chemical equilibrium conditions of $\mu_e=\mu_n-\mu_p=4\delta E_s(\rho)$ and charge neutrality requirement of $\rho_e=(1-\delta)\rho/2$,

$$P_{npe}(\rho,\delta)=\rho^2\left(\frac{dE_0}{d\rho}+\frac{dE_s(\rho)}{d\rho}\delta^2\right)+\frac{1}{2}\delta(1-\delta)\rho E_s(\rho). \tag{1}$$

The energy nucleon for symmetric nuclear matter can be well approximated as[34, 38]

$$\begin{aligned}E_0(\rho)=&\frac{8\pi}{5mh^3\rho}p_f^5+\frac{\rho}{4\rho_0}(-216.55)+\frac{B}{\sigma+1}(\frac{\rho}{\rho_0})^\sigma\\&+\frac{1}{3\rho_0\rho}(C_l+C_u)(\frac{4\pi}{h^3})^2\Lambda^2\times[p_f^2(6p_f^2-\Lambda^2)\\&-8\Lambda p_f^3\arctan\frac{2p_f}{\Lambda}\\&+\frac{1}{4}(\Lambda^4+12\Lambda^2p_f^2)\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}],\end{aligned} \tag{2}$$

and the symmetry energy can be expressed as[38]

$$\begin{aligned}E_s(\rho)=&\frac{8\pi}{9mh^3\rho}p_f^5+\frac{\rho}{4\rho_0}[-24.59+\frac{4Bx}{\sigma+1}]-\frac{Bx}{\sigma+1}(\frac{\rho}{\rho_0})^\sigma\\&+\frac{C_l}{9\rho_0\rho}(\frac{4\pi}{h^3})^2\Lambda^2[4p_f^4-\Lambda^2p_f^2\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}]\\&+\frac{C_u}{9\rho_0\rho}(\frac{4\pi}{h^3})^2\Lambda^2[4p_f^4-p_f^2(4p_f^2+\Lambda^2)\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}],\end{aligned} \tag{3}$$

where $p_f=\hbar(3\pi^2\rho/2)^{1/3}$ is the Fermi momentum for symmetric nuclear matter, and the values of the parameters are $\sigma$=4/3, $B$=106.35 MeV, $C_l$=–11.70 MeV, $C_u$=–103.40 MeV and $\Lambda=p_f^0\equiv p_f(\rho_0)$, where $\rho_0$ is the saturation density. The value of $E_{\rm sym}(\rho)$ is 31 MeV[38-40]. The parameter $x$ was introduced to vary the density dependence of the symmetry energy without changing any property of symmetric nuclear matter. It has been shown that $x$=1 can reproduce the FOPI/GSI pion production data[33] within the transport model analysis, and corresponds a super-soft symmetry energy. The corresponding EOS is denoted as MDIx1.

In the traditional framework, MDIx1 EOS cannot support the observed masses of pulsars because of its super-soft symmetry energy. To support the observations of neutron stars, the non-Newtonian gravity was introduced[34]. The non-Newtonian gravity can be described by adding a Yukawa term to the conventional gravitational potential between two objects[41-43], that is,

$$V(r)=-\frac{Gm_1m_2}{r}(1+\alpha e^{-r/\lambda}), \tag{4}$$

where $G$ is the universal gravitational constant, $\alpha$ is a dimensionless strength parameter, and $\lambda$ is a length scale. In the scalar/vector boson exchange picture $\alpha=\pm g^2/(4\pi Gm_b^2)$ and $\lambda=1/\mu$, where $g$, $\mu$ and $m_b$ are the boson-baryon coupling constant, the boson and baryon mass, respectively. Within the mean-field approximation, the non-Newtonian gravitational effects can be described by the equation of state of dense matters, but Einstein equation remains unchanged. The extra energy density due to the non-Newtonian gravity can be calculated by[42,43]

$$\varepsilon_{\rm UB}=\frac{1}{2V}\int\rho(\vec{x}_1)\frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}\rho(\vec{x}_2)d\vec{x}_1d\vec{x}_2=\frac{1}{2}\frac{g^2}{\mu^2}\rho^2, \tag{5}$$

where $V$ is the normalization volume, $\rho$ is the baryon number density, and $r=|\vec{x}_1-\vec{x}_2|$. The corresponding addition to the pressure is then

