Available online at www.sciencedirect.com
ScienceDirect

NUCLEAR SCIENCE AND TECHNIQUES

Nuclear Science and Techniques 19 (2008) 325–328

Simulation on control of beam halo-chaos by power function in the hackle periodic-focusing channel

YU Haijun[1,*] WENG Jiaqiang[2] BAI Long[3]

[1] *Department of Physics, Huainan Normal University, Huainan 232001, China*

[2] *College of Physics and Information Technology, Guangxi Normal University, Guilin 541004, China*

[3] *College of Science, China University of Mining and Technology, Xuzhou 221008, China*

Abstract The K-V beam through a hackle periodic-focusing magnetic field is studied using the particle-core model. The beam halo-chaos is found, and a power function controller is proposed based on mechanism of halo formation and strategy of controlling halo-chaos. Multiparticle simulation was performed to control the halo by using the power function control method. The results show that the halo-chaos and its regeneration can be eliminated effectively. We also find that the radial particle density evolvement is of uniformity at the beam's centre as long as appropriate parameters are chosen.

Key words Beam halo-chaos, Hackle periodic-focusing magnetic field, Power function

CLC number TL501^{+}.5

1 Introduction

High intensity particle beams are utilized widely due to their attractive features in possible applications, such as clean activity nuclear power systems, nuclear physics, and medical radioisotope production. The halo of a particle beam, however, may reduce the accelerator efficiency, and cause damages to human body and the environment. It is necessary to control the halo-chaos based on mechanisms of halo formation [1-3] and strategy of chaos control [4-8].

At present, according to stability analysis for the dimensionless envelope equation of the beam propagating through a periodic-focusing field, using the Poincare-Lyapunov theorem [4], some nonlinear feedback functions have be proposed to control the halo-chaos, and the particle-in-cell (PIC) simulations have shown that these controllers are effective in suppressing beam halo-chaos [5-14]. However, these studies focused on the beam via a rectangle periodic-focusing channel or a uniform-focusing channel. In fact, a hackle periodic-focusing magnetic field is close to practical condition in a high current accelerator. So studying the beam propagating through the hackle periodic-focusing field is valuable and realizable in engineering application. In this work, halo-chaos of a beam in a hackle periodic-focusing magnetic field was studied, and a power function for controlling the halo was proposed. Multiparticle numerical simulations were performed. The results show clearly that the power function control method is effective.

2 Numerical methods

For simulation studies of time evolution of charged-particle bunches in an accelerator, the self-consistent treatment of space-charge forces in beam macro-particles is needed necessarily for describing

Supported by the National Natural Science Foundation of China (Grant No. 10247005), the Natural Science Foundation of the Anhui Higher Education Bureau (Grant No. KJ2007B187) and the Scientific Research Foundation of China University of Mining and Technology for the Young (Grant No. OK060119).

* Corresponding author. *E-mail address:* haijun20030@163.com

Received date: 2008-03-30

quantitatively the beam dynamics. The charged-particle dynamics in an accelerator can be described by the Vlasov equation[7]:

$$\frac{\partial f}{\partial t}+\dot{r}(\frac{\partial f}{\partial r})+\dot{p}(\frac{\partial f}{\partial p})=0, \tag{1}$$

where f is the particle distribution in phase space, r is the spatial position, and $\dot{p}=F=F_{\text{ext}}+F_{\text{sc}}$ is the momentum. The F includes the contributions from both the external periodic focusing fields (F_{ext}) and the space-charge force F_{sc}. The space-charge force in this equation is a mean-field approximation of the N-body microparticle Coulomb force. In the moving frame, the space-charge force can be obtained from the solution of Poisson's equation:

$$\nabla^2\phi(r)=-\rho(r)/\varepsilon_0, \tag{2}$$

$$F_{\text{sc}}=-q\nabla\varphi(r), \tag{3}$$

where ϕ is the electrostatic potential in the moving frame, ρ is the particle spatial charge density, ε_0 is the vacuum permittivity. The charge density can be calculated from the distribution function f by $\rho(r)=\int \mathrm{d}^3 p f(r,p)$. When using z as the independent variable, $\varphi(r)=\Phi^s(x,y,s)$.

