Gamma ray absorption of cylindrical fissile material with dual shields

WU Chen-Yan[1], TIAN Dong-Feng[2,3], CHENG Yi-Ying[4], HUANG Yong-Yi[2], LU Fu-Quan[2]*,YANG Fu-Jia[2]

([1] *Institute of Mathematics, Fudan University, Shanghai 200433;*

[2] *Institute of Modern Physics, Fudan University, Shanghai 200433;*

[3] *Institute of Applied Physics and Computational Mathematics, Beijing 100088;*

[4] *Department of Physics, University of Rhode Island, 02881-0817, USA)*

Abstract This work analyzed the gamma ray attenuation effect from the self-absorption and shield attenuation perspectively. An exact mathematical equation was given for the geometric factor of the cylindrical fissile material with dual shields. In addition, several approximation approaches suitable for real situation were discussed, especially in the radial and axial directions of the cylinders, since the *G*-factors have simple forms. Then the space distribution patterns of the *G*-factor were analyzed based on numerical result and effective ways to solve the geometric information of the cylindrical fissile material, the radii and the heights, were deduced. This method was checked and verified by numerical calculation. Because of the efficiency of the method, it is ideal for application in real situations, such as nuclear safeguards, which demands speed of detection and accuracy of geometric analysis.

Keywords Fissile material, Isotopic composition, Gamma ray self-absorption, Dual shields

CLC numbers TL352, TL375.6

1 Introduction

With the surging demand of nuclear energy from both military and civil industries, large quantities of nuclear fissile fuel, such as U and Pu, have been produced and stockpiled. ^{235}U and ^{239}Pu, with the potential risk of proliferation, have been the focal point of the international society. The International Atomic Energy Agency (IAEA) has been engaged in the nuclear supervisory technology for many years. Since increased nuclear safeguards need more measurements, rapid measurement methods that do not alter the state of items being investigated are very desirable. Thus it is of practical significance to develop rapid nondestructive detection techniques for identifying, analyzing and even tagging.

Different radioactive nuclides are identifiable by their distinctive characteristic gamma spectra. Using Ge detector to examine the characteristic gamma spectra of the nuclear apparatus, we can determine what the constituent nuclide is. Another objective in the nuclear supervisory technology is to gather geometric information of the nuclear apparatus through measurements of the varying intensity of gamma rays. Since the attenuation coefficients, together with the thickness of attenuation layers, dictate the pattern of gamma ray intensity in the space, this approach is powerful in dissecting the geometric information, i.e, size, shape, shield thickness, etc.

In 1989, a joint nuclear supervision group of U.S. and the former Soviet Union examined the SSN-12 nuclear warhead on board the Slava cruiser. Their measurements and analysis were published in magazine *Science*.[1] They concluded that through measurements of gamma spectra, warhead composition, whether it was a U warhead or Pu one, could be known. However, due to limitation of the number of measurements and time, they failed to discern the information concerning its design (geometric information). In fact, their calculation assumed the shape was

* E-mail address: fqlu@fudan.edu.cn

Received date: 2005-06-24

sphere in either bulk form or surface form.

Based on the fact that nuclear material itself attenuates the gamma rays it emits, Tian et al.[2,3] showed how the geometric information of the nuclear apparatus could be derived. They conducted exact analysis for the sphere structure and approximately applied their method to the detection in axial direction of the cylindrical nuclear apparatus.

In 2001, Lu et al.[4] studied the geometric problem of the cylindrical nuclear apparatus, taking measurements along axial direction and its perpendicular direction. They gave the analytical expression of the self-absorption coefficient in those directions.

Huang et al.[5] solved the geometric problem, taking measurements in arbitrary direction of the cylindrical nuclear apparatus and gave the exact analytical expression for the self-absorption coefficient. Also in their article, they proposed one way to determine the size of the nuclear apparatus by means of the self-absorption coefficient.

However, due to safety reason, shields are commonly used in nuclear apparatus. A further attenuation of the gamma ray's intensity will influence the gamma ray intensity pattern significantly. Thus the previous analysis when applied to shielded nuclear apparatus needs modification. In this article, we will investigate how to derive the geometric information of the shielded nuclear apparatus from detection data concerning the distribution of gamma ray intensity in the space. Our extension better suits the reality and will have positive effect on the safeguard techniques against nuclear proliferation.

