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α-Decay half-life screened by electrons

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Abstract In this paper, by considering the electrons in different external environments, including neutral atoms, a metal, and an extremely strong magnetic-field environment, the screened α-decay half-lives of the α emitters with proton number $Z = 52$–105 are systematically calculated. In the external environment, the decay energy and the interaction potential between α particle and daughter nucleus are both changed due to the electron screening effect and their variations are both very important for the electron screening effect. Besides, the electron screening effect is found to be closely related to the decay energy and its proton number.

Keywords Electron screening effect · α-Decay half-life · Density-dependent cluster model

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1 Introduction

Since the pioneering work of Gamow in 1928 [1] where α decay was successfully explained as a quantum tunneling effect, much attention has been paid to the α decays of unstable nuclei and several analyses have been performed to calculate the half-lives of α emitters throughout the nuclide chart with the shell model [2], the cluster model [3–5], the liquid-drop model [6], and the fission-like model [7]. By combining the two-potential approach (TPA) [8] and a microscopic potential, we investigated the α-decay half-lives of both spherical and deformed nuclei by using the density-dependent cluster model (DDCM) [9–13].

Although lots of theoretical studies of α decays have been conducted, α-decay half-life screened by electrons has not been systematically studied. The electron screening effect is discussed only in a few theoretical works in several external environments, such as in neutral atoms within different approaches [14–18], in a metal environment [19–22], on nuclear decays and reactions at astrophysical energies [23, 24], and in dense astrophysical plasmas and super strong magnetic fields [25–30]. Previous research only focuses on the screening effects on α decays in one specific environment. In our recent study [31], the screened α-decay half-lives are systematically calculated with the DDCM in external environments, namely neutral atoms, a metal, and an extremely strong magnetic-field environment. A brief review is given here.

2 General analysis of α decays screened by electrons

In an external environment, the interaction potential, $V(R)$, between α particle and daughter nucleus and the decay energy, Q are both changed, resulting in a variation in the potential barrier that the α particle penetrates, as shown in Fig. 1.

It is obvious that the screened potential barrier is different from the non-screened (bare) one so that the α-decay half-life $T_{1/2}$ will be changed in an external environment. In the DDCM, $T_{1/2}$ can be calculated by [11–13]

$$T_{1/2} = \frac{\hbar \ln 2}{P_\alpha F \frac{\hbar^2}{4\mu} \exp\left[-2\int_{R_2}^{R_3} \mathrm{d}R \sqrt{\frac{2\mu}{\hbar^2}|P(R)|}\right]}, \tag{1}$$

where P_α is the α-particle preformation factor and F is a factor well defined by the TPA [8]. R_2 and R_3 are two turning points. $P(R)$ is defined as $P(R) = V(R) - Q$, where $V(R)$ is the interaction potential and Q is the decay energy. For a bare nucleus and in an external environment, $P(R)$ can be expressed as

$$P_\mathrm{B}(R) = V_\mathrm{B}(R) - Q_\mathrm{B}, \tag{2}$$

$$P(R) = V_\mathrm{equ}(R) - Q_\mathrm{B}, \tag{3}$$

where $V_\mathrm{equ}(R)$ is an equivalent interaction potential in an external environment,

$$\begin{aligned} V_\mathrm{equ}(R) &= V_\mathrm{B}(R) + \Delta V(R) - \delta Q \\ &= V_\mathrm{B}(R) + \delta V(R) + [\delta V - \delta Q] \\ &= V_\mathrm{B}(R) + \delta V(R). \end{aligned} \tag{4}$$

The quantities $\Delta V(R) = \delta V + \delta V(R)$ and δQ are the variations in $V(R)$ and Q in an external environment, respectively [16–18]. The condition $\delta V = \delta Q$ presented by Karpeshin et al. [16, 17] is also applied here. Equations (2–4) show that the term $\delta V(R)$ results in the difference of the α-decay half-life between the bare nucleus and the external environment. In the following section, we will present the terms $\delta V(R)$ in different external environments.

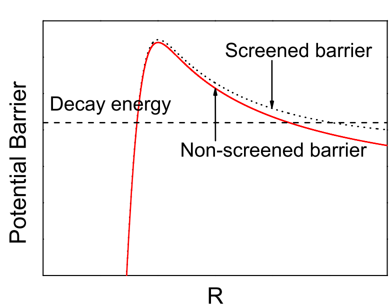

Fig. 1 Comparison of non-screened and screened potential barriers that α particle penetrates

3 α decay in external environments

3.1 α decay screened by electrons in neutral atoms

In neutral atoms, the variation in the Coulomb potential $\delta V(R)$ is analytically derived in Ref. [18],

$$\delta V(R) = \frac{2(2Z)^{2\gamma+1}}{\Gamma(2\gamma+2)\gamma} \frac{e^2}{a_0} \left(\frac{R}{a_0}\right)^{2\gamma}, \tag{5}$$

where a_0 is the Bohr radius and the factor $\gamma = \sqrt{1 - \beta^2 Z^2}$. $\beta = e^2/\hbar c$ is the fine-structure constant.

