Shear viscosity to entropy density ratio in BUU transport model

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Abstract Shear viscosity (η) is a basic transport coefficient of the medium. In this work, we calculate shear viscosity to entropy density ratio (η/S) of an equilibrated system in intermediate energy heavy ion collisions within the framework of the Boltzmann-Uehling-Uhlenbeck model (BUU) model. After the equilibration of Au + Au system at central collision in a fixed volume is reached, temperature, pressure and energy density are extracted by the phase space information and then η/S is calculated using the Green-Kubo formulas. The results show that η/S drops with the incident energy and its value is not so drastically different from the RHIC results.

Key words BUU model, Liquid-gas phase transition, Shear viscosity, Entropy density, η/S

1 Introduction

Phase transition and critical phenomenon is an extensively debatable subject of natural sciences. There are two classes of strong interaction phase transition in nuclei. One is the nuclear liquid-gas phase transition which usually occurs in immediate heavy -ion collisions (HIC)[1–7], while another is quark gluon plasma (QGP) phase transition in ultra- relativistic heavy-ion collision[8–12]. Recently, the shear viscosity to entropy density ratio (η/S) was applied to study the QGP transport coefficient and a minimum value of η/S was found around 170 MeV[13,14], but limited investigations have been done on the ratio of η/S in immediate-energy heavy ion collision [15–18].

In this paper, we study the thermodynamic and transport properties of nuclear reaction system in intermediate-energy HIC, to find how η/s evolves with the beam energy, with the equilibration of nuclear system in a finite volume using Boltzmann-Uehling-Uhlenbeck (BUU) model. To have a system of enough nucleons in a fixed spherical volume, we choose Au + Au system in head-on collision (b = 0 fm).

The central collisions are studied in this paper, and the system can approach a local equilibration if the system has a long-enough time to evolve. The equilibrium in intermediate energy heavy ion collisions can be judged by using the temperature and other dynamical variables. After the system is in equilibrium, the thermodynamic parameters (pressure, energy density and entropy density) are calculated from phase space information of the system. Shear viscosity coefficient is calculated from stress tensor fluctuations around the equilibrium state using Green-Kubo formula. The η/S in different incident energies are compared and discussed.

2 BUU model and system equilibrium

To study equilibration of nuclear system, microscopic calculations in immediate heavy ion collision are performed by using BUU model [19, 20]. The BUU equation reads [21]

$$\partial f/\partial t+v\nabla_r f-\nabla_r U\nabla_p f=[4/(2\pi)^3]\int d^3p_2 d^3p_3 d\Omega(d\sigma_{nn}/d\Omega)V_{12}\ [f_3f_4(1-f)(1-f_2)-ff_2(1-f_3)(1-f_4)]\ \delta^3(p+p_2-p_3-p_4) \quad (1)$$

where, $d\sigma_{nn}/d\Omega$ is medium nucleon-nucleon cross section and V_{12} is relative velocity, for the colliding nucleons; and U is the mean field potential including the isospin-dependent term:

$$U(\rho,\tau_z)=a(\rho/\rho_0)+b(\rho/\rho_0)^\sigma+C_{sys}[(\rho_n-\rho_p)/\rho_0]\tau_z \quad (2)$$

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where ρ_0 is the normal nuclear matter density; ρ, ρ_n, and ρ_p are densities of the nucleon, neutron and proton, respectively; τ_z = 1 for neutrons and τ_z = –1 for protons; and a, b and σ are parameters for nuclear equation of state. In this paper, mean field parameters are used, namely the hard EOS with K=380 MeV (a=–124 MeV, b=70.5 MeV, σ=2), and C_{sys}= 32 MeV, which is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium.

In this paper, we focus on the thermodynamic and transport properties of a nuclear system. For this purpose, we investigate the process of the head-on Au +Au collision in a spherical volume of 5 fm in radius.

First we check the evolution of equilibration situation and temperature. The anisotropy ratio, which is a measure of the degree of equilibration in a heavy-ion reaction, is defined as:

$$R_p = (2/\pi)\ R_{//}/R_{\perp} \tag{3}$$

where $R_{//}$=$<(p_x^2+p_y^2)^{1/2}>$ and $R_{\perp}$=$<p_z>$ are calculated by the momentum of nucleons in the given sphere. As an example, the time evolutions of R_p for Au + Au systems within a 5-fm radius sphere at 50, 70 and 100 AMeV are shown in Fig.1.

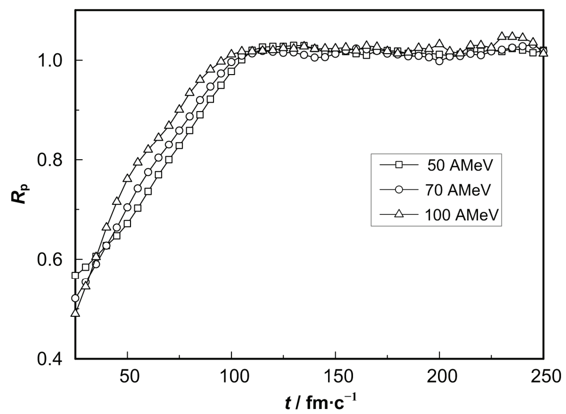

Fig.1 R_p values at 50, 70 and 100 AMeV as a function of time. When R_p approaches to 1,the system tend towards equilibrium.

