Evaluation of external quality factor of the superconducting cavity using extrapolation method

MA Zhenyu[1,2,*] MA Guangming[1,2] HOU Hongtao[1] YU Haibo[1,2] ZHANG Rui[1,2] LIU Jianfei[1]

[1] *Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China*

[2] *Graduate University of Chinese Academy of Sciences, Beijing 100049, China*

Abstract The estimation of the external quality factor is important for designing coupling devices for the cavities. A new representation of the external quality factor calculations for single-cell cavity coupled to a coaxial transmission line is derived based on analytic analysis and numeric analysis with the help of 3D electromagnetic code, and verified with experimental measurements at room temperature. In logarithmic scale the results for the external quality factor were quasi-linear over the limited range, and the simulated and measured data could be used and extrapolated to the superconducting case. For the unpolished 1.5 GHz 3rd harmonic superconducting cavity, the discrepancy between the evaluation value and measurement result is less than 25% within an acceptable deviation.

Key words External quality factor, Vertical testing, Superconducting cavity, Coaxial input coupler

1 Introduction

A coaxial type of input coupler has been designed for the single-cell superconducting cavity on a vertical test stand at Shanghai Synchrotron Radiation Facility (SSRF). The RF power, however, will be transferred efficiently from the coupler to the cavity only when the coupling factor between the cavity and transmission line is unity, that is, the external quality factor of coupler-cavity system, Q_{ext}, is equal to the unloaded quality factor of cavity. Therefore, to estimate the Q_{ext} is important for designing coupling devices for the cavity. For the third harmonic superconducting cavity of SSRF[1], expected Q_{ext} of the input coupler is around 5×10^8 to make the cavity critically coupled and the inner conductor/antenna movement range is about 20mm.

The external quality factor can be calculated with several methods, either analytical approximations with the equivalent circuit theory[2], or numerical simulations using the techniques of frequency domain[3] and time domain[4]. Perhaps, the simplest way to estimate Q_{ext}, which works for both strong and weak couplings, is to calculate directly the decay time constant from the induced field or energy in a cavity. For a high Q_{load} ($>10^6$) system, nevertheless, it is impractical to run a 3D electromagnetic code in time domain to obtain an accurate value for the decay time until a steady-state field amplitude is reached. In this work, the Q_{ext} factor of 10^2–10^5 was simulated, and the results were verified with measurements. The Q_{ext} factor of the single-cell cavity fits approximately to a natural exponential function, which will be used in the vertical test to adjust the antenna penetration depth, so that the cavity can be critically coupled.

2 Theoretical analysis

For wave propagation inside a hollow waveguide of uniform cross section, the longitudinal component of magnetic and electric fields for both TM and TE waves are related by[5]

$$Z(z) = A^{+}\mathrm{e}^{-\gamma z} \tag{1}$$

where A^{+} is the magnitude of electric or magnetic field, z is the distance from origin, and γ is the propagation constant, which is defined as:

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* Corresponding author. *E-mail address:* mazhenyu@sinap.ac.cn

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$$\gamma = \sqrt{k_c^2 - k^2} = \alpha + j\beta \quad (2)$$

where α is the attenuation constant, and β is phase-shift constant.

We can rewrite equation (2) by defining the wave-number as:

$$k = 2\pi / \lambda = 2\pi f / c$$

and

$$k_c = 2\pi / \lambda_c = 2\pi f_c / c$$

So that we can write

$$\gamma = \sqrt{\frac{2\pi}{c}\left(f_c - f\right)} = \alpha + j\beta$$

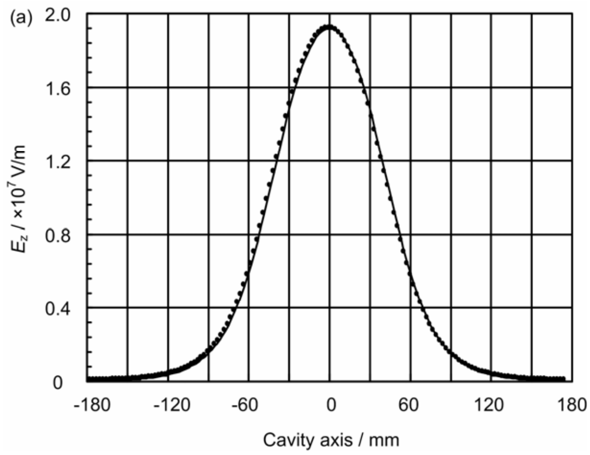
where f_c is the cutoff frequency and f is the mode frequency of electromagnetic wave in waveguide.

