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# Application of grey model on analyzing the passive natural circulation residual heat removal system of HTR-10

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**Abstract** Using the grey correlation analysis, it can be concluded that the reactor pressure vessel wall temperature has the strongest effect on the passive residual heat removal system in HTR (High Temperature gas-cooled Reactor), the chimney height takes the second place, and the influence of inlet air temperature of the chimney is the least. This conclusion is the same as that analyzed by the traditional method. According to the grey model theory, the GM(1,1) and GM(1, 3) model are built based on the inlet air temperature of chimney, pressure vessel temperature and the chimney height. Then the effect of three factors on the heat removal power is studied in this paper. The model plays an important role on data prediction, and is a new method for studying the heat removal power. The method can provide a new theoretical analysis to the passive residual heat removal system of HTR.

**Key words** Residual heat removal system, Grey model, HTR-10

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## 1 Introduction

High temperature gas-cooled reactor (HTR), one of the six reactor technologies selected for research and development under the Generation-IV program, is China’s important type of nuclear reactor to study[1], according to the Medium-long Term Outline on Science and Technology Development in 2006-2020 of China. The first demo-project of HTR nuclear power plant in China, HTR-10, will be built in Rongcheng, Shandong Province.

In theoretical studies on its residual heat removal system[2], an important part of the project, traditional heat transfer theory or experiment model with certain mathematic correlations[2-7], were used[8]. In the traditional analysis, averaging method is adopted to treat data fluctuations, hence unreliable analysis results. This shall be improved by adopting the grey system theory, which was proposed by Deng Julong[9] in 1982 and has been used widely in economy, science, industry etc. A main model of the theory, the grey model GM(1,1), i.e. a single variable first order grey model that works with four data or more, can be used to obtain fine forecasting result.

An important difference between the grey model of prediction and other forecasting models is accumulating generation[10-14], which is fatal to the heat removal system, as its heat transfer is affected by complicated correlative factors. Having typical characteristics of a grey system, the residual heat removal system can be studied using the grey mathematics, which well treats incomplete information, unstable data and inaccurate number. In the grey system theory, the rule of "what exists is reasonable" considers sufficiently the contribution of every data point, attaching importance to concentrating

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conditions and dispersed data points. A grey model was established for predicting and analyzing the thermal power removal from the residual heat removal system in HTR-10. In this paper, the grey theory and dynamic mathematical model of the heat removal system are presented. The study will provide theoretical basis for calculating heat removal rate to prevent any accident from happening to the HTR-10.

## 2 Grey relation analysis method

By calculating correlation coefficient and correlated degree of the reference columns and parallel columns in the grey relation analysis method, the relation of every factor in the system can be found for comparing the development tendency of the system and spotting the key influence factor of target value. The equations[9] are as follows.

Firstly, data conversion and normalization to$\{X_1(t)\}$ and $\{X_j(t)\}$is carried out:

$$X_1(t)=X_1(t)/[X_1(t)]_{\max} \quad (1)$$

$$X_i(t) = X_j(t)/[X_j(t)]_{\max} \qquad i=2,\ldots,N;\ j=2,\ldots,N \quad (2)$$

Secondly, correlation coefficient is calculated:

$$\xi_{1i}(k) = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\left|X_1(k) - X_i(k)\right| + \rho\Delta_{\max}} \quad (3)$$

Thirdly, correlated degree is calculated:

$$r_{1i} = \frac{1}{N}\sum_{k}^{N}\xi_{1i}(k) \quad (4)$$

where

$\{X_1(t)\}$— the reference column;

$\{X_j(t)\}$ — the parallel column;

N — sequence number;

$t$, $k$ —time;

$\Delta_{\max}$— maximum of the absolute value of differences; for the two columns each moment;

$\Delta_{\min}$— minimum of the absolute value of differences for the two columns each moment;

$\rho$— resolution coefficient, 0.5 generally.

And, Eq.(3) is used to calculate the correlation coefficients of the two columns when $t=k$.

