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Calculation of the ex-core neutron noise induced by individual fuel assembly vibrations in two PWR cores

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Abstract Calculation of the neutron noise induced by fuel assembly vibrations in two pressurized water reactor (PWR) cores has been conducted to investigate the effect of cycle burnup on the properties of the ex-core detector noise. An extension of the method and the computational models of a previous work have been applied to two different PWR cores to examine a hypothesis that fuel assembly vibrations cause the corresponding peak in the auto power spectral density (APSD) increase during the cycle. Stochastic vibrations along a random two-dimensional trajectory of individual fuel assemblies were assumed to occur at different locations in the cores. Two models regarding the displacement amplitude of the vibrating assembly have been considered to determine the noise source. Then, the APSD of the ex-core detector noise was evaluated at three burnup steps. The results show that there is no monotonic tendency of the change in the APSD of ex-core detector; however, the increase in APSD occurs predominantly for peripheral assemblies. When assuming simultaneous vibrations of a number of fuel assemblies uniformly distributed over the core, the effect of the peripheral assemblies dominates the ex-core neutron noise. This behaviour was found similar in both cores.

Keywords Neutron noise · Fuel vibration · Ex-core noise · APSD · PWR

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1 Introduction

Ex-core neutron noise is the fluctuation of neutron leakage from the reactor core to the detectors located outside of the core due to in-core fluctuations. Monitoring of the reactor core internal vibrations, e.g., core barrel vibration and fuel assembly vibration, through analysing the ex-core neutron noise has long been conducted effectively at pressurized water reactors (PWRs). The in-core fuel assembly vibrations contribute significantly to the ex-core detector signals in a wide range of frequency of 0.1–50 Hz [1, 2]. Analysis of measurement data indicated that the greatest change in the ex-core neutron noise occurred in the frequency range of 5–10 Hz which corresponds to the frequency of core barrel vibration (beam mode) and the second mode of fuel assembly vibration [2]. Numerical simulation remains a challenge to reproduce and interpret the measurement data. In modelling of core barrel motion, the vibration of the core is usually considered as the vibration of a solid block, and hence, the contribution of the vibration of individual fuel assembly is not taken into account. This may lead to some uncertainty in the result. One of the limitations of the ex-core noise calculation is that the ex-core detectors (ion chambers) are actually located far from the reactor vessel, while core physics and kinetics codes do not allow simulating further than the outer reflector vessel. Therefore, in the simulation, it is assumed that the ex-core detectors are located at the outer reflector region, and then, the behaviour of the noise at the outer reflector is used to represent that of the ex-core noise.

In the measurement of neutron noise at the Ringhals pressurized water reactors (PWRs), it was found that the amplitude of the peak in the ex-core neutron auto power spectral density (APSD), corresponding to the frequency of the beam mode vibrations of the core barrel, increases during the cycle, but returns to the initial value after refuelling at the beginning of the next cycle [3]. The reasons for this behaviour are not clearly understood. The scaling factor between the core barrel displacement and the normalized neutron noise during burnup was investigated with a simple 1D model of an PWR core using the noise simulator CORE SIM [4]. However, no increase in the normalized noise was found with constant vibration amplitude of the core barrel during the cycle, though such kind of investigation of the core barrel motion should be conducted further with a more realistic model [5]. Analysis of recent measurement at Ringhals PWRs recognized that the 8-Hz peak in the ex-core spectra, corresponding to the beam mode, consists of two peaks close to each other in frequency [3, 6]. The two peaks have different origins and different time evolution during the cycle. The peak closer to 7 Hz is induced by the core barrel vibrations, and its amplitude does not change significantly during the cycle. The peak close to 8 Hz is due to the individual fuel assembly vibrations, and it is the amplitude of the peak corresponding to this frequency which increases monotonically during the cycle [6]. A hypothesis suggested by Sweeney et al. [2] is that vibrations of individual fuel assemblies cause the ex-core noise increase during the cycle due to the effect of fuel burnup and the change in soluble boron concentration.

A previous work investigated the possibility to confirm the validity of this statement by Ref. [2] through calculations of the neutron noise in a realistic PWR core model [7]. An extensive survey was performed by calculating the noise induced by individual fuel assemblies at various core positions, various vibration trajectories, and models of the perturbation induced by the vibrations, as a function of the burnup. Two possible models were assumed regarding the amplitude of the displacement of the vibrating assembly. The first one corresponds to the vibration with a large amplitude, i.e. the trajectory of the vibrating assembly overlaps with the location of its neighbouring assembly. The second one corresponds to the vibrations with a smaller amplitude as compared to the first model, i.e., the assembly just vibrating within its location without overlapping other locations. The APSD of the normalized noise at the four ex-core detectors, which for simplicity are assumed to be located in the outer periphery of the reflector, is evaluated at the beginning, middle, and end of cycle in order to investigate the effect of fuel burnup. The effect of the vibrations of individual fuel assemblies is investigated separately. The results of these calculations do not show a universal increase in the amplitude for all fuel positions and for both vibration types, only for peripheral fuel assembly positions and with the noise source model for small-amplitude vibrations. In the case of a group of fuel assemblies vibrating simultaneously, the effect of peripheral assembly dominates the ex-core noise and a universal monotonic increase in the noise amplitude was found with the model of small-amplitude vibrations.

To confirm the conclusions of the previous work [7], similar calculations in several other reactor cores would be beneficial. In the present paper, an extension work has been carried out to extend the method and the calculation models to the Ringhals-4 PWR core and compared with the results obtained with the Ringhals-3 PWR core. The two PWR cores have similar configurations but different fuel arrangement. Since the burnup effect could also be influenced by the fuel loading pattern, the calculation of the ex-core neutron noise based on the two different core loading patterns is considerably beneficial. In the numerical calculations, we focus on the stochastic vibration of individual fuel assemblies in one octant core and evaluate the APSD of the closest ex-core detector. The APSD of ex-core noise induced by individual fuel assembly vibrations and also the simultaneous vibrations of a group of fuel assemblies distributed evenly throughout the core have been investigated.

The paper is organized as follows. Section 2 starts with a brief description of the principle of the frequency-dependent neutron noise equations in two-group diffusion theory to be solved in a neutron simulator CORE SIM. Then, the description of the reactor cores including the kinetic parameters of the core and the calculation method including the modelling of the fuel vibration is presented in Sect. 3. In order to determine the noise source in case of the fuel assembly vibration, two models of stochastic vibrations regarding the amplitude of the displacement of the vibrating fuel assembly were assumed. The first one corresponds to a large displacement and the second one corresponds to a small displacement of the vibrating fuel assembly. Numerical results and discussion on the effect of in-core fuel vibrations on the APSD of the ex-core detector during burnup in comparison with the two PWR cores are presented in Sect. 4. Finally, some concluding remarks are given in Sect. 5.

