

Systematics of α -preformation factors in closed-shell regions

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Abstract The α -preformation factors of medium and heavy-mass nuclei are calculated by using the cluster-formation model. The obtained preformation factors of even-even, odd-A, and odd-odd nuclei consist in both magnitude and trend with the general features predicted by standard calculations. The variation of α clustering affected by the evolution of nuclear structure is observed from different behaviors of preformation factors. We typically analyze the variation of preformation factors in the closed-shell $N = 126$ and $Z = 82$ regions, and discuss in detail the structural effects on α -cluster formation. This work shows the strong correlation between α -preformation factors and the shell structure, which would be a useful reference for microscopic cluster-model calculations of α -decay half-lives.

Keywords α -Preformation factors · α Clustering · α Decay · Medium and heavy-mass nuclei

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1 Introduction

As one of the fundamental decay modes of unstable nuclei, α decay receives constant attention due to its importance in investigations of nuclear stability and structure. Experimentally, the identifications of newly synthesized superheavy elements are mostly achieved by detecting α -decay chains [1]. Meanwhile, measurement of α decay energy also provides reliable information for the study of nuclear mass and excitation level [2–4]. On the other hand, since α -decay half-life is closely related to the structural properties of the decaying system [5–8], α decay is used as a powerful tool to extract the structural information such as deformation [9, 10], nucleon density distribution [11, 12], α -preformation probability [13–17], etc. As a kind of microscopic theory of α decay, the cluster model, which considers the decaying nucleus as an α -core system, has been able to reproduce the half-lives of most known α -radioactive nuclides within permissible range of deviation [18, 19]. Furthermore, the α -decay fine structure (branching ratios to different excitation levels) was well described in recent coupled-channel calculations [20, 21]. Although the present cluster-model description can well reproduce most experimental data, the calculation still can be improved because the α -preformation factor employed in the theory is not consistently evaluated due to the complexity of nuclear many-body problem. More importantly, this factor is considered to carry most information of nuclear structure, which is essential to understand how α clustering happens in heavy nuclei.

Within the cluster-model description, the preformation factor (P_α), which is also named spectroscopic factor in some literatures, is introduced to describe the probability of an α cluster forming inside the parent nucleus. Usually, this

factor is presumed as a constant for a certain kind of nuclei (even–even/odd–odd) whereby the value is determined through minimizing the deviation between theoretical and experimental decay widths. This simple treatment yields satisfactory results for most open-shell nuclei, and also there are statistical analyses suggesting that the P_α factor should vary smoothly in the open-shell region [22, 23]. As for nuclei around closed magic shells, however, such an assumption may cause large discrepancy of α -decay half-life between theoretical predictions and the experimental data [24, 25].

Progress on microscopic calculation of P_α has been made on a typical nucleus ^{212}Po , which can be regarded as an α cluster plus a double-magic core ^{208}Pb . Varga et al. successfully reproduced the experimental decay width of ^{212}Po by using a wavefunction with combined shell-model and cluster-model components as the initial state of parent nuclei, and the P_α factor was consistently obtained as 0.3 [26, 27]. Recently, Röpke et al. attempted to describe the transition of α -cluster state in ^{212}Po from an unbound four-nucleon shell model state to an intrinsic bound α -like state as the baryonic density decreases at the nuclear surface [28]. The α formation is considered to take place at a critical density as 0.03 fm^{-3} . The microscopic calculations above were performed due to the simplicity of structure for ^{212}Po , but as more valence nucleons are involved, the complexity of depicting the α correlation in nuclei would largely increase. Therefore, it would be very difficult to pursue a fully microscopic description of α -preformation process for nuclei without an α -core structure.