2 Expression of self-absorption correction coefficient

2.1 Configuration

Let the gamma rays go through three layers of absorptive materials. The innermost cylinder is radioactive, while the rest of the cylinders are absorptive materials only. We can generalize to the situation of an arbitrary number of cylinders, but in real situations the core is usually U or Pu first surrounded by Pb shield and then by steel or iron. Fig.1 shows the configuration. An infinitesimal volume is chosen to demonstrate how gamma rays emit and traverse through the cylinders.

To simplify the problem, we propose the following assumptions:

1) The detector is relatively far away from the nuclear apparatus. Thus every part of the nuclear apparatus can be regarded as located at the same distance to the detector. In other words, we consider the nuclear apparatus as a point when attenuation due to its geometric feature is not involved.

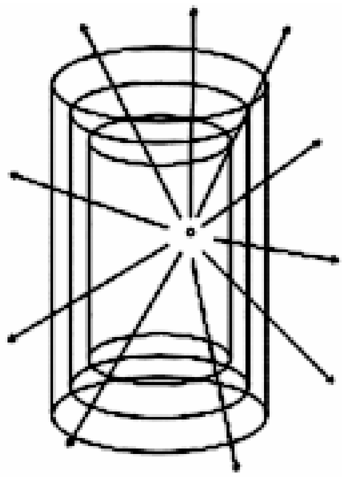

Fig.1 Configuration of nuclear apparatus.

2) The nuclear material and the shields are homogeneous. Thus the gamma flux is in proportion to the volume of the nuclear material.

Then the geometric factor is as follows:

$$G = \frac{1}{V}\iiint_{\Omega} \mathrm{e}^{-\sum_i \mu_i t_i}\, \mathrm{d}\Omega \qquad (1)$$

where Ω is the volume of the innermost cylinder (the nuclear material), μ_i the attenuation coefficient of the i-th cylinder (the innermost being the first), t_i the i-th absorption distance.[6]

2.2 Absorption distances (Fig.2)

Given a cylinder and two points in the space, one inside and the other outside, we need to determine how much of the line segment connecting the two points is contained in the cylinder. First we set up the Descartes coordinates and introduce some variables.

The centre of the cylinder is located at the origin and its generatrix is parallel to the z axis. Some variables and constants are listed as follows:

P_0: the point outside the cylinder, with coordinates (x_0, y_0, z_0), where the detector is located;

P: a point inside the cylinder, with coordinates (x, y, z);

R: the radius of the cylinder;

H: the height of the cylinder.

Note that the detector is placed far away from the nuclear apparatus. Thus $|PP_0|$ is much greater than the size of the nuclear apparatus.

Using equations to represent the side and the top and bottom surface of the cylinder, we can calculate the intersection point of the aforementioned line segment and the cylinder.

The side of the cylinder is represented as

$$\begin{cases} \xi^2 + \eta^2 = R^2 \\ \dfrac{-H}{2} \le \zeta \le \dfrac{H}{2} \end{cases} \tag{2}$$

whereas the top and bottom of the cylinder as

$$\begin{cases} \zeta = \dfrac{+H}{2} \\ \xi^2 + \eta^2 \le R^2 \end{cases} \tag{3}$$

The line segment connecting P and P_0 with s being the parameter is

$$\begin{cases} \xi = x + s(x_0 - x) \\ \eta = y + s(y_0 - y), s \in [0,1) \\ \zeta = z + s(z_0 - z) \end{cases} \tag{4}$$

Solving the intersection point of the line segment and the side face, we get

$$\begin{aligned} s = {} & \frac{1}{2[(x-x_0)^2 + (y-y_0)^2]} \times \\ & \{-[2x(x_0 - x) + 2y(y_0 - y)] + \\ & [(2x(x_0 - x) + 2y(y_0 - y))^2 - \\ & 4((x-x_0)^2 + (y-y_0)^2)(x^2 + y^2 - R^2)]^{1/2}\} \end{aligned} \tag{5}$$

the negative root is already discarded. We denote this s as s_1.

Solving the intersection point of the line segment and the top and bottom face, we get

$$s = \frac{\dfrac{\pm H}{2} - z}{z_0 - z} \tag{6}$$

We preserve the s in the range of [0, 1) and denote it as s_2.

Then $s=\min\{s_1, s_2\}$ is the parameter representing the intersection point. Substituting s in Eq.(4) with $s=\min\{s_1, s_2\}$, we can determine the coordinates of the intersection point. The absorption distance, t, is the Descartes distance between two known points.

$$t = s\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right] \tag{7}$$

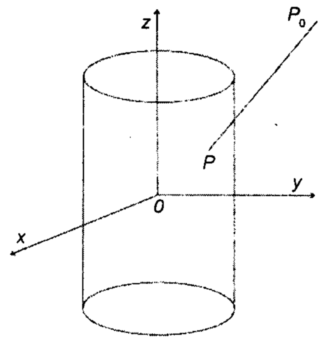

Fig.2 Absorption distance.

