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Nuclear alternating parity bands and transition rates in a model of coherent quadrupole–octupole motion in neutron-rich barium isotopes

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Abstract Deformed even–even nuclei Barium isotopes with quadrupole–octupole deformations are investigated on the basis of a collective model. The model describes energy levels of the yrast band with alternating parity in the neutron-rich ^{140,142,144,146,148}Ba. The structure of the alternating parity bands is examined by odd–even ($\Delta I = 1$) staggering diagrams. An analytical method of the collective model is proposed for the calculation of $E2$ transition probabilities in alternating spectra of the nuclei ^{140,142,144,146}Ba.

Keywords Collective Hamiltonian · Quadrupole–octupole deformations · Staggering effect · Electric transition probability

1 Introduction

In atomic nuclei, the simultaneous manifestation of quadrupole and octupole degrees of freedom is correlated with typical spectroscopic characteristics of nuclear collective motion. The quadrupole mode can be applied in all regions of deducing vibrational, rotational, and transitional structures of the spectra. The display of octupole degrees of freedom is superposed in some regions. This leads to complicated shape properties and parity effects in the spectrum of the system [1].

It is commonly thought that the core issue of quadrupole–octupole collectivity is to resolve the breaking of reflection symmetry [2], this is chiefly because of the difficulty in determining the total inertia tensor of the system. Based on this situation, if simplifying assumptions that the axial symmetry is still preserved and the octupole deformations are fixed suitably with the principal axes of the quadrupole shape, both degrees of freedom are separated adiabatically. In situations like this, the collective motion can be related to the reflection asymmetric shape in reference to an octupole variable in a double-well potential [3], and the tunneling through the potential barrier can reasonably explain the parity shift effect observed in nuclear alternating parity bands. The above concept has been generalized for the case of simultaneously contributing quadrupole and octupole modes. The double-well potential was defined in accordance with a variable bringing contribution to not the absolute values of each deformation variables, but the different degrees of freedom [4]. In this way, the explicit form of the original potential according to the quadrupole and octupole deformation variables was not given. Another important issue is, if and to what extent, one may take into account a tunneling effect existed in the space of the octupole variable, β_3, after the quadrupole coordinate, β_2, is made to vary. In order to clarify the above question, it has proposed a collective model [5] for the quadrupole–octupole vibration and rotation motion of even–even nucei.

The purpose of the present work is to apply this theoretical model to explain the properties of quadrupole–octupole deformations [6–8] in even–even nuclei: barium isotopes [9–12]. It can also obtain basic characteristics of energy levels, parity shift, and electric transition properties

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in nuclei (^{140,142,144,146,148}Ba) [13] with collective bands built on coherent quadrupole–octupole vibrations.

2 Theoretical descriptions

Now we begin with a brief review of the theoretical framework. The quadrupole–octupole Hamiltonian [14] for the collective motion is given by

$$H_{\mathrm{qo}} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I), \tag{1}$$

and the potential is

$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2\beta_2^2 + \frac{1}{2}C_3\beta_3^2 + \frac{X(I)}{d_2\beta_2^2 + d_3\beta_3^2}, \tag{2}$$

with $X(I) = I(I+1)/2$. Here I is the collective angular momentum, β_2 and β_3 are the axial deformation variables, B_2 and B_3 are the mass parameters, C_2 and C_3 are the stiffness parameters, and d_2 and d_3 are the moment of inertia parameters. The last term in Eq. (2) is a coupling between quadrupole and octupole degrees of freedom.

If a condition for the simultaneous presence of nonzero coordinates of the potential minimum is applied, the inertial and stiffness parameters are correlated as $d_2/C_2 = d_3/C_3$. On this occasion, the potential bottom is an ellipse that surrounds the internal potential core. If the prolate quadrupole deformation $\beta_2 > 0$ is taken into account, the motion in the octupole coordinate between positive and negative β_3 values along the ellipse surrounding the potential core. And it also uses polar variables $\beta_2 = \rho\cos(\theta)/\sqrt{d_2/d}$, and $\beta_3 = \rho\sin(\theta)/\sqrt{d_3/d}$, with $d = (d_2 + d_3)/2$. According to the above condition, the potential energy hinges on the deformation variable, ρ, and on the angular momentum, I, and not on the angular variable, θ. The potential is expressed as