As a simple method to obtain the information of P_α for known α emitters, one can roughly extract the P_α factors through dividing the experimental decay width by the theoretical value. Numbers of works are devoted to this approach and reasonable results are obtained [11, 29–33]. However, in some cases such a simple treatment cannot ensure a smooth variation of P_α for adjacent nuclei in open-shell regions, because the decay width is strongly dependent on the chosen α -daughter potential. To obtain a reasonable P_α variation, the recently proposed cluster-formation model (CFM) suggests that the P_α factors can be phenomenologically extracted from the experimental binding energies [34, 35]. Based on simple quantum-mechanical considerations, the relation between P_α and binding energy is derived by considering the interaction between surface nucleons as a predominant factor to the formation of α cluster. More recently, we successfully generalized the model to odd- A and odd–odd nuclei by including the effect from unpaired nucleons [36, 37]. The results obtained reasonably agree with the major features predicted in previous researches. In the present study, in order to obtain a comprehensive view of the systematics of

P_α in the closed-shell region, the calculation of P_α factors is extended by including more experimental data. Up to 505 nuclei are investigated and analyzed within the CFM, and we analyze in detail the behavior of P_α in the typical $N = 126$ and $Z = 82$ shell regions.

The paper is organized as follows. In Sect. 2, we briefly present the theoretical formulism of the CFM, specially the content of extracting the α -cluster formation energy. In Sect. 3, we show the numerical results and discuss in detail the correlations between P_α variation and its associated structural effects. A summary of the results is given in Sect. 4.

2 Theoretical descriptions

Within the CFM, the clusterization state, Ψ_i , is introduced to describe different kinds (i) of possible clustering of nucleons. The clusterization state is the eigenstate of its corresponding clusterization Hamiltonian, H_i , which is derived from the theoretical separation of the many-body Hamiltonian for the parent nucleus into the one for a specific clusterization configuration [34]. Accordingly, the initial state, Ψ , of the parent nucleus can be defined as the superposition of all these possible clusterization states, and the total Hamiltonian for the system, H , can be written as the summation of corresponding clusterization Hamiltonians,

$$\Psi = \sum_i^N a_i \Psi_i, \quad (1)$$

$$H = \sum_i^N H_i, \quad (2)$$

where a_i is the superposition coefficient for Ψ_i . Since all these clusterization states describe the same system, they are assumed to share a same eigenenergy, which is equal to the eigenenergy, E , for the total wavefunction. Consequently, considering the orthonormality of the clusterization wavefunctions, one can derive the following relation in terms of energy [34],

$$E = \sum_i^N |a_i|^2 E = \sum_i^N E_{fi}, \quad (3)$$

where E_{fi} is the formation energy for the cluster in clusterization state Ψ_i . Specifically in the case of α clusterization, the preformation probability of α cluster can be properly defined as

$$P_\alpha = |a_\alpha|^2 = \frac{E_{f\alpha}}{E}, \quad (4)$$

where a_α denotes the superposition coefficient for α clusterization state Ψ_α . In principle, to accurately determine

this coefficient, one should calculate the overlap integral between α clusterization state and the initial state wavefunction by solving the time-independent Schrödinger equation. However, according to Eq. (4), the P_α factor can be alternatively evaluated if the ratio of the formation energy to the total energy is determined. For medium and heavy-mass nuclei, it is suggested that the formation of α cluster happens at the nuclear surface [38–40]. One can reasonably consider the α correlation of surface nucleons contributes predominantly to the cluster formation. Since the α correlation is based on the pairing and n–p correlations, which are widely investigated on the basis of binding energy [41–44], it would be possible to determine these two energy values by extracting from experimental binding energies. Within the CFM, the α formation energy, $E_{f\alpha}$, represents the intrinsic correlation energy of the α cluster, whereas the energy, E , corresponds to the total energy of the considered four-nucleon system. For even–even nuclei, both these energies were determined by Ahmed and co-workers after they systematically analyzed the nucleon–nucleon correlations in the heavy-mass region [34]. In our recent research, we generalized the model to odd- A and odd–odd nuclei by expressing these two energies in a separation-energy systematics [36, 37],

$$E_{f\alpha} = \begin{cases} 2S_p + 2S_n - S_\alpha & (\text{even–even}) \\ 2S_p + S_{2n} - S_\alpha & (\text{even–odd}) \\ S_{2p} + 2S_n - S_\alpha & (\text{odd–even}) \\ S_{2p} + S_{2n} - S_\alpha & (\text{odd–odd}) \end{cases}, \quad (5)$$

$$E = S_\alpha(A, Z), \quad (6)$$

where $S_p(A, Z)$, $S_n(A, Z)$, and $S_\alpha(A, Z)$ are single-proton separation energy, single-neutron separation energy, and α -cluster separation energy, respectively. The definitions of these separation energies are given by