With the preceding formulas, we can calculate the absorption distance of the innermost cylinder. The absorption distances of the shields are also soluble, since the intersection points are known by applying the foregoing argument. Now we denote t_i as the absorption distance of the i-th cylinder, the innermost one being the first.

3 Analysis of *G* factor

3.1 Relative size of cylinders

Transforming Eq.(1) by switching to cylindrical coordinates, we get

$$G = \frac{1}{V}\int_{\frac{-H}{2}}^{\frac{H}{2}} \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta \int_0^R r\mathrm{e}^{-\sum_i \mu_i t_i} \mathrm{d}r \tag{8}$$

Fixing the radius of the cylinder, R and its height, H, we may calculate the G factor along the arc. If the cylinders are scaled, we will investigate the relationship between the new G factor and the old one.

Writing out the independent variables explicitly, we have

$$\begin{aligned} G(P_0, R, H) = {} & \frac{1}{\pi R^2 H} \times \\ & \int_{\frac{-H}{2}}^{\frac{H}{2}} \mathrm{d}z \int_0^{2\pi} \mathrm{d}\theta \int_0^R r\mathrm{e}^{-\sum_i \mu_i t_i (P_0, r, \theta, z)} \mathrm{d}r \end{aligned} \tag{9}$$

and

$$G(P_0,kR,kH)=\frac{1}{\pi(kR)^2(kH)}\times \int_{-\frac{H}{2}}^{\frac{H}{2}}\mathrm{d}z\int_0^{2\pi}\mathrm{d}\theta\int_0^{kR} r\mathrm{e}^{-\sum_i \mu_i t_i(P_0,r,\theta,z)}\mathrm{d}r \quad (10)$$

where k is the scale factor.

Set $\overline{r}=\frac{r}{k}$ and $\overline{z}=\frac{z}{k}$ and we get

$$G(P_0,kR,kH)=\frac{1}{\pi R^2 H)}\times \int_{-\frac{H}{2}}^{\frac{H}{2}}\mathrm{d}\overline{z}\int_0^{2\pi}\mathrm{d}\theta\int_0^{R} \overline{r}\mathrm{e}^{-\sum_i \mu_i t_i(P_0,k\overline{r},\theta,k\overline{z})}\mathrm{d}\overline{r} \quad (11)$$

If we substitute the corresponding absorption distance $t_i(P_0,k\overline{r},\theta,k\overline{z})$ with $kt_i(P_0,\overline{r},\theta,\overline{z})$, since the cylinder is scaled by a factor of k, we have

$$G(P_0,kR,kH)=\frac{1}{\pi R^2 H}\times \int_{-\frac{H}{2}}^{\frac{H}{2}}\mathrm{d}\overline{z}\int_0^{2\pi}\mathrm{d}\theta\int_0^{R} \overline{r}(\mathrm{e}^{-\sum_i \mu_i t_i(P_0,\overline{r},\theta,\overline{z})})^k\mathrm{d}\overline{r} \quad (12)$$

that is

$$G=\frac{1}{V}\iiint_{\Omega}\mathrm{e}^{-\sum_i k\mu_i t_i}\mathrm{d}\Omega \quad (13)$$

where Ω is the volume of the cylinder with R as its radius and H its height.

From Ref.[2], if there is no shielding material, we have $G\propto 1/\mu$. Thus we can regard the scaled cylinders as the original set except μ has changed to $k\mu$ and we can approximate the G factor as

$$G=\frac{1}{kV}\iiint_{\Omega}\mathrm{e}^{\mu_i t_i}\mathrm{d}\Omega \quad (14)$$

If we set a reference point and calculate the relative G factor, the space distribution of the relative G factor remains the same. Thus the problem is further simplified, because only the ratio of H and R is significant. We can fix R and see how the G factor pattern changes due to varying H . However, when shielding layers are added, no simple relationship is found yet. We will fix R when analyzing shielded nuclear material, but a changed R just means a modification of coefficients in the empirical formulas deduced in later sections.