## 3 The grey model establishment method

In the grey system theory[9], based on the concepts of correlation space and smooth scatterings function, the grey derivative and differential equations are defined to build dynamic model of differential equation using scattering data column, which is expressed as GM (Grey Model). The general equation of grey theory model is GM ($n$, $h$), which means that the variable $h$ is expressed by $n$-order differential equation.

In the following text, the modeling method is established with specified time sequence.

Firstly, the correlation sequences are calculated as follows.

The time sequences is given by

$$\left\{X_i^{(0)}(t)\right\},\ i=1,2\ldots h;\ t=1,2\ldots N \quad (5)$$

Corresponding accumulative value sequences is calculated by

$$\left\{X_i^{(1)}(t)\right\},\ i=1,2\ldots h;\ t=1,2\ldots N \quad (6)$$

where

$$X_i^{(1)}(t) = \sum_{k=1}^{i} X_i^{(0)}(k)\,,\quad \hat{X}^{(0)}(1) = \hat{X}^{(0)}(0)$$

The generated sequences of corresponding means value is calculated by

$$Z^{(1)}(t) = 0.5X^{(1)}(t) + 0.5X^{(1)}(t-1) \quad (7)$$

Secondly, the model is built according to the following equations.

The differential equation of GM (1,1) is

$$\frac{\mathrm{d}X^{(1)}}{\mathrm{d}t} + aX^{(1)} = u \quad (8)$$

The algebra equation of GM (1,1) is

$$\hat{X}^{(1)}(t+1) = (X^{(0)}(1) - \frac{u}{a})\mathrm{e}^{-at} + \frac{u}{a} \quad (9)$$

The differential equation of GM (1, h) is:

$$\frac{\mathrm{d}X_1^{(1)}}{\mathrm{d}t} + aX_1^{(1)} = b_2X_2^{(1)} + b_3X_3^{(1)} + \ldots + b_hX_h^{(1)} \quad (10)$$

The algebra equation of GM (1, $h$) is:

$$\hat{X}_1^{(1)}(t+1) = \left[X_1^{(0)} - \frac{1}{a}\sum_{i=2}^{N} b_iX_i^{(1)}(t+1)\right]e^{-at} + \frac{1}{a}\sum_{i=2}^{N} b_iX_i^{(1)}(t+1) \quad (11)$$

The general equations for â and $Y_N$ are parameters are as follows

$$\hat{a} = (B^TB)^{-1}B^TY_N \quad (12)$$

$$Y_N = [X^{(0)}(2), X^{(0)}(3), \cdots, X^{(0)}(N)]^T \quad (13)$$

where $\hat{a}$— the general equation.

In model GM (1,1),

$$\hat{a}=(a,u)^T \tag{14}$$

In model GM (1,h),

$$\hat{a}=(a,b_2,b_3,\ldots,b_h)^T \tag{15}$$

$Y_N$— the general equation of value.

$B$ in model GM (1,1) is

$$B=\begin{bmatrix} -Z(2) & 1 \\ -Z(3) & 1 \\ \vdots & \vdots \\ -Z(N) & 1 \end{bmatrix} \tag{16}$$

$B$ in model GM (1, $h$) is

$$B=\begin{pmatrix} -\frac{1}{2}\left(X_1^{(1)}(2)+X_1^{(1)}(1)\right) & X_2^{(1)}(2)\ldots X_h^{(1)}(2) \\ -\frac{1}{2}\left(X_1^{(1)}(3)+X_1^{(1)}(2)\right) & X_2^{(1)}(3)\ldots X_h^{(1)}(3) \\ \vdots & \vdots \\ -\frac{1}{2}\left(X_1^{(1)}(N)+X_1^{(1)}(N-1)\right) & X_2^{(1)}(N)\ldots X_h^{(1)}(N) \end{pmatrix} \tag{17}$$

# 4 Analysis of the calculated example

## 4.1 Grey correlation analysis

Assuming that the residual heat removal power is $X_1'$, the air entrance temperature in the chimney ($X_2'$), the reactor pressure vessel temperature ($X_3'$) and the chimney height ($X_4'$) are the correlated factors of $X_1'$. The data[15] are listed in Table 1 (other corresponding parameters keep almost constant).