$$S_{2p}(A, Z) = B(A, Z) - B(A - 2, Z - 2), \quad (7)$$

$$S_{2n}(A, Z) = B(A, Z) - B(A - 2, Z), \quad (8)$$

$$S_\alpha(A, Z) = B(A, Z) - B(A - 4, Z - 2). \quad (9)$$

Figure 1 explains schematically the detailed components of the formation energy shown in Eq. (5). In the α clusterization state, each nucleon of the α cluster contributes a S_p (for proton) or S_n (for neutron) to the α formation energy. The summation of four single-nucleon separation energies doubly counts the internal interactions of the α cluster when compared to the total energy, S_α . As a result, the formation energy can be extracted by this summation minus the α -cluster separation energy. In the case of even–even nuclei, the single-nucleon separation energies of the two protons (neutrons) are not distinguished because all nucleons are paired in the ground state. But for odd- A and odd–odd systems, the existence of unpaired

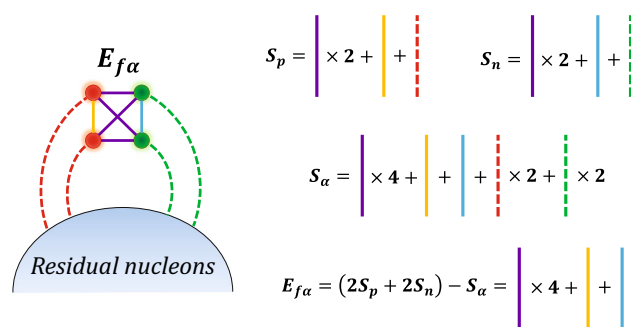


Fig. 1 (Color online) A schematic illustration of the components of the α -cluster formation energy in Eq. (5). The formation energy of an α cluster is denoted by summation of all *solid lines*, while the total energy of the system is a combination of all *solid and dashed lines*. Each nucleon of the α cluster contributes a single-nucleon separation energy which includes three kinds of interaction: the interaction between like nucleons within the α cluster (*one yellow or blue solid line*), the neutron–proton interaction within the α cluster (*two purple solid lines*), and the interaction with residual nucleons outside the α cluster (*one red or green dash line*)

nucleon can largely suppress the cluster formation. To incorporate the influence from the unpaired nucleon(s), the term $2S_p$ ($2S_n$) in formation energy should be replaced with a two-nucleon separation energy S_{2p} (S_{2n}). In this way, the hindrance caused by the unpaired nucleon can manifest itself as a reduction in formation energy, and consequently the P_α would decrease if more odd nucleons are included.

It should be noted that α decays of odd- A and odd–odd nuclei are split into favored and unfavored transitions. The P_α factors calculated by using Eq. (5) are mainly for the favored case, because only the influence from binding energy is considered in the present model. For unfavored decays, the α formation process becomes extremely complicated due to the change of spin-parity between parent and daughter states. This makes the P_α factor highly correlated to the details of the wavefunctions which are still too difficult to be acquired. Therefore, the P_α given by the CFM basically correspond to favored α decays.

3 Results and discussion

In this work, the P_α of 505 nuclides (138 even–even nuclei, 254 odd- A nuclei, and 113 odd–odd nuclei) are calculated using Eqs. (4), (5), and (6). All the investigated nuclei have a positive Q_α value, and their corresponding binding energies are taken from the newest atomic mass evaluation (AME2012) tables [45]. As is known, the P_α factors also can be evaluated indirectly by extracting from experimental α -decay half-lives. Calculations based on this approach have been performed by models such as the generalized density-dependent cluster model (GDDCM) [11], generalized liquid drop model (GLDM) [29, 30], and

Hamiltonian energy density approach in terms of the SLy4 Skyrme-like effective interaction (SLy4) [17, 32]. Before we present the details of our results, it would be interesting to compare the calculated P_α factors with these models to examine the reliability of the CFM. In Fig. 2, we show the P_α factors of Po isotopes calculated by different models. It can be clearly observed that the trends of P_α variation predicted by these four models are very similar. The large shell effect for $N = 126$ is reasonably reproduced by all these models. Furthermore, the P_α factors by the CFM vary more smoothly than by the others. This is mainly due to the fact that the binding energy almost increases homogeneously with nucleon number in this region. The major difference in this figure is the magnitude of the P_α factors. It can be seen that the results by GLDM and SLy4 come close to each other and are about one order of magnitude smaller than by the CFM and GDDCM, whose magnitudes are basically of the same order. It should be noted that the P_α factor of ^{212}Po was microscopically evaluated as 0.3 in previous study [26]. This value is considered reliable and comes very close to the results 0.22 and 0.25 by CFM and GDDCM, respectively. Therefore, the P_α obtained by the CFM and GDDCM might be more approximate to the realistic values.