$$U_I(\rho) = \frac{1}{2}C\rho^2 + \frac{X(I)}{d\rho^2}, \tag{3}$$

where C is defined as $1/C = d_2/(dC_2) = d_3/(dC_3)$. Then we assume that the quadrupole and octupole modes in the collective motion have the same oscillation frequencies. Then the mass and inertia parameters have the relation $1/B = d_2/(dB_2) = d_3/(dB_3)$. The model Hamiltonian and the quadrupole–octupole oscillation wave function are obtianed. and the wave function can be taken in a separable form $\phi(\rho,\theta) = \psi(\rho)\varphi(\theta)$. The Schrödinger equation is separated into two equations for the variables ρ and θ

$$\frac{\partial^2}{\partial\rho^2}\psi(\rho) + \frac{1}{\rho}\frac{\partial}{\partial\rho}\psi(\rho) + \frac{2B}{\hbar^2}\left[E - U_I(\rho) - \frac{\hbar^2}{2B}\frac{k^2}{\rho^2}\right]\psi(\rho) = 0; \tag{4}$$

$$\frac{\partial^2}{\partial\theta^2}\varphi(\theta) + k^2\varphi(\theta) = 0, \tag{5}$$

where k is the separation quantum number. Equation (4) with the potential (3) is similar to the equation for the Davidson potential [15, 16], which is analytically solvable. Equation (4) is solved analytically and gets the explicit expression for the energy spectrum [17]

$$E_{n,k}(I) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + bX(I)}\right], \tag{6}$$

where $\omega = \sqrt{C/B}$, $n = 0, 1, 2, \ldots$, and $b = 2B/(\hbar^2 d)$. The eigenfunctions $\psi(\rho)$ are obtained in terms of the Laguerre polynomials

$$\psi_n^I(\rho) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+2v+1)}}(\rho^2 a)^v e^{-a\rho^2/2} L_n^{2v}(a\rho^2), \tag{7}$$

where $a = \sqrt{BC}/\hbar$ and $v = (1/2)\sqrt{k^2 + bX(I)}$. Under the boundary condition $\varphi(\pi/2) = \varphi(-\pi/2) = 0$, the Eq. (5) has two different solutions with positive and negative parities, $\pi_\varphi = (+)$ and $\pi_\varphi = (-)$, respectively, $\varphi^+(\theta) = \sqrt{2/\pi}\cos(k\theta), k = \pm1, \pm3, \pm5\cdots$; $\varphi^-(\theta) = \sqrt{2/\pi}\sin(k\theta), k = \pm2, \pm4, \pm6\cdots$. The lowest states of the system in the variable θ is considered, one has $k = k_+ = 1$ for φ^+ and $k = k_- = 2$ for φ^-. With the parity dependent number, k, the Eq. (6) determines the structure of an alternating parity spectrum. The energy levels $E_{n,k}(I)$, with $n = 0$, are equal to the yrast alternating parity sequence. With $n \neq 0$, the levels are equal to higher energy bands, in which the rotation states are built on quadrupole–otcupole vibrations of the system.

The odd–even staggering [18–20] is referred to as a zigzagging behavior of the nuclear inertial parameter between the odd and even angular momentum states of a rotation band. It provides some information about the fine structure of the nuclear collective spectrum in different regions of the nuclear chart. The odd–even staggering patterns can be determined as

$$\begin{aligned} Stg(I) &= E(I+3) - 4E(I+2) + 9E(I+1) \\ &\quad - 10E(I) + 5E(I-1) - E(I-2). \end{aligned} \tag{8}$$

The model formalism which allows the calculation of $E2$ transition probabilities for the energy spectra in the system with coherent quadrupole–octupole motion is widely applied to even–even nuclei. Then we give the formulas for calculation of $E2$ transition probabilities [5] between levels with $|n_i I_i\rangle$ and $|n_f I_f\rangle$

$$B(E2, I_i \to I_f) = b_2\langle I_i 020|I_f 0\rangle^2 S^2(E2, I_i \to I_f), \tag{9}$$