**Table 1** Values of residual heat removal power and correlated factors

| I | $X_1$ / kW | $X_2$ / K | $X_3$ / K | $X_4$ / m |
|---|---|---|---|---|
| 1 | 138.7 | 321 | 480 | 15 |
| 2 | 158 | 295 | 495 | 54 |
| 3 | 172 | 273 | 505 | 83 |

Normalizing the data in Table 1 with Eq.(1), we have the $X_1$, $X_2$, $X_3$ and $X_4$ in Table 2.

**Table 2** Values of residual heat removal power and correlated factors after normalization

| I | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| 1 | 0.8064 | 1 | 0.9505 | 0.1807 |
| 2 | 0.9186 | 0.9190 | 0.9802 | 0.6506 |
| 3 | 1 | 0.8505 | 1 | 1 |

Using Eq.(3) and the data in Table 2, the correlation coefficient $\xi_{1i}$ ($k$) can be obtained (Table 3).

**Table 3** Correlation coefficients

| I | $\xi_{12}(k)$ | $\xi_{13}(k)$ | $\xi_{14}(k)$ |
|---|---|---|---|
| 1 | 0.3347 | 0.3333 | 0.3333 |
| 2 | 1.0 | 0.5391 | 0.5386 |
| 3 | 0.3946 | 1.0 | 1.0 |

By using Eq. (4), the correlated degree values can be obtained as $r_{12}$=0.5764, $r_{13}$=0.6242 and $r_{14}$=0.6240. These indicate that temperature in the reactor pressure vessel has the strongest effect on the passive residual heat removal system regarding the removal power diversification of HTR, the chimney height takes the second place and influence of inlet air temperature in the chimney is the least. But the orders of their dimensionless correlation coefficients are corresponsive, and they play a similarly important role to affect the residual heat removal power.

From Table 1, it can be found that the linear variance ratio of residual heat removal power with reactor pressure vessel temperature is 1.3143 kW/K, and the linear variance ratio with inlet air temperature of the chimney is –0.7056 kW/K. It is obvious that the reactor pressure vessel temperature has stronger effect on the residual heat removal power than inlet air temperature of the chimney.

The correlated coefficients after a linear regression of the data in Table 1 are $r_{12}$= –0.1137, $r_{13}$=0.5268 and $r_{14}$=0.1805.

## 4.2 The GM(1, 1) model

Considering the reactor pressure vessel temperature with the strongest effect on the residual heat removal power, the GM(1,1) model is established. Data of the second and the third column in Table 4 are the reactor pressure vessel temperature and residual heat removal power when the inlet air temperature is 295 K.

**Table 4** Comparison of data and results calculated by model

| I | $X_4$ / K | $X_1$ / kW | $\hat{X}_1^{(1)}(t)$ | $\hat{X}_1^{(0)}(t)$ | Error $e$ |
|---|---|---|---|---|---|
| 1 | 475 | 132.5 | 132.5 | 132.5 | 0 |
| 2 | 485 | 145.0 | 277.4 | 144.9 | 0.1 |
| 3 | 495 | 158.0 | 435.3 | 157.9 | 0.1 |
| 4 | 505 | 172.0 | 607.3 | 172.0 | 0 |

With the data in Table 4, using Eqs.(7), (9), (13) and (16), the following equation can be obtained:

$$B=\begin{bmatrix}-205.0 & 1\\ -356.5 & 1\end{bmatrix},\quad Y_N=\begin{bmatrix}145.0 & 158.0\end{bmatrix}^T$$

Using Eqs.(8) and (9), the following equation is obtained:

$$\frac{dX_1^{(1)}}{dV}-0.0858X_1^{(1)}=127.4092$$

$$X_1^{(1)}(t+1)=1617.4558\,\mathrm{e}^{0.0858t}-1484.9557$$

From Table 4, where $e$ is the error, i.e. $X_i-\hat{X}_i^{(0)}(t)$, using $X_4$ to predict $X_1$, we have $t$=4, $\hat{X}_1^{(1)}(4)=607.3$ and $\hat{X}_1^{(0)}(4)=172.0$ .As $X_4$=505 K, $X_1$=172 kW, then $e(4)$=0.