$$P_{\rm UB}=\varepsilon_{\rm UB}=g^2\rho^2/(2\mu^2), \tag{6}$$

where a constant boson mass independent of the density is assumed[34,43]. Including the non-Newtonian gravity the EOS becomes $P=P_{npe}+P_{\rm UB}$, where $P_{npe}$ is the conventional pressure for neutron star mainly consisting of neutrons, protons and electrons. Shown in Fig.1 is the EOS of MDIx1, where the non-Newtonian gravitational parameter $g^2/\mu^2$ is taken as 75 GeV$^{-2}$ or 100 GeV$^{-2}$ (denoted as MDIx1-75, MDIx1-100). In order to show the effect of the non-Newtonian gravity, the typical EOS, APR[44] is used for comparison. The mass-radius relation is displayed in Fig.2. It is easy to see that for the EOS MDIx1, the radius of the neutron star is far larger than that of the typical EOS APR.

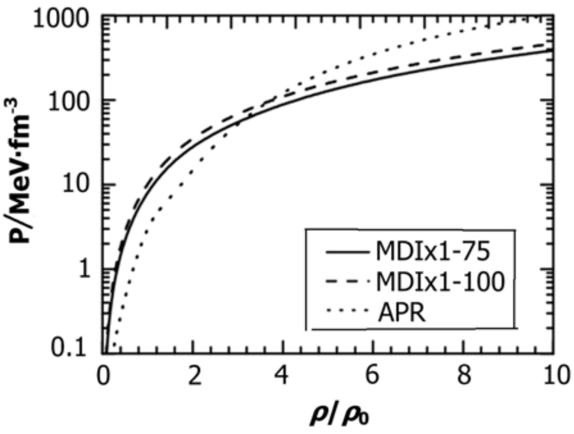


**Fig.1** The equation of states versus the reduced baryon number density $\rho/\rho_0$.

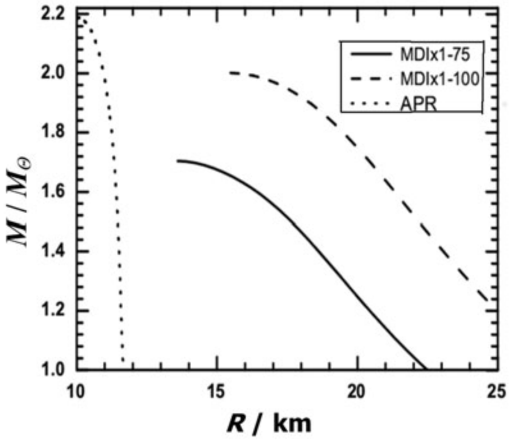


**Fig.2** The mass-radius relations.

## 3 The gravitational waves from the axial *w*-modes of neutron stars with super soft EOS and considering non-Newtonian gravity

First, we briefly introduce the formalism for the axial *w*-modes. Here we use geometrical unit ($G$=$c$=1). According to Chandrasekhar & Ferrari[3], the axial perturbation equations for a static neutron star can be simplified by introducing a function $z(r)$, constructed from the radial part of the perturbed axial metric components. It satisfies the following differential equation

$$\frac{d^2z}{dr_*^2}+[\omega^2-V(r)]z=0, \tag{7}$$

where $\omega$ (=$\omega_0$+$i\omega_i$) is the complex eigen-frequency of the axial *w*-mode, $r_*$ is the tortoise coordinate[3]. Inside the star, the potential function $V$ is defined by

$$V=\frac{e^{2\nu}}{r^3}[l(l+1)r+4\pi r^3(\rho-p)-6m] \tag{8}$$

where $l$ is the spherical harmonics index, $\rho$ is the density, $p$ is the pressure, $m$ is the mass inside radius $r$, and $e^{\nu}$ and $e^{\lambda}$ are the metric functions given by the line element for a static neutron star. Outside the neutron star, Eq.(8) reduces to

$$V=\frac{r-2M}{r^4}[l(l+1)r-6M], \tag{9}$$

where $M$ is the total gravitational mass of neutron star. The solutions to this problem are subject to a set of boundary conditions (BC) constructed by Chandrasekhar and Ferrari[3]. The numerical calculations are performed by applying the continued fraction method[7,45] together with the EOSs described in Section 2.