where b_2 is scaling constant related to effective charges,

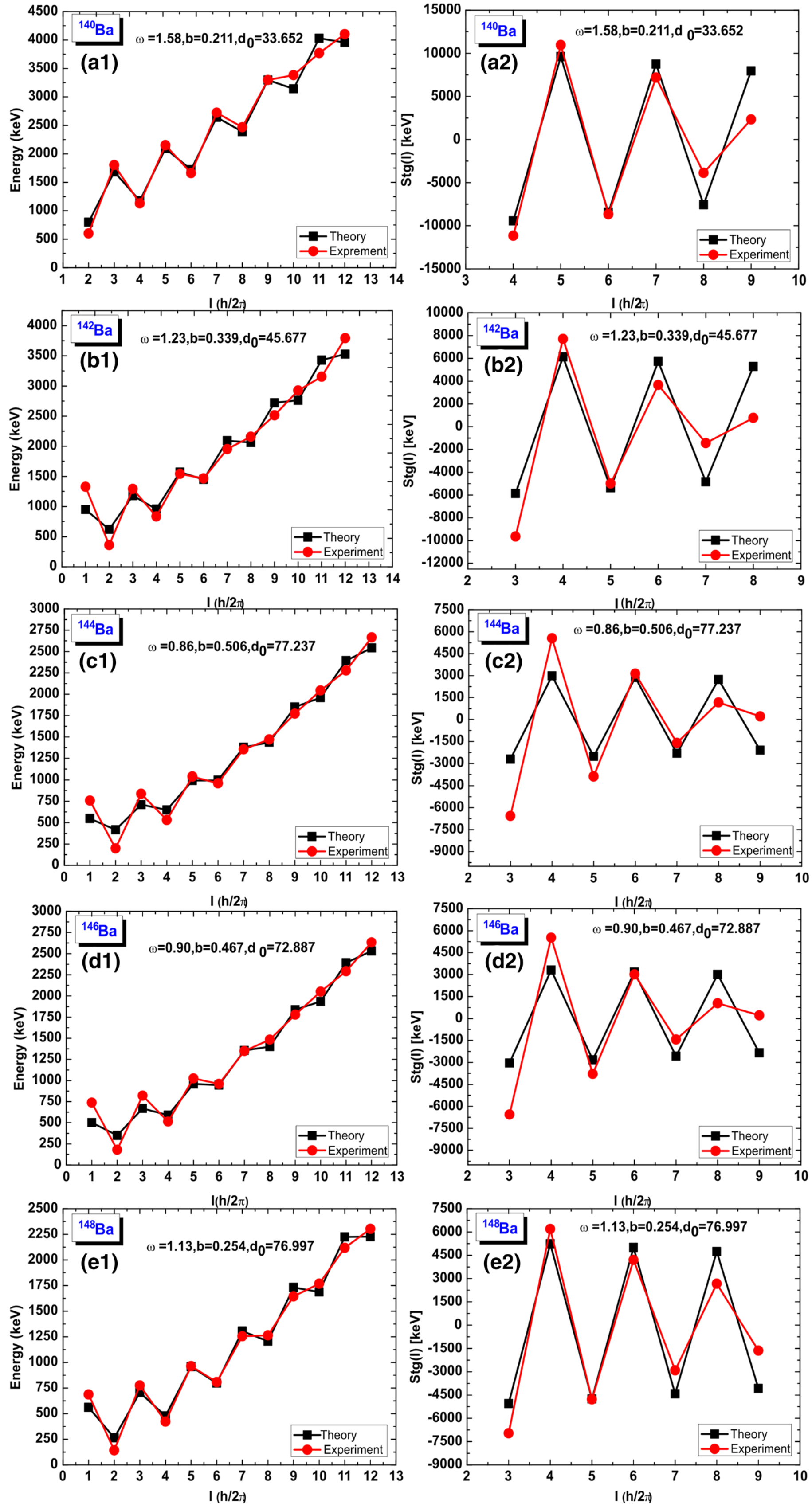

Fig. 1 Theoretical and experimental energy levels (*left*) and staggering patterns (*right*) for the alternating parity bands in ^{140,142,144,146,148}Ba. Experimental data are taken from Refs. [9, 22], the theoretical results are obtained by Eqs. (6) and (8), respectively

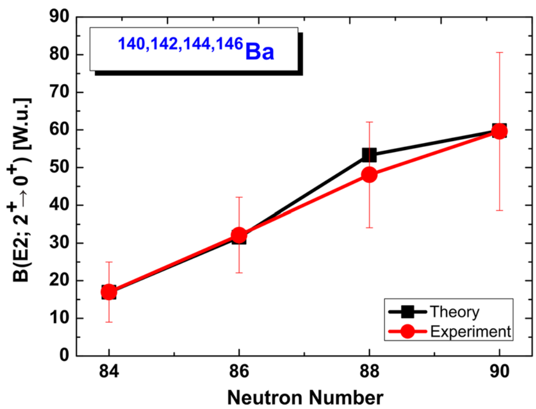

Fig. 2 The values of theoretical and experimental transition probabilities ($B(E2; I+2 \to I)$, here $I=0$) in the alternating parity spectrum of Barium isotopes ^{140,142,144,146}Ba (the values of the parameter a are 0.327,0.324,0.272,0.239). Experimental data from Ref. [10]. The theoretical results are obtained by Eqs. (9) and (11)