When R_p approaches to 1 at around 100 fm/c, the nuclear system is under equilibrium. Time evolution of temperature is used to judge the state of equilibration. Temperature of the system can be derived from the momentum fluctuations of particles in the center of mass frame of the fragmenting source [22,23]. The σ^2 is obtained from the Q_z distribution

$$\sigma^2 = \left\langle Q_z^2 \right\rangle - \left\langle Q_z \right\rangle^2 \tag{4}$$

where Q_z is the quadruple moment defined, and p_x; p_y and p_z are three components of momentum vector extracted from the phase space of BUU model. If the mean equals zero, the second term vanishes. Q_z^2 is described by

$$\left\langle Q_z^2 \right\rangle = \int d^3 p (2p_z^2 - p_x^2 - p_y^2)^2 f(p) \tag{5}$$

Assuming a Maxwellian distribution for the momentum distribution, i.e.

$$f(p)=\exp[-(p_x^2+p_y^2+p_z^2)/(2mT)]/(2\pi mT)^{3/2} \tag{6}$$

we can obtain after Gaussian integral,

$$\left\langle Q_z^2 \right\rangle = 4m^2 A^2 T^2 \tag{7}$$

where m is the mass of a nucleon and A is the mass number of the fragment. For a nucleonic system of A = 1, we can calculate the evolution of temperature using Eq(7). Fig. 2 shows the evolution of temperature after 25 fm/c at 50, 70, and 100 AMeV. One finds that temperature reaches a maximum around 50 fm/c when the system is in the most compressible stage, and then it starts to cool down when the system expands, towards a thermodynamic equilibrium of the system. Fig.2 also shows that higher temperature can be reached by higher beam energy.

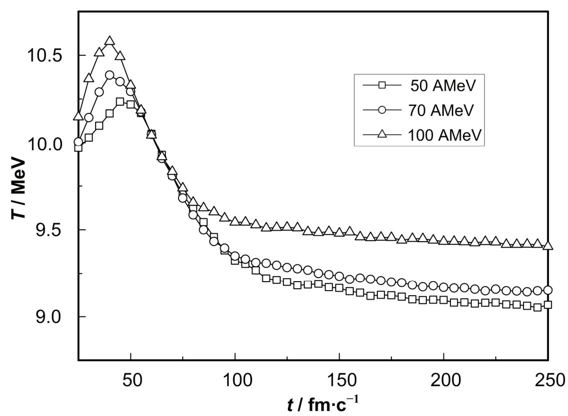

Fig.2 Temperature as a function of time (after 25 fm/c) for the head-on Au+Au collision within 5-fm radius sphere at 50 AMeV. When t≥90 fm/c, temperature approaches to an asymptotical value.

For an equilibrated system, the kinetic energy distributions approach the Boltzmann distribution as time increases[24]. After the expansion process, the system will equilibrate, so we can investigate the viscosity coefficient and entropy density in the system.

3 Calculation of viscosity and entropy density

Transport coefficients, such as viscosity, diffusions, and conductivity, characterize the dynamics of fluctuation of dissipative fluxes in a medium. Transport coefficients can be measured, as in the case of condensed matter applications. However, in principle they should be calculable theoretically. Monte Carlo simulations for transport coefficients are a powerful tool when studying transport coefficients using Green-Kubo relations[25,26]. In high energy heavy-ion collisions, transport coefficients of shear viscosity for a binary mixture was calculated in Ref. [27], and the coefficient of a hadrons gas was calculated in Ref.[28]. In this work, we employ the Green-Kubo formula to extract the shear viscosity of the system in intermediate-energy HIC[29]. The formula relates linear transport coefficients to near-equilibrium correlations of dissipative fluxes and treats dissipative fluxes as perturbations to local thermal equilibrium. The Green-Kubo formula for shear viscosity is defined by

$$\eta = \frac{1}{T}\int \mathrm{d}r^3 \int_0^\infty < \pi_{ij}(0,0)\pi_{ij}(\boldsymbol{r},t) > \mathrm{d}t \qquad (8)$$

where T is temperature of the system, t is the post-equilibration time (t =0 when the system equilibrates), and $<\pi_{ij}(0,0)\pi_{ij}(\boldsymbol{r},t)>$ is the shear component of the energy momentum tensor. The energy momentum tensor is defined by $\pi_{ij} \equiv T_{ij} - \delta_{ij} T_i^i/3$ and $T_{ij} = \int \mathrm{d}^3 p (p^i p^j / p_0) f(x,p,t)$[30], where $f(x,p,t)$ is the phase space density of the particles in the system. Assuming that the nucleons are uniformly distributed in space and volume is fixed, the viscosity can be calculated by

$$\eta = V / <\pi_{ij}(0)^2> \tau_\pi \qquad (9)$$

where $\pi_{ij}(0)$ is the energy momentum tensor when the system equilibrates and τ_π is the entropy density. We extract the energy density, pressure and temperature when the system is under equilibrium, and use the Gibbs formula, $S = (\varepsilon + p - \mu_n \rho)/T$ to get the entropy density, where μ_n is the nucleon chemical potential and ρ is nucleon density of system within the given sphere. Here we assume μ_n= 20 MeV in the calculation[31].

