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Characteristics of a reactor with power reactivity feedback

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Abstract The point-reactor model with power reactivity feedback becomes a nonlinear system. Its dynamic characteristic shows great complexity. According to the mathematic definition of stability in differential equation qualitative theory, the model of a reactor with power reactivity feedback is judged unstable. The equilibrium point is a saddle-node point. A portion of the trajectory in the neighborhood of the equilibrium point is parabolic fan curve, and the other is hyperbolic fan curve. Based on phase locus near the equilibrium point, it is pointed out that the model is still stable within physical limits. The difference between stabilities in the mathematical sense and in the physical sense is indicated.

Key words Point-reactor, Stability, Center manifold, Phase locus

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1 Introduction

The point-reactor model with power and temperature reactivity feedback is a nonlinear system of complex dynamic characteristics. Stability of several models taking no delayed neutrons into account was analyzed by FU Longzhou[1] with Lyapunov method. It is well known, however, that Lyapunov function is difficult to construct.

Moreover, resolution of the point kinetic equation with temperature feedback can be found only under certain assumptions[2-4]. CHEN Wenzhen[5] analyzed the point-reactor neutron-kinetics equation under the conditions of small step reactivity ($\rho_0<\beta$) and temperature feedback. Let $d^2C/dt^2=0$, he derived the analytic expressions of the reactivity and output power with time for any initial power.

In this paper, a model based on the mathematic definition of stability in differential equation qualitative theory is analyzed. Single group of delayed neutrons and power reactivity feedback are taken into account. The difference between stabilities in the mathematical sense and in the physical sense is explained in terms of phase locus near the equilibrium point.

2 Stability analyses

The neutron kinetics equations of one group are given by

$$\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}t} = \dfrac{\rho-\beta}{\Lambda} n + \lambda C \\ \dfrac{\mathrm{d}C}{\mathrm{d}t} = \dfrac{\beta}{\Lambda} n - \lambda C \end{cases} \tag{1}$$

where n is the neutron density or the reactor power, C is the average density of delayed neutron precursors, ρ is the reactivity, λ is the radioactive decay constant of delayed neutron precursors, Λ is the prompt neutron lifetime and β is the total fraction of the delayed neutron.

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The general relationship between reactivity feedback and power is $\rho=\rho_0-\alpha n$, where ρ_0 is inserted reactivity, and $\alpha > 0$. There are two equilibrium points of the system. One is $(n, C) = (0, 0)$ and ρ is arbitrary. The other equilibrium point is $(n, C) = (\rho_0/\alpha, \rho_0\beta/(\lambda\alpha\Lambda))$ and $\rho=0$.

While the inserted reactivity $\rho_0 = 0$, two equilibrium points coincide with each other. This condition is discussed as follows.

Substituting $\rho = -\alpha n$ into Eq.(1) gives

$$\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}t} = \dfrac{-\beta}{\Lambda} n - \dfrac{\alpha}{\Lambda} n^2 + \lambda C \\ \dfrac{\mathrm{d}C}{\mathrm{d}t} = \dfrac{\beta}{\Lambda} n - \lambda C \end{cases} \tag{2}$$

Let $a= -\beta/\Lambda$ and $b=\lambda$, we have the corresponding linear equations

$$\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}t} = an + bC \\ \dfrac{\mathrm{d}C}{\mathrm{d}t} = -an - bC \end{cases} \tag{3}$$