The variation δQ can be obtained from the difference of the electron binding energies of the three particles [19]:

$$\delta Q = B(Z, Z) - B(Z-2, Z-2) - B(2, 2), \tag{6}$$

where $B(Z, Z)$ denotes the electron binding energy of an atom with Z protons and Z electrons [19] and the value is given in Ref. [32]. For neutral atoms, the decay energies, Q, are given in the atomic mass evaluation [33]. So the decay energies for bare nuclei can be calculated by $Q_\mathrm{B} = Q - \delta Q$. Then the half-lives for bare nuclei and neutral atoms can be calculated.

3.2 α decay screened by electrons in a metal environment

In a metal environment, the variation in $\delta V(R)$ can be divided into two parts [18],

$$\delta V(R) = \delta V_1(R) + \delta V_2(R), \tag{7}$$

where $\delta V_1(R)$ is the same as in Eq. (5) from electrons of the mother nucleus and $\delta V_2(R)$ comes from the metal [18],

$$\delta V_2(R) = \frac{8e^2}{\pi} \int_0^{q_\mathrm{F}} \mathrm{d}q \int_0^{qR} \frac{\mathrm{d}y}{y^2} \int_0^{y} \mathrm{d}x \left[F^2(x) - \frac{x^2 q_\mathrm{F}^2}{3q^2}\right], \tag{8}$$

where $F(x)$ is the radial function [18] and the Fermi vector, q_F, is determined by the average electron density, n_0 [18],

$$q_\mathrm{F} = (3\pi^2 n_0)^{1/3}. \tag{9}$$

Here we take the metal copper (Cu) as an example with $n_0 = 8.48 \times 10^{22} \mathrm{cm}^{-3}$.

3.3 α decay screened by electrons in an extremely strong magnetic-field environment

In an extremely strong magnetic-field environment, one usually introduces the function $\phi(x)$ to obtain the screened Coulomb potential $V_\mathrm{C}(R) = \frac{Z_1 Z_2 e^2}{R}\phi(x)$ [26–29]. The function $\phi(x)$ fulfills the equation $\frac{\mathrm{d}^2\phi(x)}{\mathrm{d}x^2} = (x\phi)^{1/2}$ [25–30] with two boundary conditions: $\phi(0) = 1$ and $\phi'(0) = -0.938966$, where $x = R/R_\mathrm{s}$ is the screening factor with $R_\mathrm{s} =$

$1.041863Z^{1/5}b^{-2/5}a_0$ [27–30]. $b = B/B_0$ is a dimensionless strength [26], where B is the strength in the environment and $B_0 = m_e^2 e^3 c/\hbar^3 = 2.3505 \times 10^9$ G is the typical value in neutron stars [26]. The first few terms can be found by applying Baker's small-x expansion [34]

$$\phi(x) = 1 + Sx + \frac{4}{15}x^{2.5} + \frac{2}{35}Sx^{3.5} - \frac{1}{126}S^2x^{4.5}, \qquad (10)$$

where $S = \phi'(0) = -0.938966$ [27, 29]. Then the two parts of $\Delta V(R)$ can be expressed as follows

$$\delta V = \frac{Z_1 Z_2 e^2}{R} Sx, \qquad (11)$$

$$\delta V(R) = \frac{Z_1 Z_2 e^2}{R}\left[\frac{4}{15}x^{2.5} + \frac{2}{35}Sx^{3.5} - \frac{1}{126}S^2x^{4.5}\right]. \qquad (12)$$

4 Numerical results and discussion

By applying the DDCM, we perform systematic calculations of the electron-screened α-decay half-lives of nuclei with the proton number Z = 52–105 in different external environments. Here we only consider the favored α transitions to avoid the uncertainties coming from the nonzero angular momentum. The difference between the screened α-decay half-life, T_{sc}, and non-screened one, T_{nsc}, is defined by Δ_{sc},

$$\Delta_{sc} = \frac{T_{sc} - T_{nsc}}{T_{nsc}}, \qquad (13)$$

which includes Δ_{Atom}, Δ_{Metal}, and Δ_{Mag}, corresponding to neutral atoms, a metal, and a magnetic-field environment.

In Fig. 2, the variation Δ_{sc} is given in Fig. 2a for neutral atoms, Fig. 2b for a metal environment, and Fig. 2c–f all for a magnetic environment, but with different strengths: $b = 10^3$ (c), $b = 10^4$ (d), $b = 10^5$ (e), and $b = 10^6$ (f). It can be seen that Δ_{sc} values are all positive. So the α-decay half-lives are all increased by the electrons in external environments because of slightly higher potential barrier as shown in Fig. 1. Then the α-particle penetration probability is relatively smaller compared to bare nuclei, leading to longer α-decay half-life. For neutral atoms and a metal environment in Fig. 2a, b, the screened α-decay half-life is varied moderately, but in a magnetic-field environment the variation can be very large, and depends closely on the strength b.