As an example, Fig.3 shows the energy density (ε), pressure (P) and entropy density (S) versus temperature for 50 AMeV Au+Au system in the sphere of 5-fm radius after 40 fm/c. All variables show a reasonable increasing behavior with the temperature.

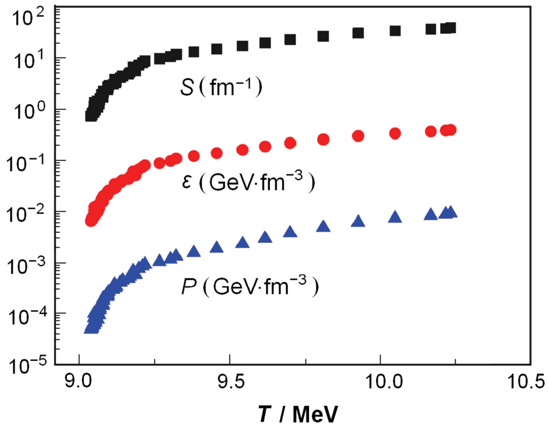

Fig.3 Different thermodynamic variables (entropy density, energy density and pressure) as a function of temperature after t = 40 fm/c for the head-on Au + Au collision within 5fm-radius sphere at 50 AMeV.

Using the aforementioned techniques to calculate the shear viscosity and entropy density, we present the value of shear viscosity to entropy density (η/S) as a function of incident energy for Au + Au system in 5-fm sphere under equilibrium (Fig.4).

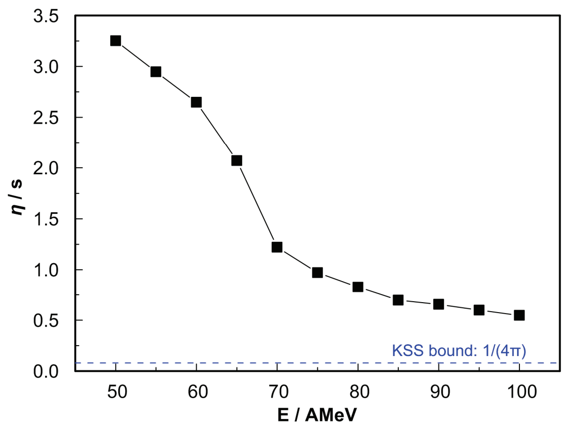

Fig.4 The ratio of shear viscosity to entropy density(η/S) as a function of incident energy for the head-on Au + Au collision within 5-fm radius sphere at 50 AMeV. The dotted line represents a KSS bound of η/S=1/(4π).

The η/S value decreases rapidly with increasing incident energy up to E < 70 AMeV, where it begins to drop slowly to about 0.5. Since the BUU equation is one-body transport theory, fragmentation which originates from the fluctuation and correlation cannot be treated in the present model. In this case, the phase transition behavior cannot be predicted in the BUU model. The continuous drop of the η/S ratio does not show a minimum at a certain beam energy, which indicates no obvious phase change or critical behavior in the present model. This is a shortcoming of the BUU model itself, especially when it is applied to higher beam energy. In the present BUU calculation, all calculated values of η/S are well above the conjectured KSS lower bound of $1/(4\pi)$[32,33]. Comparing these values of η/S for our finite nuclei in BUU model, they do not differ drastically from either the RHIC results[27,34] or results of the usual finite nuclei at low temperature from the widths of giant vibrational states in nuclei[18]. As pointed out in Ref. [18], it is possible that the strong fluidity is a characteristic feature of the strong interaction of the many-body nuclear systems in general and not just of the state created in the relativistic collisions.

4 Conclusion and outlook

In summary, we studied thermodynamic variables and the shear viscosity/entropy density ratio for a finite nuclear system after the system tends to equilibrate in intermediate energy heavy ion collisions by using the BUU model. The Green-Kubo relation was applied for the nucleonic matter in a central region with a moderate volume when the system was in equilibrium stage for central heavy-ion collisions of Au+Au. The system's ratio of shear viscosity to entropy density η/S decreases quickly before 70 AMeV, and drops slowly towards η/S = ~0.5 at beam energies of >70 AMeV. The η/S does not differ drastically from either the RHIC results or results of the usual finite nuclei at low temperature. However, no obvious minimum η/S value occurs within the energy range under investigation. This suggests that there may no be liquid-gas phase transition by the present model, which lacks dynamical fluctuation and correlation. Therefore, other models which can incorporate liquid gas phase transition should be checked for the shear viscosity and entropy density. For instance, it will be interesting to use quantum molecular dynamics-type model to check if a minimum of η/S will occur around liquid-gas phase transition. The work along this direction is in progress.

Acknowledgments

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