3.2 Approximation in special condition

The G factor values along the y - and z –axis are the special cases we are to consider. Along the y -axis, all the gamma rays intersect the side of the cylinders. As d is much greater than the size of the nuclear apparatus, every cross section parallel to xy -plane has identical gamma ray attenuation pattern. Thus by Eq.(8), we have the approximation

$$G=\frac{H}{V}\int_0^{2\pi}\mathrm{d}\theta\int_0^{R} r\mathrm{e}^{-\sum_i \mu_i t_i}\mathrm{d}r = \frac{1}{\pi R^2}\int_0^{2\pi}\mathrm{d}\theta\int_0^{R} r\mathrm{e}^{-\sum_i \mu_i t_i}\mathrm{d}r \quad (15)$$

Calculating one slice of the nuclear apparatus, we can easily get the G factor value without laborious computation.

Along the z -axis, all the gamma rays intersect the top surface of the cylinders. Still because d is much greater than the size of the nuclear apparatus, we consider all the rays traversing the cylinders perpendicularly to the top surfaces. Thus we have

$$G=\frac{1}{V}\int_{-\frac{H}{2}}^{\frac{H}{2}}\mathrm{d}z\int_0^{2\pi}\mathrm{d}\theta\int_0^{R} r\mathrm{e}^{-\mu_i(\frac{H}{2}-z)}\mathrm{e}^{-\sum_{i>1}\mu_i T_i}\mathrm{d}r \quad (16)$$

where T_i is the top thickness of the i-th shield. The above equation can be written more explicitly,

$$G=\frac{1}{\mu_1 H}\mathrm{e}^{-\sum_{i>1}\mu_i T_i}(1-\mathrm{e}^{-\mu_1 H}) \quad (17)$$

The term $\mathrm{e}^{-\mu_1 H}$ is generally much smaller than 1 and thus can be ignored. We have a very concise expression for the G factor value along the z-axis,

$$G=\frac{1}{\mu_1 H}\mathrm{e}^{-\sum_{i>1}\mu_i T_i} \quad (18)$$

4 Numerical analysis

We consider the situation that the shielding materials are Pb and Fe. We calculate the G factor along an arc in the yz -plane where y and z are greater than zero, with its centre at the origin and radius d=150cm, which is a typical distance between the detector and the nuclear apparatus (Fig.3).

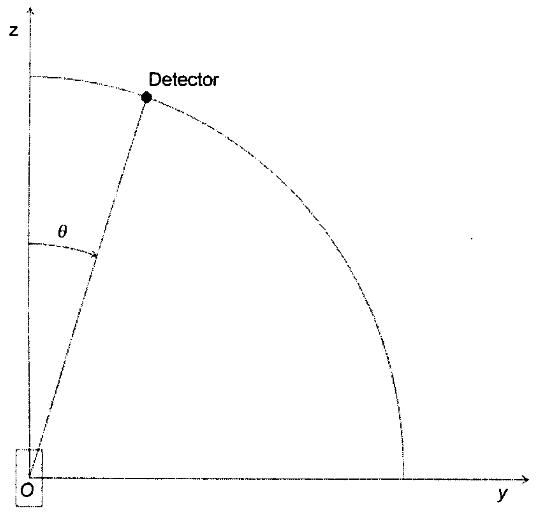

Fig.3 Detector along an arc in the first quadrant.

To simplify the problem, we suppose the shield is homogeneous in thickness, which in reality is generally the case. In the following discussion we assume that R=1cm. A further restriction is that the two shields are of the same thickness to allow a simpler solution. Vary H/R from 0.5 to 6 with a step of 0.25 and θ from 0 to $\pi/2$ with a step of $\pi/40$, where θ is the angle between z -axis and the line connecting the detector and the origin. The linear attenuation coefficient can be determined from data provided by Ref.[7,8].

The values we use in our calculation are:

μ_U=28.9135 g/cm^2,

μ_{Pb}=13.8393 g/cm^2,

μ_{Fe}=1.24 g/cm^2.

Results are plotted in the following figures (Figs.4-8).

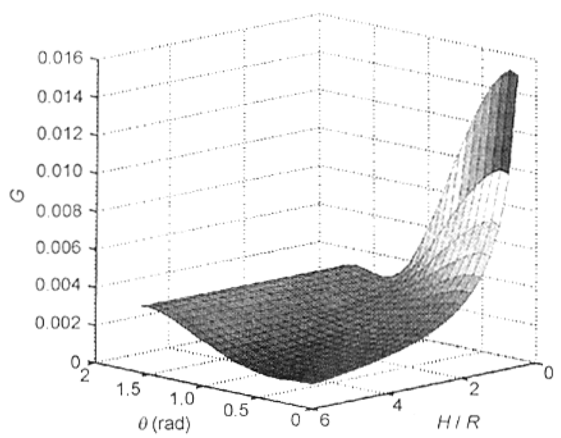

Fig.4 Pb shield (0.1R) and Fe shield (0.1R).