It is seen that the wave behaviors depend on the propagation constant γ. For mode frequencies higher than the cutoff frequency, the propagation constant is imaginary, i.e. the waves propagate in the guide. For mode frequencies lower than the cutoff frequency, the propagation constant is real, and the waves cannot propagate and evanesce as $e^{-\alpha z}$ along the cavity axis Z, hence the name of cutoff modes or evanescent modes. In our type of cavity the resonant mode used for acceleration is the TM_{010}, with resonant frequency of 1.5 GHz, less than the cutoff frequency of the beam pipe. Therefore, the field of fundamental mode in the cavity decays exponentially along the beam pipe, and the propagation constant is defined as,

$$\gamma = \alpha = \sqrt{\left(\frac{2.405}{R}\right)^2 - \left(\frac{2\pi f}{c}\right)^2} \quad (3)$$

where R is the radius of the beam pipe, f is the resonant frequency of cavity, and c is the speed of light in vacuum. Here the beam pipe is 41.5 mm in radius, and the propagation constant is calculated as 0.049 mm^{-1}.

The propagation constant of electric field is also calculated using numerical method to compare with the above analytical method. The longitudinal electric field along the cavity axis and the field attenuation curve along the beam pipe were calculated with the electromagnetic code, as shown in Fig.1. It can be calculated from the curve that, in a linear relationship, the attenuation coefficient is 0.0487 mm^{-1}, the same as the one calculated from Eq.(3).



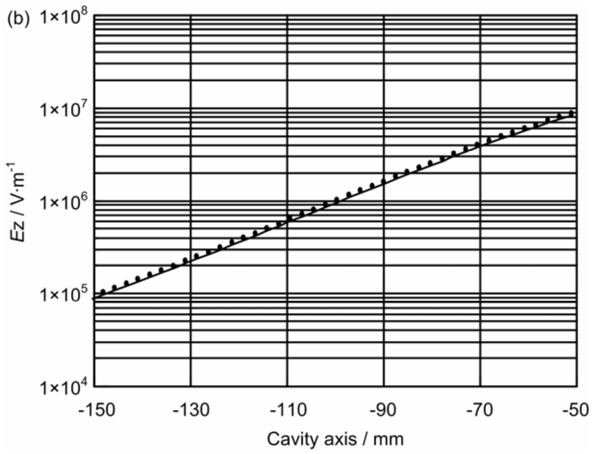


Fig. 1 Longitudinal electric field along the cavity axis (a) and the field attenuation along beam pipe (b).

The RF power is transmitted to the cavity via a coaxial coupler in a TEM mode. The center conductor protrudes into the beam pipe of the cavity and forms the outer conductor. In this case, the TEM mode propagating in the input coupler couples weakly to the cavity fields via an intermediate non-propagating mode. The coupling strength can be adjusted by changing the penetration of the center conductor. Assuming a lossless cavity containing RF energy U at a resonant frequency ω and if the cavity is weakly coupled to an infinite line, the line drives out a certain RF power P and the energy stored in the cavity decreases gradually. The Q_{ext} factor can be defined as

$$Q_{ext} = \frac{\omega U}{P_e} = \frac{\omega \iiint_{cavity} |E|^2 \, dv}{c \iint_{linesect} |E|^2 \, ds} \quad (4)$$

where ω is the angular frequency of resonant cavity, U is the stored energy in the cavity which is the integral over the cavity volume, and P_e is the emission power through the coupler which can be computed either from electric or magnetic field amplitude by integrating over the line cross-section. Since the lectric field decays exponentially along the beam pipe, and it's assumed that the energy stored in the cavity is constant and the center conductor does not change field distribution in the cavity remarkably, the following equations can be deduced

$$Q_{\text{ext}} \propto e^{2\alpha z} \tag{5}$$

$$\beta = \frac{Q_0}{Q_{\text{ext}}} \propto e^{-2\alpha z} \tag{6}$$

It could be interpreted that, since the cavity field for the fundamental mode fall off exponentially with distance in the cutoff region, the external quality factor of the cavity-coupler system increase exponentially with its departure from the center of cavity and its propagation constant is twice as the attenuation coefficient of electric field which is calculated above, i.e. 0.098 mm^{-1}.