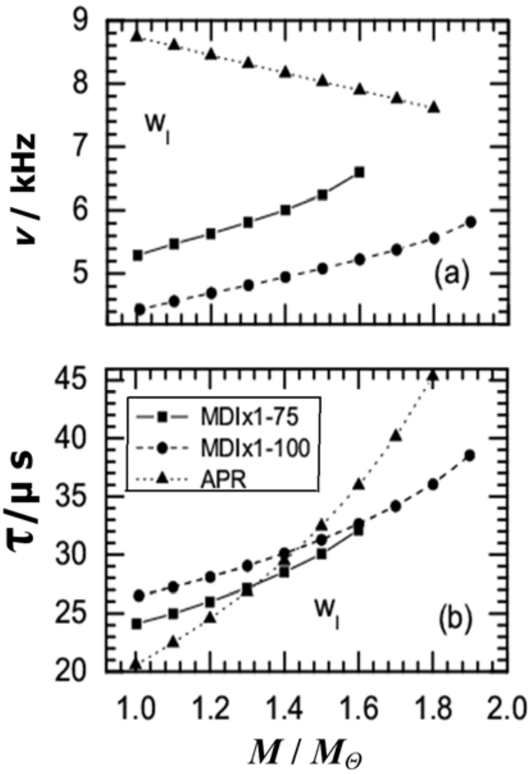


**Fig.3** Frequency (a) and damping time (b) of $w_I$-mode as a function of the neutron star mass $M$.

In what follows we present our numerical results for the axial *w*-modes of neutron stars described by our models. Shown in Fig.3 are the frequencies (a) and the damping times (b) of the $w_I$-mode as a function of the stellar mass. In order to show the effect of the non-Newtonian gravity on the *w*-mode, we also plot the corresponding results of typical EOS, APR, which is often used as a comparing criterion in the neutron star studies. Fig.3(a) shows that at the same stellar mass, the frequency of the $w_I$-mode in the neutron star described by MDIx1 with non-Newtonian gravity is smaller than that of the typical EOS APR. Moreover, for our model, the frequencies increase with the stellar mass, which is contrary to that of the typical EOS APR. The difference of the frequencies between our model and the typical EOSs may come from the neutron star globe structure, as shown in Fig.2, that is, the neutron stars described by our model have larger radii. And a larger non-Newtonian gravitational parameter $g^2/\mu^2$ corresponds with a small frequency. The damping time of $w_I$-mode showing in Fig.3(b) indicates that there is no notable distinguishability among the three considered EOSs for the neutron star with canonical mass $1.4M_{\odot}$.

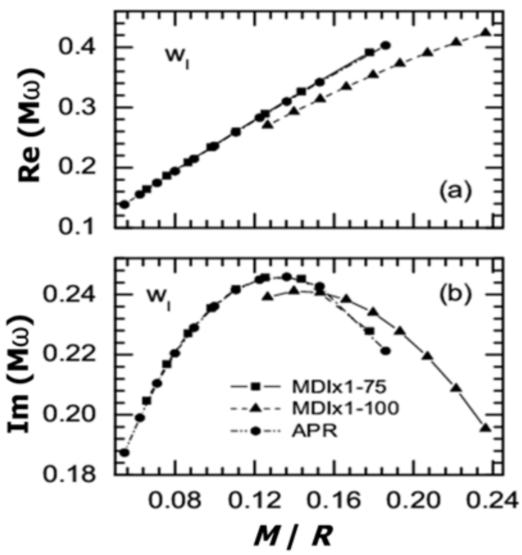


**Fig.4** Frequency (a) and damping time (b) of $w_I$-mode scaled by the stellar mass $M$ as a function of compactness $M/R$.

Previous work has shown that the eigen-frequency of $w$-mode scaled by the stellar mass exhibits a universal behavior independent of the EOS as a function of the compactness $M/R$, which allows an accurate determination of the eigen-frequency of the gravitational waves from the $w$-mode if the masses and radii of neutron stars are accurately observed[7-10]. In Fig.4, we show the frequency (a) and the damping time (b) of $w_I$-mode scaled by the stellar mass $M$ as a function of compactness $M/R$. Also, our results have the universal behavior, and the scaled frequencies of the two models described by MDIx1-75 and MDIx1-100 have a perfect universal behavior.