In order to investigate the detailed behavior of the P_α factors in the $N = 126$ region, we plot the P_α evolution along different isotopic chains in Fig. 3. Interestingly, we find the results are mostly divided into two groups. For even- N isotopic chains, P_α smoothly decreases as N increases below $N = 126$, but a sharp increase happens for all isotopic chains when N goes beyond the neutron shell closure. This behavior shows that the stabilizing effect from $N = 126$ closed shell strongly suppresses the formation of the α cluster. The sudden enhanced P_α at $N = 128$

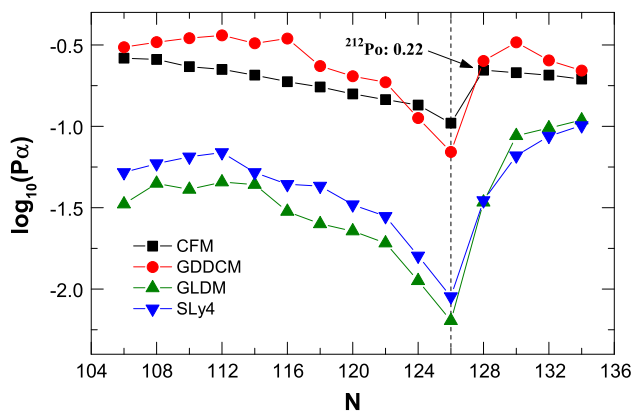


Fig. 2 (Color online) Comparison of the α -preformation factors of Po isotopes obtained by different models. The results show that all the considered models yield a similar trend of P_α variation, while the P_α by GLDM and SLy4 are about one order of magnitude smaller than by the CFM and GDDCM

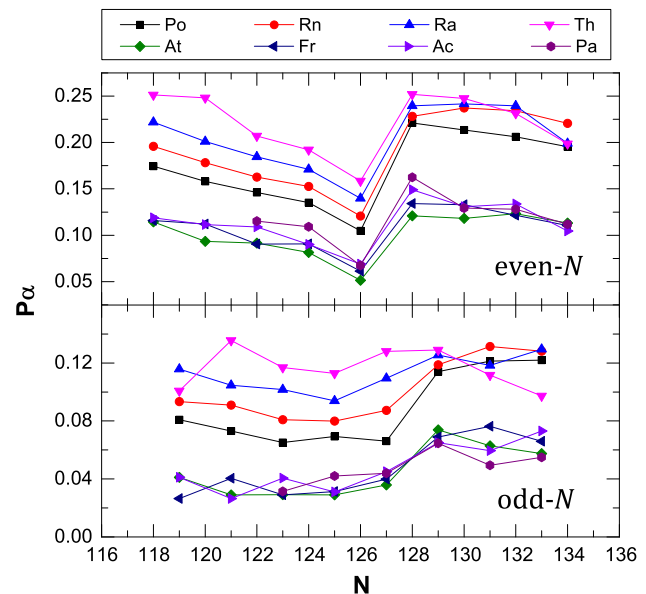


Fig. 3 (Color online) Behavior of the α -preformation factors around $N = 126$. For even- N isotopic chains, the shell effect is indicated by the sudden increase in P_α after $N = 126$, while for odd- N isotopic chains such an increase happens between $N = 127$ and $N = 129$

can be attributed to the two loosely bounded neutrons above the shell contributing predominantly to the cluster configuration, which manifests itself as a typical signature of the shell effect. However, one can observe such an increase for odd- N isotopic chains occurs at $N = 129$ instead of $N = 127$. Of a similar physics, this is because the α transition would become more favored if the neutron pair above the shell closure could be utilized to form the α cluster. In fact, the $N = 126$ shell effect in P_α evolution of even-even nuclei was revealed in previous studies [12, 17, 29, 30]. In our results, not only a consistent P_α evolution for even-even nuclei is reproduced, we also demonstrate the $N = 126$ shell effect for odd- A and odd-odd nuclei and point out that the indications of such effect for even- N and odd- N isotopic chains are characterized by a typical increase emerging at different locations in the P_α evolution.