The differences between the data and the calculated value by the GM(1,1) model is small, so the GM(1,1) model can predict the residual heat removal power well within the certain experimental range. The same method can be used to build the GM(1,1) model for reactor pressure vessel temperature, chimney height and residual heat removal power.

### 4.3 The GM(1, 3) model

Based on data in the second to fifth column of Table 5, the GM(1,3) model can be established.

**Table 5** Values of residual heat removed power and correlated factors

| I | $X_1'$ / kW | $X_2'$ / K | $X_3'$ / K | $X_4'$ / m |
|---|---|---|---|---|
| 1 | 138.7 | 321 | 480 | 15 |
| 2 | 145 | 310 | 485 | 35 |
| 3 | 158 | 295 | 495 | 54 |
| 4 | 165 | 285 | 500 | 67 |
| 5 | 172 | 273 | 505 | 83 |

Summing the data of the four variables in Table 5 by Eq.(6) and averaging them by Eq.(7), we have Table 6.

**Table 6** Summation values and mean calculation value

| I | $\hat{X}_1^{(1)}(t)$ | $\hat{X}_2^{(1)}(t)$ | $\hat{X}_3^{(1)}(t)$ | $\hat{X}_4^{(1)}(t)$ | $Z^{(1)}(t)$ |
|---|---|---|---|---|---|
| 1 | 138.7 | 321 | 480 | 15 | — |
| 2 | 283.7 | 631 | 965 | 50 | 211.2 |
| 3 | 441.7 | 926 | 1460 | 104 | 362.7 |
| 4 | 606.7 | 1211 | 1960 | 171 | 524.2 |
| 5 | 778.7 | 1484 | 2465 | 254 | 692.7 |

Using Eqs.(13), (16) and (17), the follow equations are obtained:

$$B=\begin{bmatrix}-211.2 & 631 & 965 & 50\\ -362.7 & 926 & 1460 & 104\\ -524.2 & 1211 & 1960 & 171\\ -692.7 & 1484 & 2465 & 254\end{bmatrix}$$

$$Y_N=\begin{bmatrix}145 & 158 & 165 & 172\end{bmatrix}^T$$

Using Eq. (16), the follow equation is obtained:

$$\hat{a}=(a,b_2,b_3,b_4,b_5)^T=$$

$$(1.9148,\ -1.9300,\ 1.8936,-1.2015)^T$$

From Eq.(10), the GM(1,3) model is

$$\frac{\mathrm{d}X_1^{(1)}}{\mathrm{d}t}+1.9148X_1^{(1)}=-1.93X_2^{(1)}+1.8936X_3^{(1)}-1.2015X_4^{(1)}$$

Using Eq.(11), the following equation is obtained:

$$\hat{X}_1^{(1)}(t+1)=[138.7+1.0079X_2^{(1)}(t+1)-$$

$$0.9889X_3^{(1)}(t+1)+0.6275X_4^{(1)}(t+1)]\mathrm{e}^{-1.9148t}-$$

$$1.0079X_2^{(1)}(t+1)+0.98898X_3^{(1)}(t+1)-0.6275X_4^{(1)}(t+1)$$

Analyzing the results of the GM(1,3) model, Eq.(6) can be used to obtain the results in Table 7.