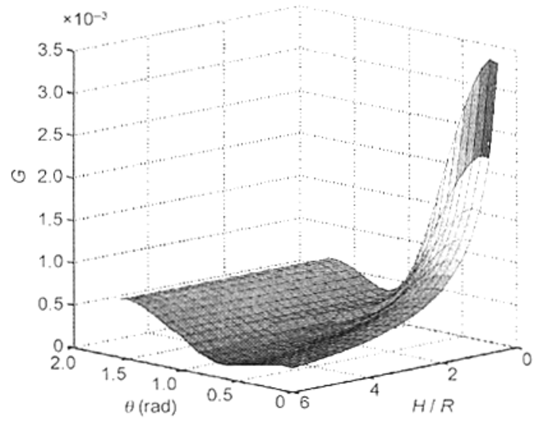

Fig.5 Pb shield (0.2R) and Fe shield (0.2R).

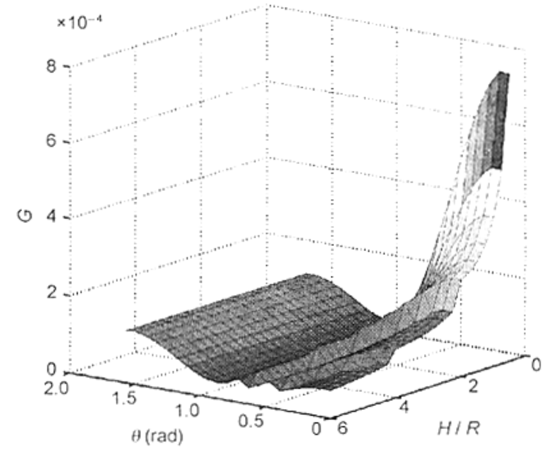

Fig.6 Pb shield (0.3R) and Fe shield (0.3R).

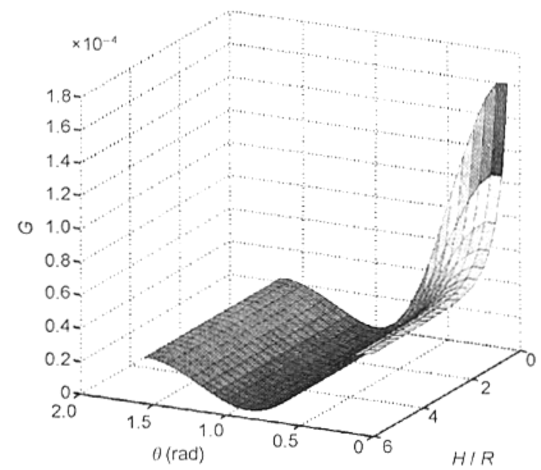

Fig.7 Pb shield (0.4R) and Fe shield (0.4R).

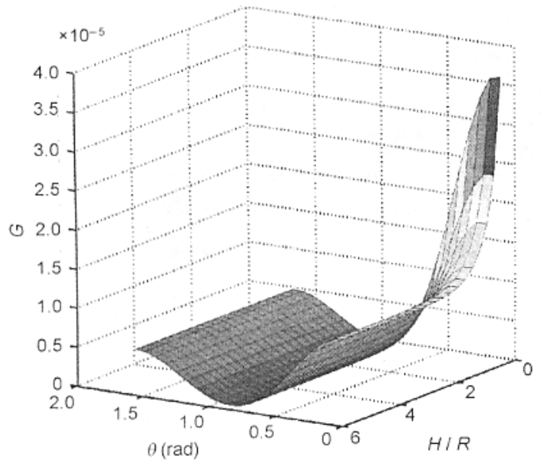

Fig.8 Pb shield (0.5R) and Fe shield (0.5R).

4.1 Examination in radial and axial directions

Now we further examine the special points, i.e. the radial and axial directions which in our coordinates are the y- and z-axes respectively. G factor values are accordingly denoted as G_y and G_z.

We find that G_y is almost a constant with the same shield thickness, which verifies the approximation in Eq.(15). This value is a very good indicator of thickness of the shield. Varying the thickness of the shield, we plot the G factor values along the y-axis (Fig.9).