Then the Poisson-Vlasov equations can be solved using the PIC approach for simulation of beam dynamics in a linac. Of course, the motion equations of the particles must satisfy the Maxwell equations in electromagnetic fields. So far, the PIC program has been developed in the transverse electromagnetic field, and the beam halo-chaos formation and the halo-chaos control in the four-dimensional phase space has been simulated [5-9].

The particle-core model has been used widely to investigate the beam halo-chaos. The model assumes that the Kapchinsky-Vladimirsky (K-V) beam is round and continuous, and the dimensionless nonlinear equation of the beam envelope and the transverse equations of motion for a single particle in a periodic-focusing system are[2,3]

$$\frac{\mathrm{d}^2 r_{\mathrm{b}}}{\mathrm{d}s^2}+\kappa_z(s)r_{\mathrm{b}}-\frac{K}{r_{\mathrm{b}}}-\frac{1}{r_{\mathrm{b}}^3}=0 \tag{4}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2}+\kappa_z(s)x+(\frac{q}{\gamma_{\mathrm{b}}^3\beta_{\mathrm{b}}^2 mc^2})\frac{\partial\Phi^s(x,y,s)}{\partial x}=0 \tag{5}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2}+\kappa_z(s)y+(\frac{q}{\gamma_{\mathrm{b}}^3\beta_{\mathrm{b}}^2 mc^2})\frac{\partial\Phi^s(x,y,s)}{\partial y}=0 \tag{6}$$

where r_{b} is the beam radius and $s=vt$ is the axial coordinate, in which v is the axial velocity of the beam particles, K is a measure of the beam self-field, c is the speed of light; $\beta_{\mathrm{b}}=v/c$, $\gamma_{\mathrm{b}}=(1-\beta_{\mathrm{b}}^2)^{1/2}$ is relativistic mass factor, q and m are the particle charge and rest mass, respectively; $\Phi^s(x, y, s)$ is the self-electric potential. The periodic function $\kappa_z(s)$ characterizes the strength of the periodic-focusing exterior magnetic field in Fig.1, $\kappa_z(s)=\kappa_z(s+S)$, in which S is a period.

In the particle-core-interaction model, the self-field force acting on a particle is given by

$$F_r=-q\nabla\Phi^s(x,y,s). \tag{7}$$

The radial space-charge field of an axis-symmetric beam is calculated from the Gauss law by counting the number of particles in cells of a finite radial grid, which extends up to five times of the beam matched radius in a multi-particle simulation.

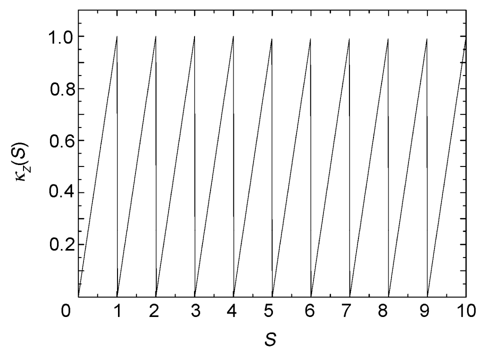

Fig. 1 Periodic function $\kappa_z(s)$.

The nonlinear feedback control method is proposed on the basis of the general strategy of chaos control. The approach is to apply a nonlinear feedback controller G to the right-hand side of Eq.(7), that is

$$F_r=-q\nabla\Phi^s(x,y,s)+G. \tag{8}$$

It is critical to select an effective controller G to control the halo-chaos in PIC simulations. In this work, we use the power function having a strong nonlinearity and excellent localization property, namely

$$f(x)=\frac{1}{x^2}, \tag{9}$$

and the power function controller G is designed as

$$G = g\left[f(r_{\mathrm{rms}}) - f(a_{\mathrm{m}})\right] = 1.5\times\left[f(r_{\mathrm{rms}}) - f(a_{\mathrm{m}})\right], \quad (10)$$

where r_{rms} is the average root-mean-square radius, a_{m} is the matching radius. When $r_{\mathrm{rms}}-a_{\mathrm{m}}\to 0$, G=0. Therefore, the purpose of controlling halo is achieved.