Let A denote the coefficient matrix of Eq.(3). The eigenvalues of A are 0 and $a-b$. Because one of the eigenvalues is zero and the other is negative, the stability of Eq.(2) cannot be determined directly, but it can be judged that some center manifolds exist in Eq.(2), and the stability of Eq.(2) on the center manifold is equal to its own stability. Therefore it is necessary to transform A into a diagonal matrix firstly. Nondegenerate linear transformation

$$(n, C)^T = P(x, y)^T \tag{4}$$

is used to obtain $P^{-1}AP = J$, where

$$P = \begin{pmatrix} -\dfrac{b}{a} & 1 \\ 1 & -1 \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} \dfrac{a}{a-b} & \dfrac{a}{a-b} \\ \dfrac{a}{a-b} & \dfrac{b}{a-b} \end{pmatrix},$$

$$J = \begin{pmatrix} 0 & 0 \\ 0 & a-b \end{pmatrix}$$

Substituting Eq.(4) into Eq.(2), one gets

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -(\dfrac{a}{a-b})\dfrac{\alpha}{\Lambda}(-\dfrac{b}{a}x + y)^2 \\ \quad = k(-\dfrac{b}{a}x + y)^2 \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = (a-b)y - (\dfrac{a}{a-b})\dfrac{\alpha}{\Lambda}(-\dfrac{b}{a}x + y)^2 \\ \quad = (a-b)y + k(-\dfrac{b}{a}x + y)^2 \end{cases} \tag{5}$$

where $k = -(\dfrac{a}{a-b})\dfrac{\alpha}{\Lambda} = \dfrac{\beta/\Lambda}{-\beta/\Lambda - \lambda}(\dfrac{\alpha}{\Lambda}) < 0$.

Since there is a local center manifold $y=h(x)$ and $h(0)=h'(0)=0$, we can assume that

$$y = h_2 x^2 + h_3 x^3 + O(x^3) \tag{6}$$

where $O(x^3)$ is an infinitesimal, which is in an order of greater than three. Substituting Eq.(6) into

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (\frac{\mathrm{d}y}{\mathrm{d}x})\frac{\mathrm{d}x}{\mathrm{d}t} = (a-b)y + k(-\frac{b}{a}x + y)^2$$

and comparing the homogeneous items of both sides of the equation with each other, we can get the solution of coefficients h_2 and h_3. But it is unnecessary in this condition. We can substitute Eq.(6) into the first line of Eq.(5) directly and yield the equation on the center manifold

$$\begin{aligned} x' &= k(-\frac{b}{a}x + y)^2 \\ &= k\left[-\frac{b}{a}x + h_2 x^2 + h_3 x^3 + O(x^3) \right]^2 \\ &= \frac{kb^2}{a^2}x^2 + O(x^2) \end{aligned}$$

It is clear that x is a sign reversal function and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{kb^2}{a^2}x^2 + O(x^2)$ is negative definite. Then, Eq.(2) is unstable on the center manifold, and unstable in itself as well. The result seems to disagree with existing engineering knowledge. The negative reactivity increases with the nuclear power and in turn reduces the nuclear power, that is, the system is stable. But attention should be paid to the difference between stabilities in the mathematical sense and in the physical sense. In order to explain the difference in detail, we analyze the phase locus of Eq.(2) in the phase plane n–C as follows.

3 Phase locus analysis

Let $y = z/(a–b)$ and $\tau= (a–b)t$, Eq.(5) becomes

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}\tau} = \dfrac{k}{a-b}(-\dfrac{b}{a}x + \dfrac{z}{a-b})^2 \\ \quad = \phi(x,z) \\ \dfrac{\mathrm{d}z}{\mathrm{d}\tau} = z + k(-\dfrac{b}{a}x + \dfrac{z}{a-b})^2 \\ \quad = z + \psi(x,z) \end{cases} \tag{7}$$

The explicit expression $z(x)$ can be derived from the following equation

$$z + \psi(x,z) = 0 \tag{8}$$

Because $z(0) = z'(x) =0$, it can be supposed that

$$z(x) = a_2x^2 + a_3x^3 + \cdots \tag{9}$$

Combining Eq.(9) and Eq.(8), we can determine the coefficient $\alpha_i(i=2, 3\cdots)$ and get

$$z(x) = -\frac{ka^2}{b^2}x^2 - \frac{2k^2b^3}{a^3(a-b)}x^3 + \cdots \tag{10}$$

Substituting Eq.(10) into $\phi(x,z)$ gives

$$\phi(x,z(x)) = \frac{k}{a-b}(-\frac{b}{a}x + \frac{z(x)}{a-b})^2 = \frac{kb^2}{(a-b)a^2}x^2 + \cdots$$