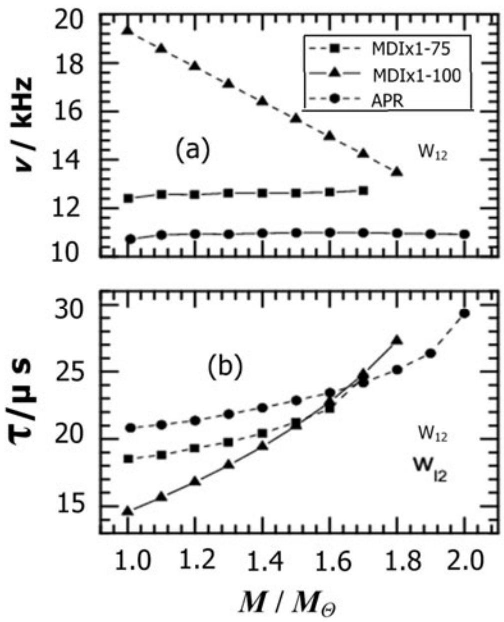


**Fig.5** Frequency (a) and damping time (b) of $w_{I2}$-mode as a function of the neutron star mass $M$.

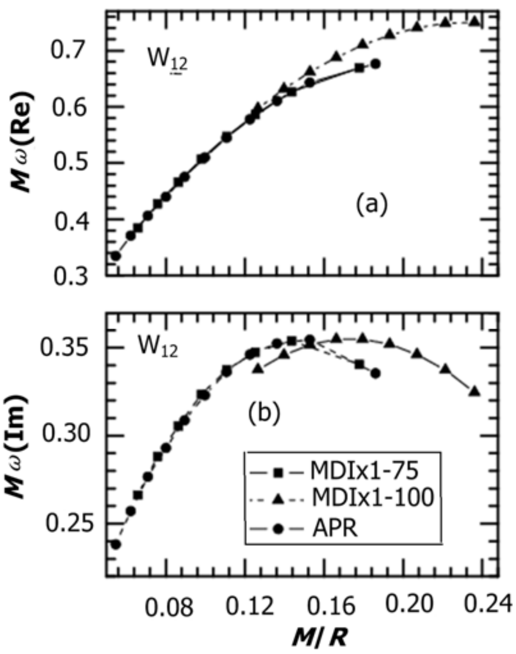


**Fig.6** Frequency (a) and damping time (b) of $w_{I2}$-mode scaled by the stellar mass $M$ as a function of compactness $M/R$.

We also investigate the eigen-frequencies and their scaled properties of $w_{I2}$-mode. The frequencies (a) and the damping times (b) of the $w_{I2}$-mode as a function of the neutron star mass $M$ are shown in Fig.5, and these quantities scaled by the stellar mass $M$ as a function of compactness $M/R$ are shown in Figs.6(a) and 6(b). It is shown in Fig.5(a) that the frequencies of $w_{I2}$-mode described by the MDIx1 are almost a constant for a fixed non-Newtonian gravitational parameter, e.g., 12.6 kHz for $g^2/\mu^2$=75 GeV$^{-2}$ and 10.9 kHz for $g^2/\mu^2$=100 GeV$^{-2}$. The frequencies of the APR model descend obviously as the stellar mass increases. Similarly, there also exists the scaled universal property for the $w_{I2}$-mode, as shown in Fig.6.

## 4 Conclusion

In this work, we calculated the eigen-frequencies of the axial $w$-modes of neutron star described by a super-soft equation of state (EOS), where the non-Newtonian gravity is considered. It is found that the at the same stellar mass, the frequencies of $w_{\mathrm{I}}$ and $w_{\mathrm{I2}}$ for our model are lower than that of the typical EOSs (such as APR). And the frequencies are increasing as the stellar mass increases, which is contrary to that of the typical EOSs. The characters of the frequencies of $w_I$ and $w_{I2}$ can be used as a probe to testify the super soft symmetry energy and the non-Newtonian gravity in the future. Moreover, we also show that this model also has the previously found universal behavior of the mass-scaled eigen-frequencies as a function of the compactness.

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