3 Results and discussion

The particle-in-cell (PIC) simulation was used to study the beam under a hackle periodic-focusing channel. The main parameters were:

the total number of particles, 5×10^5,

the vacuum phase advance $\sigma_0 = 115^{\circ}$,

filling factor Γ= 0.80,

tune-depression η =0.80, $\eta = (a_{\mathrm{m}}^2\sigma_0)^{-1}$ is the strength of space-charge effect, and

mismatch factor M=1.5, which gives the ratio of the initial beam radius to the matched radius.

From the calculation, we had the matched radius a_{m}= 0.7891642 and perveance K=0.9032079. The evolution periodic steps are 1800. The numerical simulation results are shown below.

3.1 Comparison of statistical variables of beams

In the simulation, the halo-chaos strength factor H is defined. It is the ratio of particles outside the 1.75 a_{m} to the total particles. H is a measure of the halo control. As shown in Fig.2, H is not zero before controlling. See curve (a). This means that the halo exists all times without the halo control. But after By controlling the halo, H is zero see line (b).

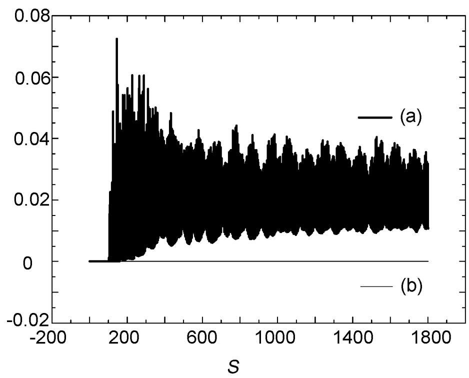

Fig. 2 The halo-chaos strength factor H vs. time s before (a) and after (b) controlling the halo-chaos.

The evolution of root-mean-square radius r_{rms} is irregular before controlling the halo (curve (a) in Fig.3). After controlling the halo by the power function controller, the amplitude of r_{rms} becomes small (b in Fig.3). These indicate that the tendency of particles escaping from the core is controlled effectively by the power function controller.

3.2 Radial density in high-intensity particle beam

In order to understand how the beam move in the hackle periodic-focusing system and how the halo can be better controlled, the radial particle density of K-V beam was investigated.

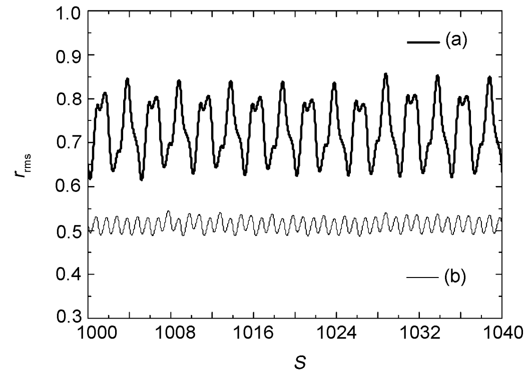

Fig. 3 Evolution of r_{rms} vs. time s before (a) and after (b) controlling the halo-chaos.

When the beam is not under halo control, the evolution of radial density of particle beam is in disordered state (Fig.4a). In one period, a number of particles accumulate disorderly in the area of r < 1.5. When 1.5 < r < 2.75，there are just a few particles staying on the beam edge. The density curve is in peaks. So the radial particle density becomes nonuniform over the cross section of beam.

From the particle-core interaction model, space-charge force is linear in the beam core, but now the linear force is destroyed rapidly, and the space-charge nonlinear effects become dominant. At the same time, particles of mismatched beam are affected by the external magnetic field. Due to the complexity of the force acting on particles, the energy exchange between the particles and the core becomes smart. Some particles can escape easily from the core to form a surrounding halo.

The halo is suppressed effectively by the power function controller; the order state appears at the center of the beam. Form Fig.4 (b), one finds that all the particles distribute uniformly in the area of r < 0.8, and the radial density of particle beam becomes larger. It is the power function controller that can offer a transverse force to suppress the emission force of particles caused by the space-charge nonlinear effects. Thus the tendency of particles escaping from the core is restrained, and the loose particles beam is compressed to the core of the beam. So the nonuniform distribution of the beam before being controlled becomes the uniform distribution after the halo-chaos is controlled by the power function controller [14].