2 The neutron noise equations in two-group theory

The noise equation in two-group diffusion theory is derived from the time-dependent diffusion equation by splitting the time-dependent quantities into mean values and fluctuations,

$$X(\boldsymbol{r},t) = X(\boldsymbol{r}) + \delta X(\boldsymbol{r},t), \tag{1}$$

removing the static parts, performing a Fourier transform, eliminating the delayed neutron precursors, and neglecting the second-order terms (linear theory). The first-order noise equation is written as follows [4, 8]

$$\left[\nabla.\overline{\overline{D}}(\boldsymbol{r})\nabla + \overline{\overline{\Sigma}}_{\mathrm{dyn}}(\boldsymbol{r},\omega)\right] \times \begin{bmatrix}\delta\phi_1(\boldsymbol{r},\omega)\\ \delta\phi_2(\boldsymbol{r},\omega)\end{bmatrix} = \begin{bmatrix}S_1(\boldsymbol{r},\omega)\\ S_2(\boldsymbol{r},\omega)\end{bmatrix} \tag{2}$$

where

$$\overline{\overline{D}}(\boldsymbol{r}) = \begin{bmatrix}D_1(\boldsymbol{r}) & 0\\ 0 & D_2(\boldsymbol{r})\end{bmatrix}, \tag{3}$$

$$\overline{\overline{\Sigma}}_{\mathrm{dyn}}(\boldsymbol{r},\omega) = \begin{bmatrix}-\Sigma_1(\boldsymbol{r},\omega) & \nu\Sigma_{\mathrm{f2}}(\boldsymbol{r},\omega)\\ \Sigma_{\mathrm{rem}}(\boldsymbol{r}) & -\Sigma_{\mathrm{a2}}(\boldsymbol{r},\omega)\end{bmatrix}, \tag{4}$$

$$\Sigma_1(\boldsymbol{r},\omega) = \Sigma_{\mathrm{a1}}(\boldsymbol{r},\omega) + \Sigma_{\mathrm{rem}}(\boldsymbol{r}) - \nu\Sigma_{\mathrm{f1}}(\boldsymbol{r},\omega), \tag{5}$$

$$\nu\Sigma_{\mathrm{f1,2}}(\boldsymbol{r},\omega) = \frac{\nu\Sigma_{\mathrm{f1,2}}(\boldsymbol{r})}{k_{\mathrm{eff}}}\left(1 - \frac{i\omega\beta_{\mathrm{eff}}}{i\omega+\lambda}\right), \tag{6}$$

$$\Sigma_{\mathrm{a1,2}}(\boldsymbol{r},\omega) = \Sigma_{\mathrm{a1,2}}(\boldsymbol{r},\omega) + \frac{i\omega}{\upsilon_{1,2}}. \tag{7}$$

In the above equations,

$\delta\phi_g$ is the neutron noise in group g with $g = 1,2$ representing for the fast and thermal energy groups, respectively; D_g is the diffusion coefficient in group g; $\Sigma_{\mathrm{a,g}}$ is the macroscopic absorption cross section in group g; $\nu\Sigma_{\mathrm{f,g}}$ is the macroscopic fission cross section in group g times the mean number of neutrons per fission; Σ_{rem} is the macroscopic scattering cross section from the fast group to the thermal group; k_{eff} is the effective multiplication factor; β_{eff} is the total delayed neutron fraction; λ is the decay constant; υ_g is the velocity of neutron in group g; ω is the frequency.

The right-hand side vector in Eq. (2) represents the noise source in the fast and thermal groups. The noise source can be modelled through the fluctuations of macroscopic cross sections as a result of mechanical or thermal processes in the core, such as absorption perturbation, core barrel vibrations, and fuel assembly vibrations and is written as follows

$$\begin{bmatrix}S_1(\boldsymbol{r},\omega)\\ S_2(\boldsymbol{r},\omega)\end{bmatrix} = \overline{\phi}_{\mathrm{rem}}(\boldsymbol{r})\delta\Sigma_{\mathrm{rem}}(\boldsymbol{r}) + \overline{\overline{\phi}}_a(\boldsymbol{r})\begin{bmatrix}\delta\Sigma_{a1}(\boldsymbol{r},\omega)\\ \delta\Sigma_{a2}(\boldsymbol{r},\omega)\end{bmatrix} + \overline{\overline{\phi}}_f(\boldsymbol{r})\begin{bmatrix}\delta\nu\Sigma_{f1}(\boldsymbol{r},\omega)\\ \delta\nu\Sigma_{f2}(\boldsymbol{r},\omega)\end{bmatrix} \tag{8}$$

where the δX_i, $i = \{\text{ rem, a1, a2, f1, f2 }\}$ stand for the fluctuations of the macroscopic cross sections, corresponding to the actual perturbation (in this case to the fuel assembly movement), and

$$\overline{\phi}_{\mathrm{rem}}(\boldsymbol{r}) = \begin{bmatrix}\phi_1(\boldsymbol{r})\\ -\phi_1(\boldsymbol{r})\end{bmatrix}, \tag{9}$$

$$\overline{\overline{\phi}}_{\mathrm{a}}(\boldsymbol{r}) = \begin{bmatrix}\phi_1(\boldsymbol{r}) & 0\\ 0 & \phi_2(\boldsymbol{r})\end{bmatrix}, \tag{10}$$

and

$$\overline{\overline{\phi}}_{\mathrm{f}}(\boldsymbol{r}) = \begin{bmatrix}-\phi_1(\boldsymbol{r}) & -\phi_2(\boldsymbol{r})\\ 0 & 0\end{bmatrix}. \tag{11}$$

ϕ_g is the neutron flux in group g.

In order to solve the neutron noise equation, the noise source in Eq. (8) must be determined via the fluctuations of the macroscopic cross sections of a specific scenario (in this case, the displacement of vibrating fuel assemblies). For pendular vibrations of a fuel assembly, the displacement function is a two-component vector $(\varepsilon_x, \varepsilon_y)$, of which the components represent the displacement of the vibrating component around the equilibrium position in the x- and y-directions, respectively, according to the (x, y)-coordinates as displayed in Fig. 1.