$$S(E2, I_i \to I_f) = \int_0^{+\infty} d\rho \psi_{n_f}^{I_f}(\rho)\rho^2\psi_{n_i}^{I_i}(\rho). \tag{10}$$

3 Results and discussion

The descriptions of the alternating parity spectra in the nuclei ^{140,142,144,146,148}Ba are obtained by taking the theoretical energy levels $\widetilde{E}_{0,k}(I) = E_{0,k}(I) - E_{0,k}(0)$ from Eq. (6). $X(I) = \frac{1}{2}[d_0 + I(I+1)]$, where the parameter d_0 indicates the characteristic of the potential $(U(\beta_2, \beta_3, I))$ in the ground state. According to the respective experimental data, the parameters ω, b, and d_0 have been adjusted to the energy levels by means of a least squares minimization procedure. In the left column of Fig. 1, the obtained numerical results for the energy levels of ^{140,142,144,146,148}Ba are compared with experimental data.

The staggering patterns illustrate that the even and odd angular momentum sequences approach each other toward higher angular momenta. However, the decrement of the staggering amplitude is not enough to provide an octupole band structure at angular momenta. For the nuclei ^{140,142,144,146,148}Ba, we observed this staggering effect. The respective experimental and theoretical staggering patterns are compared in the right column. In Fig. 1, it is illustrated that the experimental patterns demonstrate the predicted behaviors of alternating parity levels with increasing angular momentum in the nuclei ^{140,142,144,146,148}Ba.

Considering in the case of transitions between states of the yrast alternating parity band, $|0I_i\rangle$ and $|0I_f\rangle$ (with $n_i = n_f = 0$), the integrals in Eq. (10) have an analytic expression [21]

$$S(E2, I_i \to I_f) = \frac{1}{a^{\frac{3}{2}}} \frac{\Gamma(v_i + v_f + \frac{3}{2})}{\sqrt{\Gamma(2v_i+1)\Gamma(2v_f+1)}}, \tag{11}$$

where $a = \sqrt{BC}/\hbar, v_i = (1/2)\sqrt{k_i^2 + bX(I_i)}$, and $v_f = (1/2)\sqrt{k_f^2 + bX(I_f)}$. This formalism can be applied for an analysis of the electric transition rates in spectra where the collective quadrupole–octupole dynamics carry the characteristics outlined in the above cases. We calculate the $E2$ reduced transition probabilities in the spectra of ^{140,142,144,146}Ba, where the available experimental data allow to get information about the angular momentum dependence of these quantities. Due to the scarce experimental data, the $B(E2)$ value of ^{148}Ba was not calculated. The theoretical transition values have been determined after fitting the parameter a in Eq. (11). The scaling (effective charge) parameter, b_2, in Eq. (9) has been set equal to 1. The specific instructions of parameter, a, and constant, b_2, are given in the fifth part of the Ref. [5]. In Fig. 2, the vertical coordinate is the value of the theoretical and experimental transition probabilities ($B(E2; 2^+ \to 0^+)$) in the alternating parity spectra of barium isotopes. On the horizontal coordinate, the different number of neutrons signify the different barium isotopes (^{140,142,144,146}Ba). The calculated results are compared with experimental data in Fig. 2. There are also some discrepancies between theory and experiment in the $E2$ transition values in ^{142,144,146}Ba.

4 Summary

In summary, we have shown accurately the energy levels and the staggering patterns in the nuclei ^{140,142,144,146,148}Ba, as displayed in Fig. 1. From the figure we can see that the staggering patterns show the even and odd angular momentum sequences approach each other toward higher angular momenta. It signifies a trend for the forming of an octupole band in even–even nuclei: barium isotopes. The results for $E2$ transition probabilities in nuclei ^{140,142,144,146}Ba (Fig. 2) suggest further analysis of additional experimental data and tests of the formalism. The current analysis shows that, in this case, the coherent contribution of quadrupole and octupole oscillations can occur in the collective motion of nuclei.

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