**Table 7** Comparison of experimental data and the results calculated by model using all data

| $\hat{X}_1^{(1)}(t)$ | Calculated $\hat{X}_1^{(1)}(t)$ | Relative error | $\hat{X}_1^{(0)}(t)$ | Calculated $\hat{X}_1^{(0)}(t)$ | Relative error |
|---|---|---|---|---|---|
| 138.7 | 138.7 | 0 | 138.7 | 138.7 | 0 |
| 283.7 | 265.1 | 6.6% | 145 | 126.4 | 12.8% |
| 441.7 | 438.6 | 0.7% | 158 | 173.5 | 9.8% |
| 606.7 | 608.9 | 3.6% | 165 | 170.3 | 3.2% |
| 778.7 | 782.2 | 4.5% | 172 | 173.3 | 0.8% |

Using Eqs.(6)~(10) and the data in Columns 1, 3 and 5 in Table 5, the following GM(1,3) model is obtained.

$$\frac{\mathrm{d}X_1^{(1)}}{\mathrm{d}t}-0.1197X_1^{(1)}=0.1226X_2^{(1)}+0.0317X_3^{(1)}+1.2891X_4^{(1)}$$

According to Eq.(11), the following equation is obtained:

$$\hat{X}_1^{(1)}(t+1)=[138.7+1.0242X_2^{(1)}(t+1)+0.2648X_3^{(1)}(t+1)+10.7694X_4^{(1)}(t+1)]\mathrm{e}^{0.1197t}-1.0242X_2^{(1)}(t+1)-0.2648X_3^{(1)}(t+1)-10.7694X_4^{(1)}(t+1)$$

Performing result analysis of the GM(1,3) model and using Eq.(6), the GM(1,3) model results can be calculated (Table 8).

**Table 8** Comparison of experimental data and the results calculated by model using part data

| $\hat{X}_1^{(1)}(t)$ | Calculated $\hat{X}_1^{(1)}(t)$ | Relative error | $\hat{X}_1^{(0)}(t)$ | Calculated $\hat{X}_1^{(0)}(t)$ | Relative error |
|---|---|---|---|---|---|
| 138.7 | 138.7 | 0 | 138.7 | 138.7 | 0 |
| 296.7 | 363.9 | 22.6% | 158 | 225.5 | 42.5% |
| 468.7 | 971.3 | 100.8% | 172 | 607.4 | 253.1% |

From Tables 7 and 8, the relative error can reach 100% with fewer data being used in the model. A singular matrix is formed, hence the increase of the relative error. By a linear regression, we have

$$X_1^{(0)}=0.1763X_2^{(0)}+0.1594X_3^{(0)}+0.3721X_4^{(0)}$$

The calculation value and the relative errors are given in Table 9.

**Table 9** Comparison of experimental data and the results calculated by model using linear regression

| $\hat{X}_1^{(0)}(t)$ | Calculated $\hat{X}_1^{(0)}(t)$ | Relative error |
|---|---|---|
| 138.7 | 138.7 | 0 |
| 145 | 145.0 | 0 |
| 158 | 151.0 | 0.7% |
| 165 | 154.8 | 6.2% |
| 172 | 159.5 | 7.3% |

The relative error of the linear regression can become big gradually for the prediction although the started relative error is small in the Table 9. For GM mode, the relative error can be small for the prediction in Table 7.

## 5 Conclusion

The result calculated by grey method is the same as that calculated by the traditional model. So the grey model theory can be used to analyze the residual heat removal power of HTR. The factors influencing residual heat removal power are complicated. The grey correlation analysis method can be used to ascertain the grey correlated degree of every factor of the residual heat removal system. The residual heat removal power influenced by the parameters is analyzed and the main influence factor can be found.

Comparisons of the results show that the grey modeling is reliable within certain range. The grey model has a strong relation with the data point. Large numbers of data points can bring high accuracy. When $h$>2, if the data is fewer, a singular matrix will be formed in the process of GM(1,$h$) model building, and the large relative error will be produced.

Prediction accuracy of GM(1,$h$) model decreases with increasing $h$.

In order to improve prediction accuracy of the grey model, improved GM models, such as the cloud grey model SCGM (System Cloud and its Grey Model) will be studied in the future.

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