3 Simulated Q_{ext}

When the RF source is cut off, the total power being lost will be the sum of the power dissipated in the cavity walls and the power that leaks out input coupler. And the energy stored in the cavity satisfies[6]

$$\frac{dU}{dt} = -(P_c + P_e) = -\frac{\omega U}{Q_{\text{load}}} \tag{7}$$

The solution to Eq.(7) is,

$$U = U_0 \exp\left(\frac{-\omega t}{Q_{\text{load}}}\right) \tag{8}$$

where U_0 is the equilibrium value of the energy stored in the cavity for a given level of RF source. The stored energy decays exponentially with a time constant

$$\tau_L = \frac{Q_{\text{load}}}{\omega} \tag{9}$$

By calculating the decay time constant, the Q_{load}, a simple relationship between Q_0, Q_{ext} and Q_{load}, can be obtained as

$$\frac{1}{Q_{\text{load}}} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} \tag{10}$$

Since a superconducting cavity has an extreme high Q_0 (>10^8), the numeric analysis can be simplified considerably by setting cavity material as perfect conductor in simulations for strong coupling cases. In this case, power dissipation is only determined by the coupling power, therefore $Q_{\text{ext}} = Q_{\text{load}}$.

We simulated this cavity-coupler system with 3D electromagnetic code in time domain, while setting the end of the coaxial type of input coupler as a port with matched load. The cavity field was excited by applying a Gaussian pulse with the center frequency approximately around the resonant frequency of the system through the setting port. A probe to record the field changing was located at the cavity axis center where the desired fundamental mode had a strong electric field component. The energy decay curve as a function of time could be obtained from the simulation results. The decay slope K could be calculated by picking up two points in the linear decay section of the curve. Applying the slope K in Eq.(11), we could obtain the Q_{ext} factor,

$$Q_{\text{ext}} = -\frac{10\lg e \cdot \omega}{K} \tag{11}$$

We noted that, as an affecting factor on the Q_{ext} value, the mesh number should not be less than about 5×10^5 to minimize variation of the Q_{ext} value, where as further increases of the number of nodes would give rise to unacceptable increase of the computing time. To reduce simulation time and memory requirements, one can take advantage of the symmetry planes of the cavity-coupler geometry, and the boundary condition at the symmetry plane has to be set to a magnetic wall. More details about the simulation considerations can be referred to Refs.[7,8].

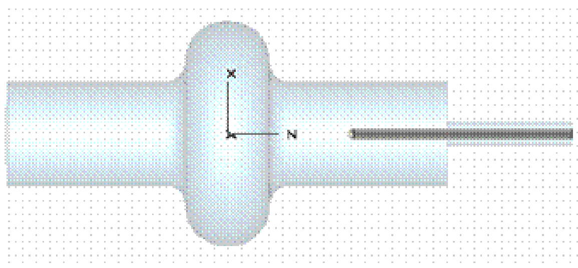


Fig.2 The model computed with 3D electromagnetic code.

The model used for calculations is shown in Fig.2. The coupler coaxial line is half the wavelength so that the solution of the fundamental transmission line mode can be stabilized. The input coupler antenna tip simulated was a rounded tip with a radius of 4.35 mm, which is the same as the inner conductor radius of the

50 Ω coaxial coupler of a 10 mm outer conductor radius. The coupler is featured by its enhanced coupling, as it penetrates the beam pipe.

The simulated Q_{ext} factors of 10^2–10^5 for various antenna penetration depths are shown in Fig.3.

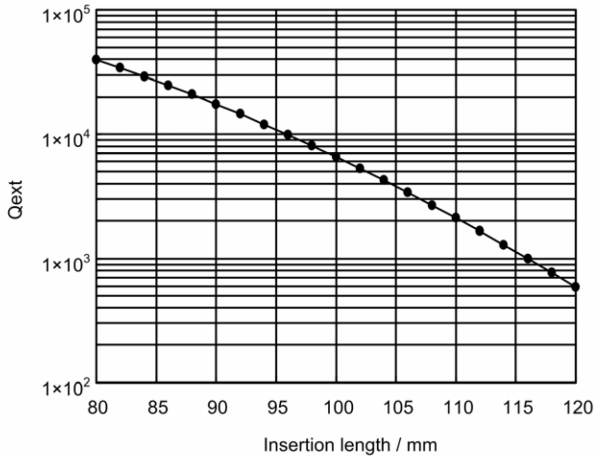


Fig.3 The simulated Q_{ext} for various antenna lengths.