In addition to the $N = 126$ shell effect, the systematic behavior of P_α around $Z = 82$ is more of our interest because the indication of $Z = 82$ shell effect for very neutron-deficient nuclei was still obscured due to the lack of precise experimental data [46, 47]. In Fig. 4, we show the P_α variation of even-even and even-odd nuclei along different isotonic chains. It can be observed clearly that the $Z = 82$ shell emerges as a typical dip at the shell closure for all isotonic chains. Moreover, the variation of P_α at $Z = 82$ is seen less significantly when compared with the $N = 126$ case in Fig. 3, which implies the proton shell closure exerts a smaller influence on the preformation probability. On the other hand, due to the investigated

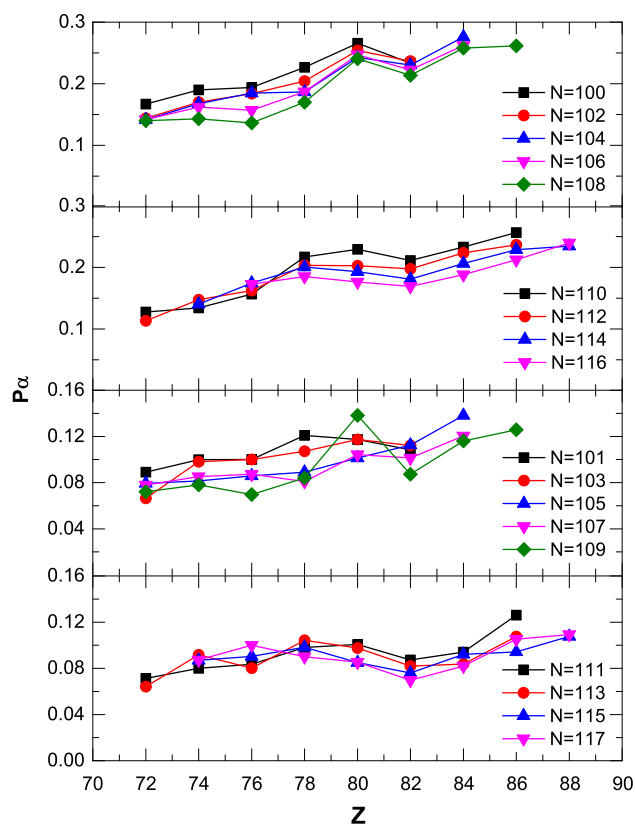


Fig. 4 (Color online) Behavior of the α -preformation factors around $Z = 82$. The $Z = 82$ shell effect manifests itself as a small dip at the shell closure for all isotonic chains

nuclei in Fig. 3 located around ^{208}Pb , the intervention of both neutron and proton shell closures may be also responsible for a larger amplitude in P_α variation at $N = 126$. It should be noted that in a recent experiment, the researchers precisely measured the α half-lives of some very neutron-deficient nuclei and then extracted the α -particle formation probability by the universal decay law (UDL) [48]. They successfully observed the signature of $Z = 82$ shell closure which also shows the above characteristics. Therefore, it is encouraging that this signature is reproduced by the CFM calculation without invoking any half-life data from experiments.

Another feature that should be noted in Fig. 4 is that the P_α factor steadily climbs with increasing Z below the proton shell closure. As can be seen from Fig. 3, the behavior of P_α below $N = 126$ is totally different, for P_α decreases rapidly due to the stabilizing effect of the closed magic shell. To understand the seemingly strange behavior observed below $Z = 82$, one should consider the excitations of protons across the $Z = 82$ shell closure. As we know, the α -cluster formation can be explained by the configuration mixing of cross-shell excitations of correlated paired nucleons [41, 42]. Below $N = 126$, the formation process is substantially suppressed because the