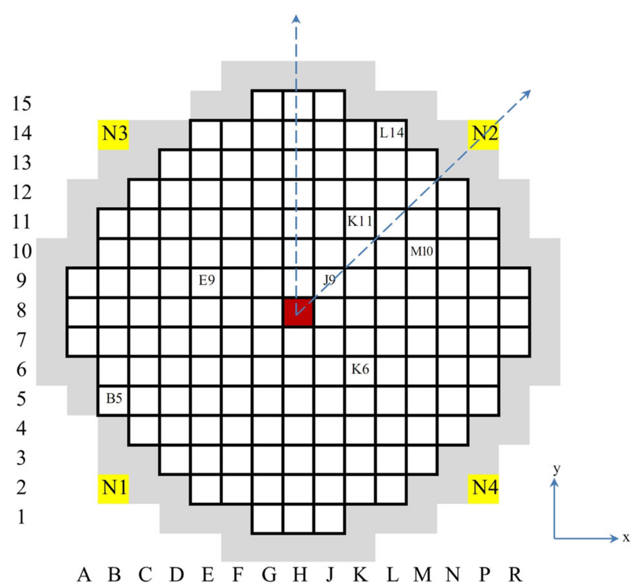

Fig. 1 Core configuration of the PWR core with four ex-core detectors N1, N2, N3, and N4

For the vibrations in both x- and y-directions, the displacements of a fuel assembly can be described by the two displacement functions ε_x and ε_y as identically distributed, independent random processes in a coordinate system [9]. The normalized total thermal noise itself is given as a linear combination of the noise induced by the vibration in x- and y-directions as

$$\widetilde{\delta\phi}_2 = \delta\phi_2/\phi_2 = A_x\varepsilon_x(\omega) + A_y\varepsilon_y(\omega), \tag{12}$$

where $|A_x|$ and $|A_y|$ refer to the scaling factors of the ex-core noise with the fuel assembly vibration in x- or y-direction, respectively. The scaling factor and the phase of the noise are parameters which can be calculated using the noise simulator. In noise analysis, the gain and phase of APSD and cross power spectral density (CPSD) are used rather than the absolute values and the phase of the detector signals.

$$\begin{aligned} APSD_{\delta\phi_2} = & |A_x|^2 APSD_{\varepsilon_x} + |A_y|^2 APSD_{\varepsilon_y} \\ & + 2A_xA_y \text{ Re } \{CPSD_{\varepsilon_x\varepsilon_y}\}. \end{aligned} \tag{13}$$

In the case of random 2D vibrations, following a model of random force acting on the assembly surface suggested by Ref. [9], one obtains that for the case of isotropic vibrations ($|\varepsilon_x| = |\varepsilon_y|$), $CPSD_{\varepsilon_x\varepsilon_y} = 0$, and the APSD of the ex-core noise can be simply calculated by

$$APSD_{\delta\phi_2} = |A_x|^2 + |A_y|^2. \tag{14}$$

3 PWR cores and modelling of fuel assembly vibration

Numerical calculations for investigating the effect of fuel burnup on the APSD of the ex-core detector noise induced by the vibrations of fuel assemblies have been performed based on the two PWR core models corresponding to the Ringhals-3 and the Ringhals-4 reactors at cycle 15 and referred to as Core 1 and Core 2, respectively. The two PWR cores have a similar configuration as illustrated in Fig. 1. Four ex-core detectors are assumed to be located at the outer periphery of the reflector to investigate the ex-core noise. It is the fact that the ex-core detectors are placed far from the core barrel. However, due to the limitation of the neutron noise simulator CORE SIM which does not allow simulating further than the reflector region, it is assumed that the neutron noise at the reflector region could represent the behaviour of the noise at the actual ex-core detector locations. To investigate the burnup effect, calculations have been performed at three burnup steps: the beginning (BOC), middle (MOC), and end of cycle (EOC). The cross-sectional data and the kinetic parameters for the 2D models of the two PWR cores are taken from previous works [7, 10]. The kinetics parameters of the two cores are given in Table 1. The difference in fuel arrangement of the two cores is illustrated by the difference in the neutron flux distribution and its evolution with burnup in the two cores. Figures 2 and 3 display the fast and thermal neutron flux distributions along the core diameter at the three burnup steps.

In the present work, similar assumption of fuel assembly vibration and the 2D simulation model as the one used in the previous work is considered [7]. One of the limitations of the 2D model is that the vibration of fuel assembly is equivalent with pendular vibration in a 3D model but not a second bending mode of fuel assembly. Since in the present work we aim at investigating only the neutron noise at ex-core detectors located on the same axial plane, the 2D model is considerably adequate. Further improvement of the simulation of second bending mode with a more realistic 3D model is being planned in the future work. In fact, there is not much information regarding the displacement of fuel assembly. According to Ref. [11], the displacement of a vibrating assembly is in the sub-millimetre range. However, in the numerical calculation, the mesh size of about 2.5 cm is used, which is too large compared to the displacement of a realistic vibration. In order to simulate the vibration with a flexible small displacement, the spatial discretization was reorganized for the vibrating assembly by adding a very fine mesh around its border while keeping the same mesh size for all other assemblies. The size of the additional meshes could be defined as small as the amplitude of the displacement. Such an approach can be applied in a 2D simulation, where a very small displacement from vibrations can be simulated while the total number of meshes does not increase appreciably and therefore no significant numerical difficulty arises from the increase in different mesh sizes. Thus, the 2D version of CORE SIM was modified to handle additional fine meshes around the vibrating assembly for the purpose of this work.

The displacement of a fuel assembly is modelled by shifting materials for a very fine mesh around its equilibrium location. It means that the fluctuations in all types of cross sections are taken into account in the noise source. If the displacement of a vibrating assembly is within the sub-millimetre range, the fluctuation of the cross sections is associated with the vibrating assembly only. However, if the amplitude of the displacement increases up to about 4 mm or more, a portion of the vibrating assembly will extend to the location of the neighbouring one, which hence also needs to be displaced. In this case, the cross-sectional fluctuations are associated with the displacement of two or more neighbouring assemblies. Therefore, two

Table 1 Kinetic parameters of the two PWR cores

Burnup	Core 1					Core 2				
	k_{eff}	β_{eff} (pcm)	λ (s^{-1})	v_1 (cm/s)	v_2 (cm/s)	k_{eff}	β_{eff} (pcm)	λ (s^{-1})	v_1 (cm/s)	v_2 (cm/s)
BOC	1.00103	596.7	0.084356	1.78631×10^7	4.17195×10^5	1.00036	595.7	0.084499	1.78631×10^7	4.15325×10^5
MOC	1.00170	551.1	0.087121	1.81783×10^7	4.13994×10^5	1.00104	554.3	0.086985	1.81434×10^7	4.15859×10^5
EOC	1.00062	520.2	0.089139	1.81658×10^7	4.04119×10^5	1.01043	523.9	0.089071	1.81459×10^7	4.03678×10^5