It can be seen from Fig.3 that in a logarithmic scale the Q_{ext} is approximately linear with the antenna penetration depths in a wide range. The curve can be fitted by,

$$\ln Q_{ext} \approx -0.105l + 19.1 \tag{12}$$

where l is the antenna penetration depths. Here, the flange port wall is taken as the original point, with the penetration depth into the beam pipe as positive distance and out as negative distance. The attenuation coefficient we got from the simulation result is a little larger than the result from analytic analysis; this difference is caused by the perturbation of antenna in the cavity electromagnetic field.

4 The measured Q_{ext}

The measurements of external quality factor were carried out on a copper mock-up of the SC cavity with coaxial input coupler (Fig.4). The network analyzer was both the RF source and RF detector. The signal was excited by the Network Analyzer through the input coupler and picked up by the probe as output coupler. The coupling could be changed by moving the electrical antenna more or less into the beam tube.

From the equivalent circuit for a cavity with one coupler, the input coupling strength and the reflection coefficient Γ for the cavity are related by,

$$\beta = \frac{1 \pm \sqrt{P_r / P_f}}{1 \mp \sqrt{P_r / P_f}} = \frac{1 \pm \Gamma}{1 \mp \Gamma} \tag{13}$$

where the upper sign is used for $\beta > 1$ (overcoupled), and the lower sign is used for $\beta < 1$ (undercoupled). The choice of sign could be determined by sweeping the frequency using the network analyzer and examining a polar plot of the reflection coefficient. If the loop encompasses the origin, the cavity is overcoupled. Otherwise, it is undercoupled. If it passes through the origin, the cavity is coupled in unity.

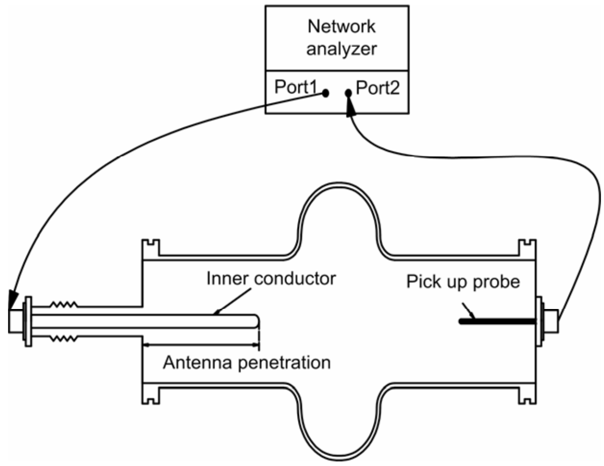


Fig.4 Q_{ext} experimental setup.

The reflection coefficient Γ could be obtained by measuring the transmission parameter S11 in dB unit directly from the network analyzer

$$\Gamma = 10^{-\frac{S11}{20}} \tag{14}$$

The loaded Q value could be obtained by measuring the bandwidth at 3 dB points of the peak in the frequency domain, i.e. S12 or directly read from the network analyzer

$$Q_{load} = \frac{f_0}{2\Delta f} \tag{15}$$

where Δf is half the resonance width.

Eq.(10) can be further expressed as

$$Q_0 = (1+\beta) Q_{load} \tag{16}$$

The Q_{ext} factor for the coupler is defined as

$$Q_{ext} = \frac{Q_0}{\beta} = \frac{(1+\beta)}{\beta} Q_{load} \tag{17}$$

Fig.5 shows the simulated and measured Q_{ext} factors for various penetrations depths of the coupler antenna. The simulation results agree well with measurements, with a discrepancy of <20 %.

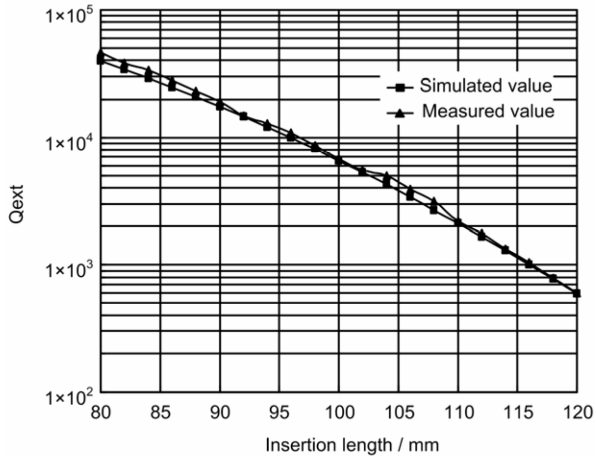


Fig.5 Simulated Q_{ext} and measured Q_{ext} for various antenna insertion.