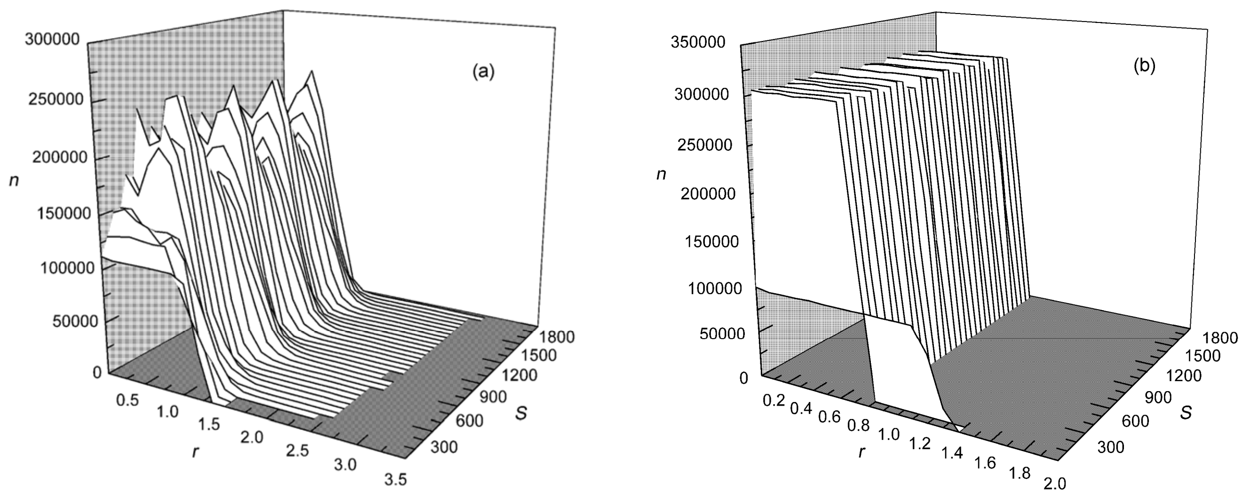

Fig.4 Evolution of radial particle density of beam before (a) and after (b) controlling the halo-chaos.

4 Summary

From the simulation results, the halo can be controlled effectively by the power function controller in the hackle periodic-focusing magnetic field, and the density uniformity of beam can be found as long as appropriate system parameters are chosen. These may be of significance for particle beam applications in the future.

References

1 Glckstern R L. Phys Rev Lett, 1994, **73**(9): 1247-1250.

2 Chiping C, Ronald C D. Phys Rev E, 1994, **49**(6): 5679-5687.

3 Chiping C, Ronald C D. Phys Rev Lett, 1994, **72**(14): 2159-2198.

4 Boccaletti S, Grebogi C, Lai Y C, *et al*. Phys Rep, 2000, **329**: 103-197.

5 FANG Jinqing, CHEN Guanrong. High Power Laser Part Beams (in Chinese), 2000, **12**(5): 647-651.

6 FANG Jinqing, CHEN Guanrong, ZHOU Liulai, *et al*. Prog Nat Sci, 2001,**11**(2):113-120.

7 FANG Jinqing, CHEN Guanrong, LUO Xiaoshu. Int J Bifurcation Chaos, 2002, **12**(5):917-930.

8 FANG Jinqing, WANG Zhongsheng, CHEN Guanrong. Commun Theor Phys (in Chinese), 2004, **42**(4): 557-560.

9 FANG Jinqing, GAO Yuan, WENG Jiaqiang, *et al*. Acta Phys Sin, 2001, **50**(3):435-440.

10 ZHOU Lunwu, WENG Jiaqiang, GAO Yuan, *et al*. Acta Phys Sin, 2002, **51** (7): 1483-1488.

11 FANG Jinqing, YU Xinghuo, CHEN Guanrong. Chin Phys Lett (in Chinese), 2003, **20** (12): 2110-2113.

12 GAO Yuan, LUO Xiaoshu, WENG Jiaqiang. Commun Theor Phys (in Chinese), 2004, **42**(1): 64-68.

13 BAI Long, ZHANG Rong, WENG Jiaqiang, *et al*. Chin Phys (in Chinese), 2006, **15**(3):1226-1230.

14 YU Haijun, BAI Long, WENG Jiaqiang, *et al*. Chin Phys Lett (in Chinese), 2008, **25**(3): 862-865.