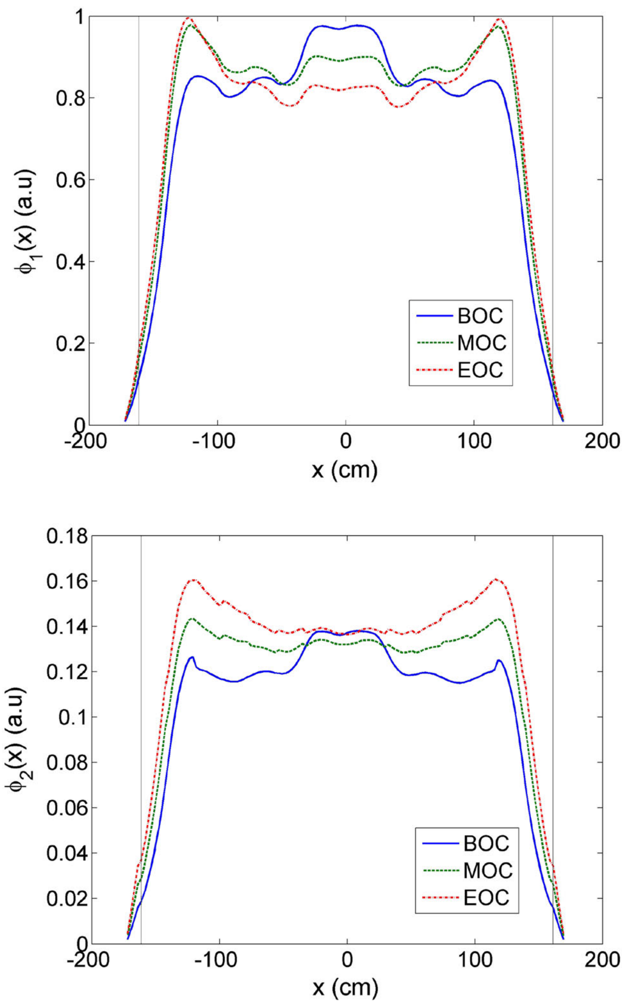

Fig. 2 Neutron fluxes in fast (*up*) and thermal (*down*) groups across the diameter of Core 1

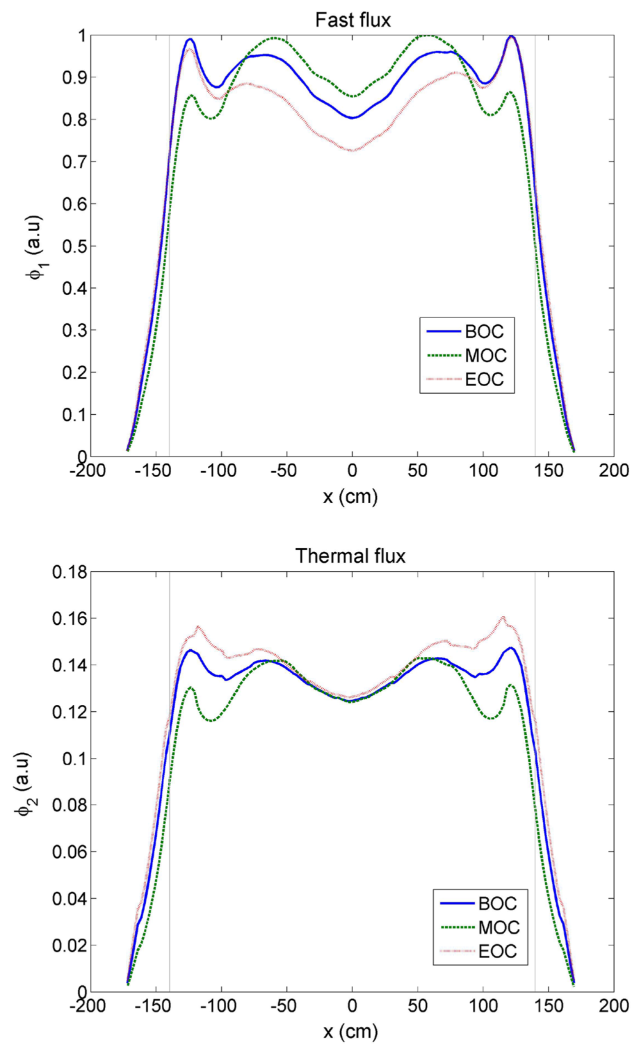

Fig. 3 Neutron fluxes in fast (*up*) and thermal (down) groups across the diameter of Core 2

possible vibration models regarding the amplitude of the displacement of vibrating assembly are as follows:

- *Model 1 (large displacement)* The displacement of the vibrating assembly overlaps the location of the neighbouring assemblies. Therefore, the real parts of the macroscopic cross-sectional fluctuations (not given here explicitly) in the meshes describing the displacement of the vibrating assembly around its equilibrium location are assumed to be the deviations of the cross sections of the vibrating assembly and its neighbouring one. In reality, this model assumes the collective vibrations of neighbouring fuel assemblies, to ensure that no space is occupied simultaneously by more than one fuel assembly.

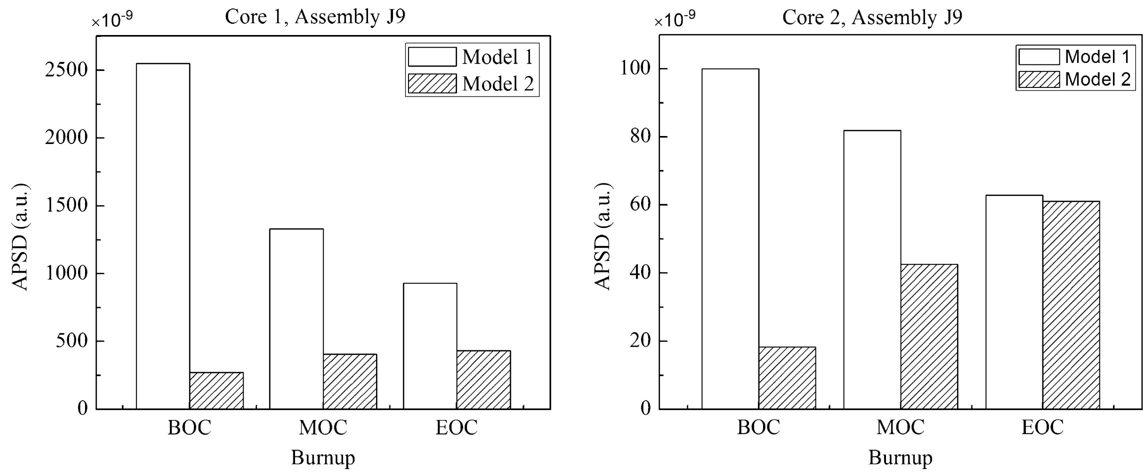

Fig. 4 APSD of detector N2 induced by vibration of assembly J9 in Core 1 (*left*) and Core 2 (*right*), respectively