In designing the input coupler, taking into account the location of the pumping port, we shifted the input coupler by a parallel displacement of 25mm in the X-axis to achieve a logical layout. As shown in Fig.6, the influence of location of input coupler on the external quality factor is uniform and can be compensated by a ±7.6 mm penetration of the antenna. Fig.7 shows simulation and measurement results of the design and the discrepancy is <22 %. The curve can be fitted by Eq.(18).

$$\ln Q_{ext} \approx -0.109l + 20.3 \tag{18}$$

The external quality factor that depends solely on the geometry is a measure of energy storage to energy propagating through the input port, and is unaffected by the conductivity of the material. Therefore, the external quality factor remains constant at different temperatures for the same penetration depth. Therefore, we can extrapolate the simulated and measured data to the superconducting case.

To prove the evaluation method, an unpolished 1.5 GHz single-cell superconducting cavity with this type of input coupler was tested at 4.2 K. It was supposed that the unloaded quality factor for the unpolished superconducting cavity at 4.2 K can reach 10^6–10^7. The external quality factor was chosen as 5×10^6, and the corresponding penetration depth was around 48.7 mm as calculated from Eq.(18). Based on the measured data, the unloaded quality factor of unpolished superconducting cavity reached 3.65×10^7, the Q_{ext} factor was 3.87×10^6, and the input coupling strength was 9.44, with a discrepancy of <25%, an acceptable deviation, between the results of evaluation and measurement.

We note that there is a limitation with the evaluation method. Since the electromagnetic field in the transmission line does not satisfy the exponential decreasing mode when the antenna pull out from the beam pipe, so in a logarithmic scale the external quality factor no longer follows the linear relation. However, the upper limit from this evaluation of the external quality factor is about 6.55×10^8, which can vertical test of the single cell superconducting cavities.

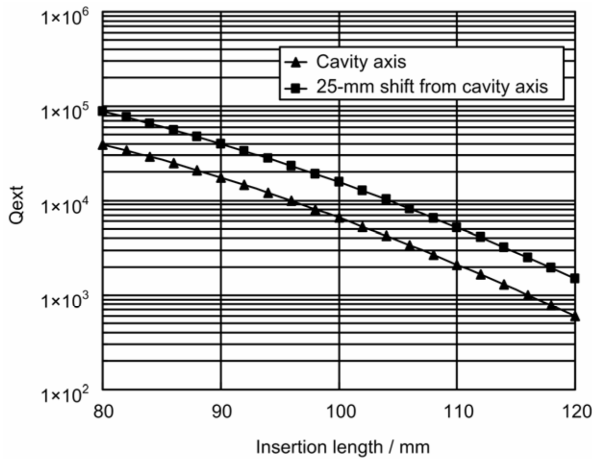


Fig.6 Simulated Q_{ext} factors of two input couplers.

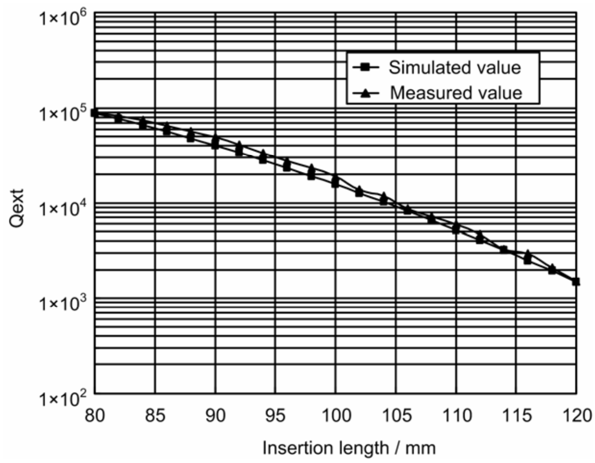


Fig.7 Simulated and measured Q_{ext} for various antenna insertion.

5 Conclusion

The external quality factor of coupler-cavity system can be easily, straightforward and efficiently computed by the evaluation method presented in this paper. The numerical simulation results and measurements results have achieved a good agreement at room temperature, and the results for the Q_{ext} of single-cell cavity with coaxial type of input coupler, were fit to a natural exponential function. We take advantage of the simulated and measured data and extrapolate the results to the superconducting case.

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