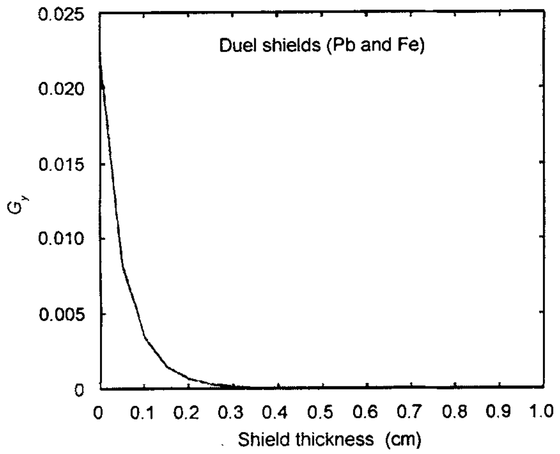

Fig.9 G factor along the y-axis.

Applying regression, we have

$$G_y = e^{-15.7675T-4.1231}$$

where T is the thickness of both shields. The coefficient of T is approximately the sum of linear attenuation coefficient (15.0832), yet a small deviation is present due to the complex absorption distances. In the z-axis, $G_z = \dfrac{1}{\mu_1 H} e^{-(\mu_2+\mu_3)T}$.

Thus we can deduce the thickness of the shields by a single measurement in the y-axis direction and from measurement in the z-axis direction we can deduce the height of the nuclear material. The result is shown below.

The errors indicate that the result is satisfactory (Fig.10), but when the restriction is relaxed, as different combination of shields may produce identical result, the situation becomes too complex to allow a simple solution.

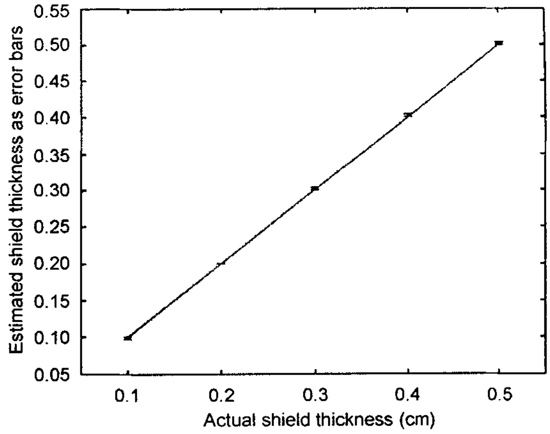

Fig.10 Shield thickness estimation.

5 Conclusion

Based on the mathematical expression for G factor, we propose a fast method to determine the geometric information of the nuclear material with shields. Specific geometric information concerning nuclear apparatus can be deduced accurately. The radius of the nuclear material can be determine from the relative intensity of the gamma rays, while the height and thickness of the dual shields can be calculated by two measurements from the radial and axial direction. The numerical analysis results show our method is satisfactory.

Acknowledgment

This research project is supported by the Institute of Applied Physics and Computational Mathematics of China and the Foundation of Wang-Dao Scholar Program of Fudan University.

References

1 Fetter S, Cochran T B, Grodzins L, *et al*. Science, 1990, **248**: 824-834

2 Tian Dongfeng, Xie Dong, Huo Yukun, *et al*. Nucl Instr Meth Phys Res, 2000, **A449**: 500-504

3 Lu Xiangdong,Tian Dongfeng, Xie Dong. Nuclear Techniques (in Chinese), 2004, **27**(11): 814-817

4 Lu Fuquan, Tian Dongfeng, Wu Shimin, *et al*. Journal of Fudan University (Natural Science) (in Chinese), 2001, **40**(3): 245-248

5 Huang Yongyi, Cheng Yiying, Tian Dongfeng, *et al*. Nuclear Science and Techniques, 2005, **16**(1): 17-24

6 Fudan University, Tsinghua University, Peking University, Experiments methods of nuclear physics (2nd ed) (in Chinese), Beijing: Atomic Energy Publishing House, 1985

7 X-ray mass attenuation coefficients, <http://physics.nist.gov/PhysRefData/XrayMass-Coef/tab3.html> Accessed Mar. 30th, 2005. Values of the mass attenuation coefficient and the mass energy-absorption coefficient as a function of photon energy, for elemental media

8 X-ray mass attenuation coefficients, <http://physics.nist.gov/PhysRefData/XrayMass-Coef/tab1.html> Accessed Mar. 30th, 2005. Material constants assumed in the present evaluations for elemental media.