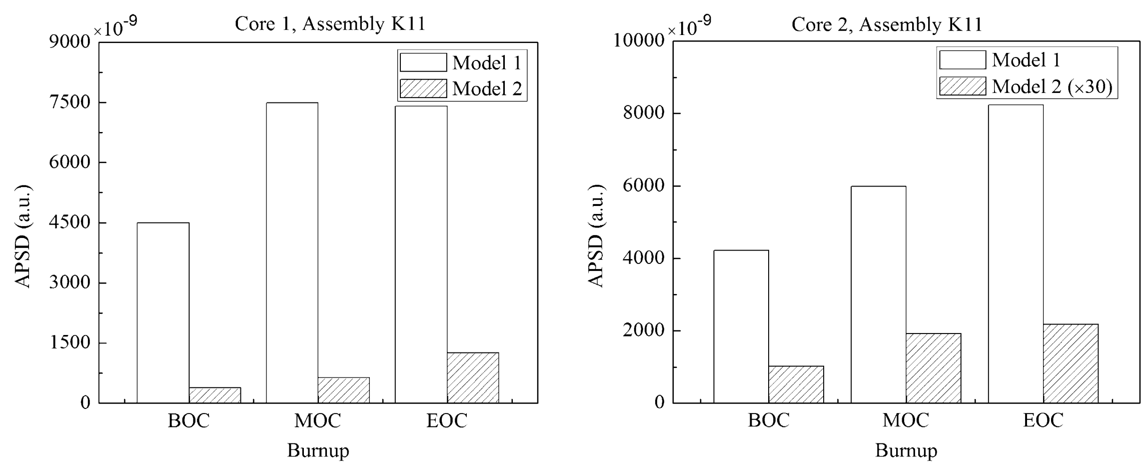

Fig. 5 APSD of detector N2 induced by vibration of assembly K11 in Core 1 (*left*) and Core 2 (*right*), respectively. The values in Model 2 of Core 2 in the lower right figure are multiplied by a factor of 30 for the sake of clarity

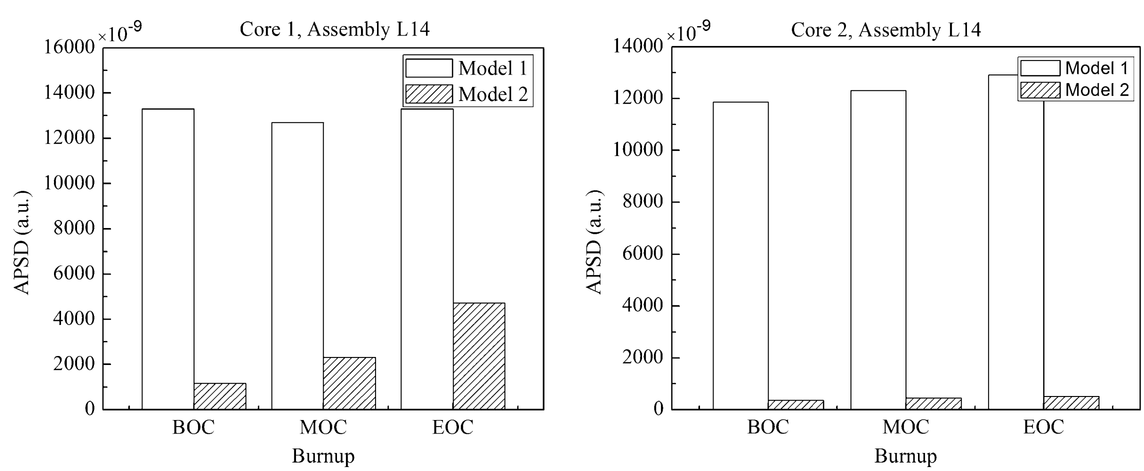

Fig. 6 APSD of detector N2 induced by vibration of assembly L14 in Core 1 (*left*) and Core 2 (*right*), respectively

Table 2 APSD ($\times 10^9$) (a.u) of detector N2 induced by stochastic fuel vibrations in Core 1 at BOC, MOC, and EOC, respectively

Assembly	Model 1			Model 2		
	BOC	MOC	EOC	BOC	MOC	EOC
H8	166	69.7	44.3	0.4	12.8	29.9
H9	1290	433	261	646	324	321
H10	149	66.4	46.8	151	184	238
H11	108	60.2	43.3	334	339	447
H12	2420	1820	1530	293	406	539
H13	73.1	247	245	37	53.2	99.2
H14	2460	2610	2210	4390	3840	3490
H15	979	1650	1750	671	1590	2980
J9	2550	1330	930	272	406	434
J10	201	259	195	53.2	253	378
J11	1580	1270	1010	185	575	864
J12	944	2190	2210	256	466	807
J13	897	1190	1250	120	131	258
J14	339	71.2	137	8700	7760	7610
J15	1160	1930	2650	373	996	2300
K10	1740	2000	1610	759	1770	2110
K11	4490	7490	7410	377	634	1260
K12	1370	1570	1360	2430	3090	4260
K13	1080	1020	1060	183	170	194
K14	2870	3600	5070	19,600	14,300	13,700
L11	151	217	322	4770	4680	6290
L12	5470	1750	1670	358	507	987
L13	997	767	636	4640	3580	3460
L14	13,300	12,700	13,300	1160	2310	4730
M12	289	94.4	97.1	1370	691	374
M13	18,700	13,800	10,300	981	1510	2330

- *Model 2 (small displacement)* The displacement of the vibrating assembly is within its own location, i.e. the vibration of fuel material does not overlap with the neighbouring assemblies. The real parts of the macroscopic cross-sectional fluctuations in the meshes describing the displacement of the vibrating assembly are assumed to be the deviations of the cross sections of fuel material and water. In this case, no collective motion of the neighbouring assemblies is required.

Regarding the vibration model, the previous work performed a systematic survey with both the directional trajectory and the stochastic trajectory vibration models [7] in which various directional vibrations in $x-$, $y-$ or both $x-$ and $y-$ directions have been considered. However, since the signals obtained in measurement provide no information about the directional vibration, it is rather a random process of the vibration. This means that the stochastic trajectory vibration model can be considered as a more realistic model. Therefore, in this work, we focus only on the stochastic trajectory vibration model and compare the results obtained with the two reactor cores.