Besides, in each chart of Fig. 2 there are several significantly larger Δ_{sc} along an isotopic chain. We find that these values are closely related to the small decay energies, Q. In Fig. 3, we plot their correlation for a typical isotopic chain, Lu. It is clearly seen that the variation in Δ_{sc} decreases with Q and the smallest decay energy of ^{159}Lu is corresponding to the biggest Δ_{sc}. This is because the electron screening effects are approximately the same for all Lu isotopes. Thus Δ_{sc} mainly depends on Q. The variation Δ_{sc} is also related to the proton number. To measure electron screening effects in experiments, α-decay candidates with relatively small decay energies and proper decay half-lives are suggested.

In a magnetic-field environment, the variation in Δ_{sc} can be very large and increases with the strength, b. Thus this environment could have a significant effect on α decays. To show the details, we plot the decay half-life ratio $f_M = T_{Mag}/T_{nsc}$ for ^{235}U in Fig. 4. As shown in Fig. 4, if only $\Delta V(R)$ is considered, the ratio, f_M, decreases sharply with b. Oppositely, the ratio, f_M, increases sharply with b if only δQ is considered. However, when both are included, the ratio, f_M, still increases with b, but the increase is much slower. Thus the variations in $V(R)$ and Q compete with each other and both are important factors for the electron screening effect.

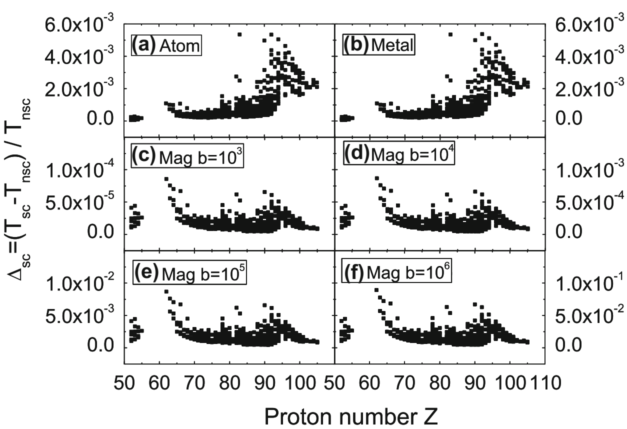

Fig. 2 Relative variation $\Delta_{sc} = (T_{sc} - T_{nsc})/T_{nsc}$ in neutral atoms (**a**), in metal (**b**), and in a magnetic-field environment with $b = 10^3$ (**c**), $b = 10^4$ (**d**), $b = 10^5$ (**e**), and $b = 10^6$ (**f**)

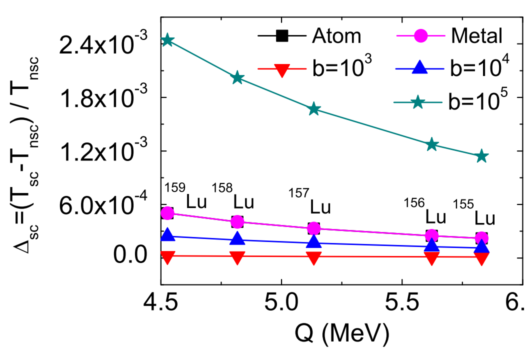

Fig. 3 Relation between Δ_{sc} and the decay energy, Q, for a typical isotopic chain of $Z = 71$

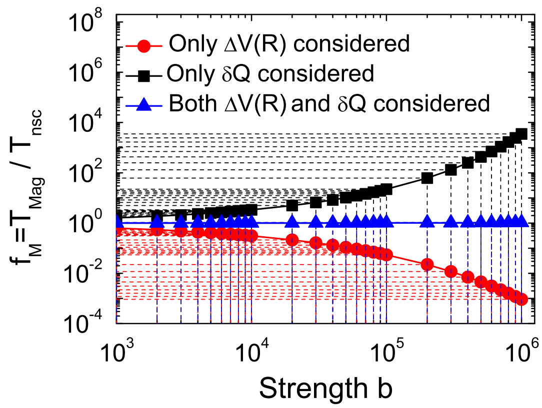

Fig. 4 Screened and non-screened half-lives ratio $f_{\mathrm{M}} = T_{\mathrm{Mag}}/T_{\mathrm{nsc}}$ for ^{235}U with different magnetic-field strengths, b

5 Summary

With the DDCM, the electron-screened α-decay half-life has been systematically calculated in external environments, including neutral atoms, a metal, and an extremely strong magnetic-field environment. From the numerical results, it can be concluded that the electron screening effects on α decays in neutral atoms and in a metal environment are very moderate. But in magnetic-field environments, the effect depends closely on the field strength. Besides, both the variations in the interaction potential and the decay energy are important for the electron screening effect. Similarly to previous studies, the electron screening effect is also closely related to the decay energy and the proton number.

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