Because the minimum order of $\phi(x,z(x))$ is even, the equilibrium point O is a saddle-node point. A portion of the trajectory of (7) in the neighborhood of the equilibrium point O is parabolic fan curve, and the other is hyperbolic fan curve (Fig. 1). For $kb^2/[(a–b)a^2]>0$, the parabolic fan curve is in the right half plane[6].

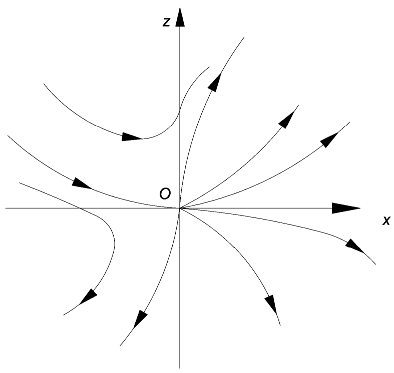

Fig.1 The trajectory distribution pattern of system in the neighborhood of equilibrium point in $x–z$ plane.

Since $n = –bx/a + y$, $C= x–z$, and $y =z/(a–b)$, x and z axis in $x–z$ plane are converted into lines of $C= –an/b$ and $C= –n$ in the $n–C$ plane, as dashed lines shown in Fig. 2. Attention should be paid to $\tau= (a–b)t$, and $\tau\to -\infty$ while $t\to +\infty$. Thus, as shown in Fig. 2, four trajectories which trace out from point O tangent to x axis in $x–z$ plane, are transformed into the other four trajectories which get into point O tangent to line $C = –an/b$ in $n–C$ plane. Two trajectories, which trace out from point O tangent to z axis, are transformed into the other two trajectories which get into O tangent to line $C = –n$. The saddle-node boundary, which gets into point O tangent to X axis, is transformed into the saddle-node boundary which traces out from O tangent to line $C = –an/b$.

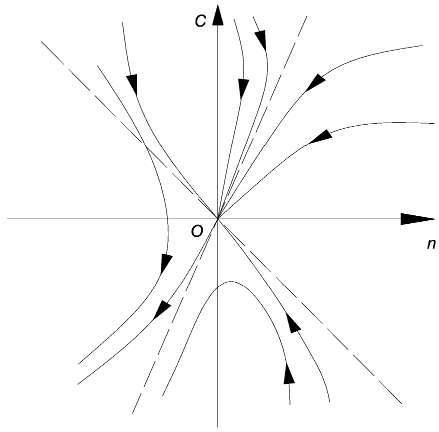

Fig.2 The trajectory distribution pattern of system in the neighborhood of equilibrium point in $n–C$ plane.

From Fig. 2, the difference between stabilities in the mathematical sense and in the physical sense can be found clearly. The equilibrium point $O(0,0)$ is unstable in the mathematic sense when the definition domain of n and C is the whole real domain. The equilibrium point is still stable when n and C limited within $n\geq 0$ and $C\geq 0$ where there is real physical meaning. The equilibrium point becomes unstable when n and C are permitted to diverge towards negative infinity.

4 Conclusions

The characteristic of neutron kinetics equation is usually studied by obtaining the resolution under special condition or getting the numeric solution[2-4]. Qualitative analysis of point kinetic equations as above is quite different from those methods. We can grasp the characteristic of point kinetic equation as a whole by this method. It offers reference for analysis of the point kinetic with another form of reactivity feedback such as $\rho = \rho_0 - \alpha T$ and $\mathrm{d}T/\mathrm{d}t = Kn$.

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