4 Calculation of the neutron noise induced by fuel assembly vibrations

Calculations were performed at three burnup steps based on a full core model for investigating the dependence of cycle burnup on the APSD of the ex-core noise. In all calculations, the frequency of 8 Hz was selected since it corresponds to the frequency of fuel assembly vibrations in the core [3, 6]. To evaluate the hypothesis that the APSD of the ex-core noise increases throughout the cycle, the calculations of the ex-core scaling factors and the phases have been performed for various locations of the vibrating assemblies in the core using CORE SIM. From this, the APSD of the noise can be obtained as in Eq. (14). Due to the symmetrical properties of the core, it is not necessary to perform the calculation for the vibration of every single assembly in the core. The detailed calculations were performed only for the individual vibrations of fuel assemblies located in 1/8th of the core close to detector N2 as shown in Fig. 1. This is because of the fact that the noise induced by vibrations of a fuel assembly in any given octant of the core, one can find the similar noise induced by another fuel assembly in the corresponding position of another octant by a rotation transformation. Hence, for individual assembly vibration, it is sufficient to consider vibrating assemblies located in one octant to map the possible tendencies as functions of the fuel assembly position. In the technical discussion, we focus only on the behaviour of the neutron noise at the detector close to the vibrating assembly. However, to evaluate the contribution of vibrating assemblies at different locations on the ex-core APSD, calculations were also performed with the assumption of a group of assemblies distributed evenly throughout the core vibrating simultaneously and independently.

It is assumed that the core can be classified into three fuel regions from the centre to the periphery: the central region, the middle region, and the outer region. Figure 4 displays the change in the APSD of detector N2 during burnup with the stochastic vibrations of assembly J9 located in the central fuel region. One can see that the variation trends of the APSDs in the two models regarding the displacement amplitude are different. The APSD decreases with burnup in Model 1 (large displacement) but increases in Model 2 (small displacement). The same behaviour was found in both PWR cores as shown in Fig. 4. Figures 5 and 6 show the same quantities with that in Fig. 4 but for the stochastic vibrations of assemblies K11 and L14 located in the middle fuel region and the outer periphery, respectively. In the vibration of assembly K11, except the

Table 3 APSD ($\times 10^9$) (a.u.) of detector N2 induced by stochastic fuel vibrations in Core 2 at BOC, MOC, and EOC, respectively

Assembly	Model 1			Model 2		
	BOC	MOC	EOC	BOC	MOC	EOC
H8	12.5	6.5	4.5	15.5	8.7	12.0
H9	420.7	224.3	146.4	4.2	1.7	1.3
H10	108.9	53.5	31.9	157	61.2	86.6
H11	90.6	57.1	44.4	48.7	22.5	45.4
H12	622.2	488.9	436.7	413.3	211.1	241.0
H13	305.3	358.8	251.1	10.4	19.1	17.7
H14	464.5	518.6	262.4	1575.9	952.1	781.6
H15	1438.1	2406.8	2342.7	18.0	16.2	1.3
J9	99.9	81.9	62.9	18.3	42.6	61.1
J10	86.5	44.7	8.9	32.2	18.1	44.5
J11	1783.4	1581.6	1320.7	51.3	265.9	350.5
J12	581.8	1557.6	2208.2	15.7	4.3	9.3
J13	1305.3	1635.0	1966.8	36.6	86.9	126.1
J14	117.8	32.9	171.6	1945.9	1043.8	1278.9
J15	982.6	1560.3	2414.1	23.4	61.4	162.7
K10	1407.6	1635.1	1302.9	421.5	627.9	660.4
K11	4218.1	5981.6	8241.7	3.4	64	72.6
K12	738.7	820.8	761.9	752.5	687.0	1031.0
K13	1047.3	1071.3	1175.9	51.4	23.6	22.1
K14	3082.9	3716.3	5739.4	4019.1	1590.5	1058.2
L11	167.5	166.6	339.2	1672.2	979.6	1648.8
L12	6718.8	3405.8	2362.6	581.6	147.6	16.4
L13	1131.0	920.6	812.7	1487.2	904.4	939.6
L14	11,860.9	12,312.6	12,916.0	361.9	447.8	510.5
M12	274	175	180	90	97	84
M13	17,831	13,849	9912	2148	2335	2254

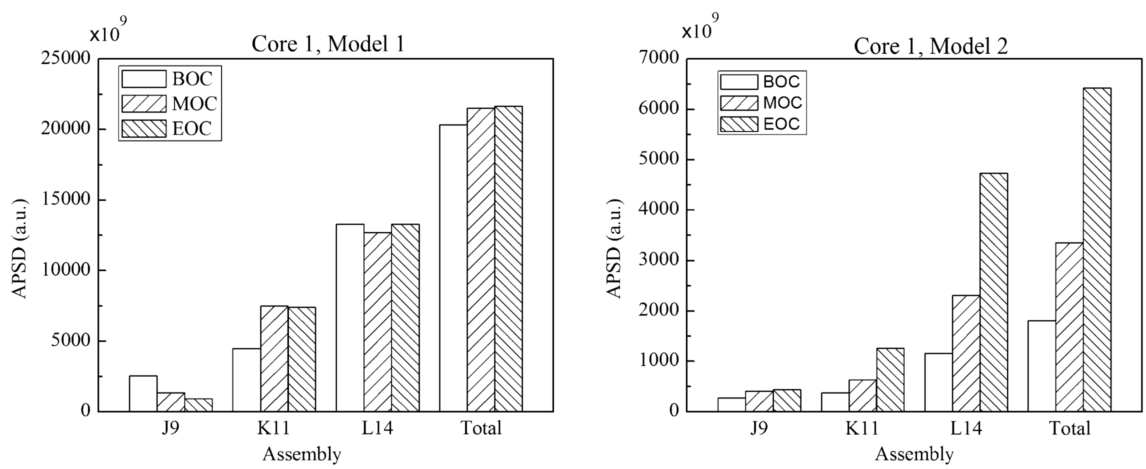

Fig. 7 APSD of detector N2 induced by simultaneous vibrations of assemblies J9, K11, and L14 in Core 1

APSD in Model 1 in Core 1 which makes a peak at MOC, the APSD of detector N2 increases with burnup monotonically. For the vibration of assembly L14 which is located in the periphery of the fuel region, Fig. 6 illustrates the increase in the APSD of detector N2 in both of the two models of the noise source, except that of N2 in Model 1 on Core 1 which has a dip at MOC. The vibrations of the three assemblies located in the three different fuel regions in the