large gap above shell closure makes it almost impossible for neutrons to be excited to a higher major shell. But as we mentioned above, the stabilizing effect of $Z = 82$ shell is much smaller. Once the protons below are excited to the $82 < Z \leq 126$ major shell, the formation probability would be enhanced because both correlated proton and neutron pairs now occupy the levels within the same major shell. The fewer valence neutrons exist below the proton shell closure, the more likely such excitations would happen. This explains why P_α increases with larger Z below the shell closure. On the other hand, the existence of this kind of excitation can also be evidenced by the shape-coexistence phenomenon around the $Z = 82$ region. This phenomenon is due to the nuclear deformation which results from cross-shell excitations of protons and the n-p interaction [49]. Therefore, the increasing trend observed in Fig. 4 is probably related to the excitations of protons across the $Z = 82$ shell.

In previous studies of α -decay half-lives of exotic isotones around $N = 126$ shell, the P_α factors are suggested to follow a linear relation with Z to achieve a better agreement with experimental data [24, 25]. This Z -dependent relation is derived from a microscopic two-level model where the shell and blocking effects can be described by considering the pairing and n-p interactions between valence nucleons [41, 42]. Interestingly, this linear behavior is also observed in our results. In Fig. 5, we show the evolution of P_α factors with Z for different isotonic chains. The P_α factors are found to be highly linear with Z according to a given linear correlation coefficient $R^2 > 0.9$. The success in reproducing such a structural effect further confirms reliability of the CFM. Besides, it should be noted that this linear behavior seems more valid for even- Z

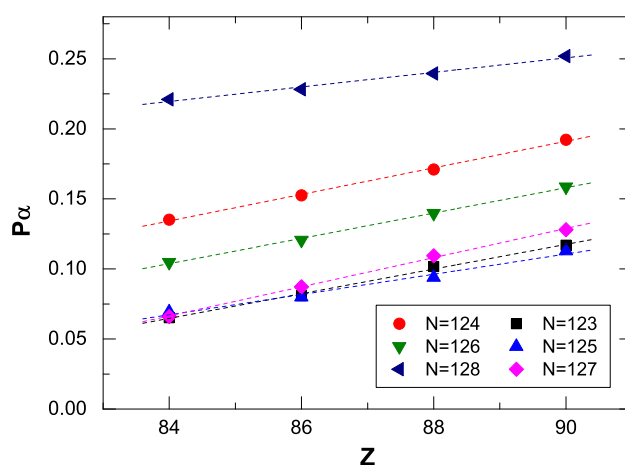


Fig. 5 (Color online) The linear behavior of α -preformation factors around $N = 126$. All the dashed lines are obtained by a linear regression analysis. The linear relation between P_α and Z suggested in Refs. [24, 25] is confirmed for the linear correlation coefficient $R^2 > 0.9$

isotones than for odd- Z isotones as can be seen from Fig 3. The vanished linear behavior for odd- Z nuclei may be attributed to the α formation being interrupted by the unpaired proton, but it still requires further investigations to reach a conclusion.

4 Summary

The evolution of α -preformation factors around $N = 126$ and $Z = 82$ shells is systematically investigated by using the cluster-formation model. The results show consistent trend and order of magnitude of P_α factors with previous statistical analyses. Strong correlation between nuclear structure and the behavior of P_α evolution is demonstrated and discussed.

In terms of the magnitude, P_α of odd- A nuclei are generally smaller than those of even-even nuclei, while for odd-odd nuclei P_α became even smaller due to the existence of both unpaired proton and neutrons. The shell effect around $N = 126$ can be observed as a sudden increase in P_α across the neutron shell closure. For even- N isotones, this increase happens between $N = 126$ and $N = 128$, but for odd- N isotones it occurs between $N = 127$ and $N = 129$. Relative to the case of $N = 126$, the $Z = 82$ shell effect behaves less notably when the smaller amplitude of P_α variation is observed. The smooth increase of P_α observed in the $Z < 82$ region probably correlates with the cross-shell excitations of protons below the proton closed shell. All these indications support the existence of $Z = 82$ shell closure for very neutron-deficient nuclei. Besides, a strong linear correlation between P_α factors and proton number is observed for even- Z nuclei around $N = 126$, which consists in the relation predicted by microscopic calculations.

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