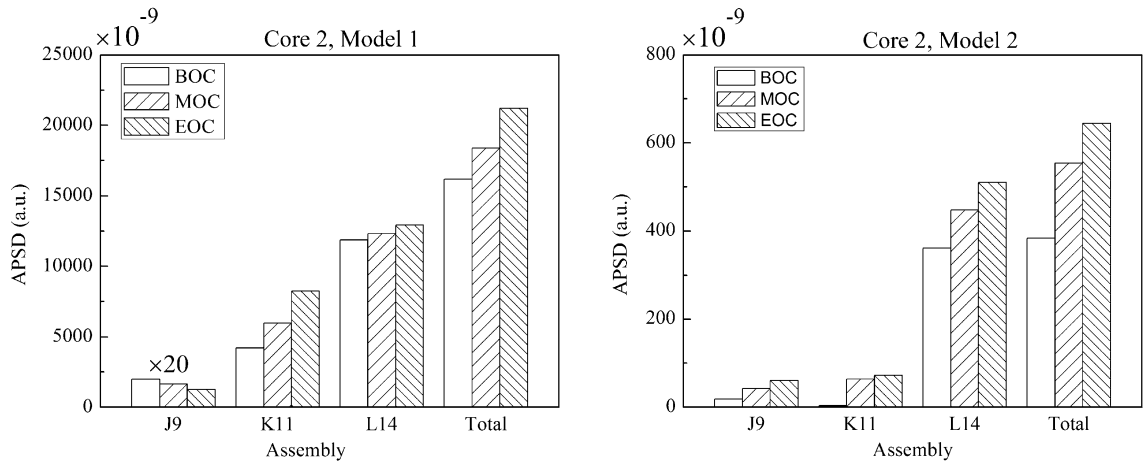

Fig. 8 APSD of detector N2 induced by simultaneous vibrations of assemblies J9, K11, and L14 in Core 2. The values of assembly J9 in the left figure are multiplied by a factor of 20 for the sake of clarity

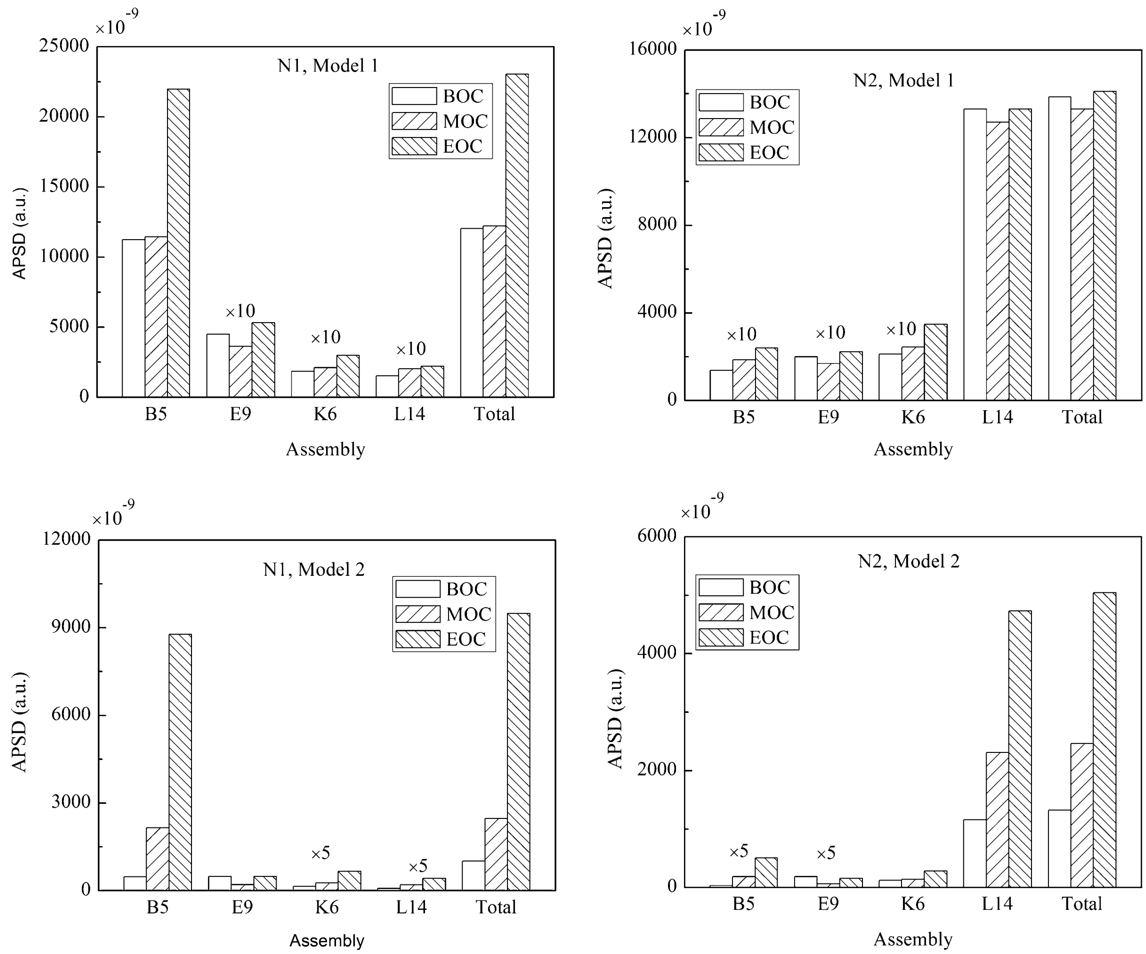

Fig. 9 APSD of detector N2 induced by simultaneous vibrations of assemblies B5, E9, K6, and L14 in Core 1. The values of some assemblies are multiplied by a factor for the sake of clarity

core show different behaviours of the APSDs of the ex-core noise, i.e., no general monotonic variation in the APSD of the ex-core detector noise is noticeable. However, for the three assemblies, the tendency of the monotonic increase in APSD occurred with Model 2 (small displacement) in both cores, which was also considered as a more realistic model

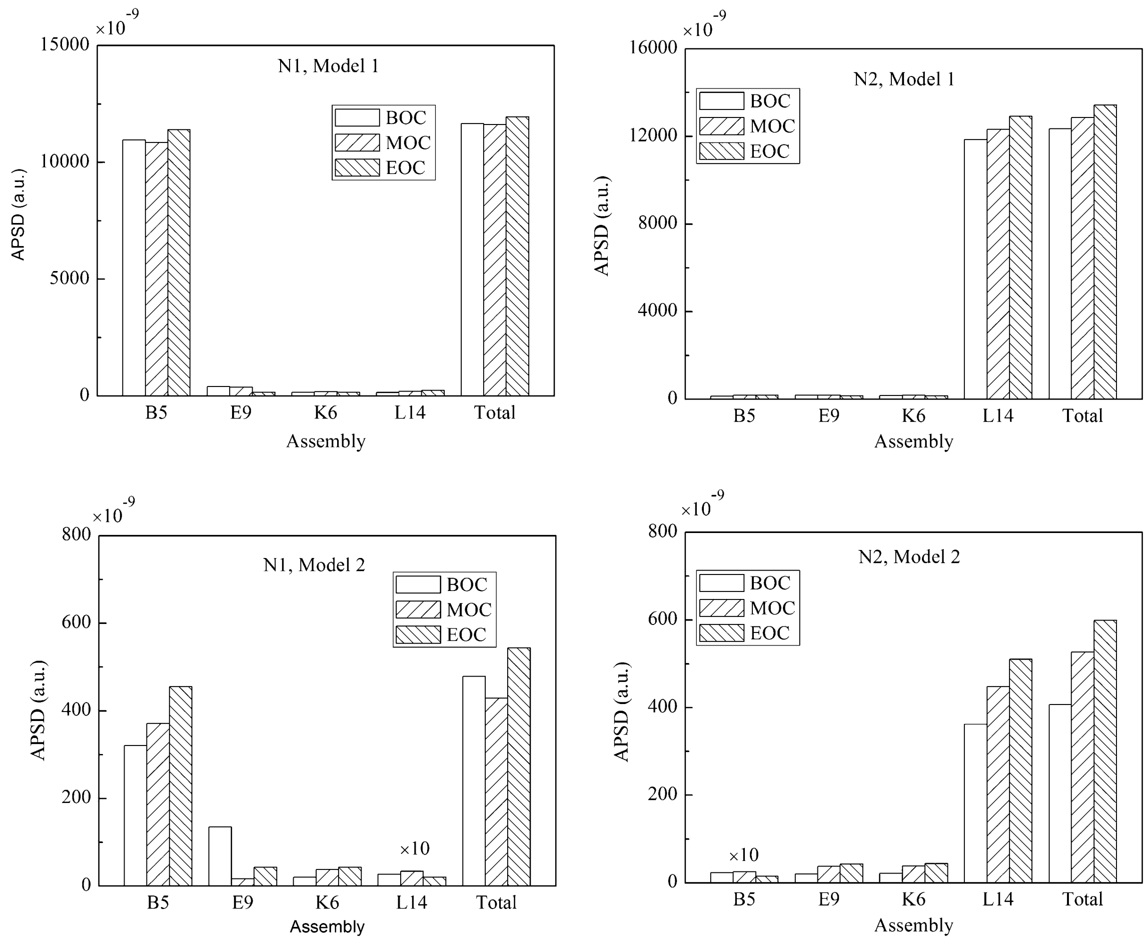

Fig. 10 APSD of detector N2 induced by simultaneous vibrations of assemblies B5, E9, K6, and L14 in Core 2. The values of some assemblies are multiplied by a factor for the sake of clarity

of in-core fuel assembly vibration. In order to reach a conclusion, further survey has been conducted with various locations of fuel assemblies in the cores.

Tables 2 and 3 show the APSDs of the ex-core detector N2 induced by individual stochastic vibrations of all fuel assemblies located in the 1/8th of the two core models, respectively. One can see that no general monotonic change in the APSDs during burnup cycle was found. Again, the two models regarding the displacement of vibrations result in different behaviour of the APSD of the ex-core noise. In Model 1, the constant decrease in APSD is found in the vibrations of assemblies located around the centre of the core, e.g. assemblies H8 to H12 and assemblies J9 to J11, whereas in Model 2, the effect of the central assemblies is not monotonic. Both the increase and the decrease in the APSD were found. Similarly, no general monotonic variation in the APSD was found with the vibrations of fuel assemblies located in the middle of the core in both PWR cores as shown in Tables 2 and 3.

Comparing the effect of different assemblies, the contribution of the peripheral assemblies to the APSD of the ex-core noise is greater than the central assemblies as shown in Tables 2 and 3. This means that the ex-core detectors are more sensitive to the vibrations of the fuel assemblies in the periphery. In order to evaluate the contribution of vibrating assemblies at different locations in the core on the ex-core APSD, calculations were performed with the assumption of a group of assemblies vibrating simultaneously and independently. Figures 7 and 8 display the APSD of detector N2 induced by simultaneous vibrations of three assemblies J9, K11, and L14 in the Core 1 and Core 2, respectively. The three assemblies are located in the octant core close to detector N2. One can see that the total APSD of detector N2 is dominated by the vibration of assembly L14, which is located at the periphery close to detector N2. The APSD of detector N2 increases monotonically with burnup in both cores and in both Model 1 and Model 2 of the displacement amplitude. Figure 9 illustrates the APSDs of detectors N1 and N2 induced by simultaneous vibrations of assemblies B5, E9, K6, and L14 in Core 1. Figure 10 displays the same quantities as in Fig. 9 but for the Core 2. It is noted that the two detectors N1 and N2 have diagonally opposite positions across the core diameter, while B5 and L14 are located in the peripheries close to N1 and N2, respectively (see Fig. 1). The assemblies E9 and K6 are located in the middle fuel region. Again, it is found that the total APSD of N1 is dominated by the vibration of the closest assembly B5, while the contribution of the other assemblies located far from N1 is small, as shown in Fig. 9. Conversely, the APSD of N2 is dominated by the vibration of the closest assembly L14. This behaviour is similar in both Core 1 and Core 2.

Thus, we will focus on the effect of some fuel assemblies located at the outer periphery, e.g. assemblies H15, J15, K14, L14, and M13. In Core 1, Table 2 shows that out of the five assemblies, the vibrations of three assemblies H15, J15, and K14 result in the increase in the APSD of detector N2 in Model 1. In Model 2, the trend of the increase in the APSD was found at four of the five peripheral assemblies (H15, J15, L14, and M13). In Core 2, the increase in the APSD of detector N2 was found at 3 peripheral assemblies in Model 1 (J15, K14, and L14) and two in Model 2 (J15 and L14) as shown in Table 3. This means that the trend of increase in the APSD with burnup is predominantly with the peripheral assemblies. However, this tendency is also not monotonic with every peripheral assembly.

5 Conclusion

Extension of the calculation model and method used in a previous work [7] has been conducted for investigating the effect of the vibrations of individual fuel assemblies on the ex-core detector noise as a function of burnup and for confirming the results obtained with Ringhals-3 PWR core. The calculations were based on the 2D models of the two PWR cores which have different fuel arrangements represented by the different neutron flux distribution and its evolution with burnup. Stochastic vibration of individual assemblies was assumed with the two models regarding the displacement amplitude of the vibrating assemblies. Analysis of the results shows that no general monotonic variation in the APSD of the ex-core detector noise with burnup was found in both cores. In the case of simultaneous vibrations of several assemblies distributed throughout the core, the noise induced by the peripheral assemblies dominates the ex-core detector signals. This behaviour is similar in both cores. The trend of the noise amplitude increase with burnup is predominant for fuel assemblies located at the periphery of the two cores. The result is supportive of the conclusions based on Ringhals-3 core in the previous work.

Since there are still a number of limitations of the simulation models used in the present works, further improvement and investigation should be conducted in order to assess more concrete conclusion regarding the questions on the fuel assembly vibration.

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