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Spin polarization and production rate studies of surface muons in a novel solenoid capture system based on CSNS

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Abstract A novel surface muon capture system with a large acceptance was proposed based on the China spallation neutron source (CSNS). This system was designed using a superconducting solenoid where a long graphite target was put inside it. Firstly, the spin polarization evolution was studied in a constant uniform magnetic field. As the magnetic field can interact with the spin of the surface muon, both the spin polarization and production rate of the surface muons collected by the new capture system were calculated by the G4beamline. Simulation results showed that the surface muons could still keep a high spin polarization (>90%) with different magnetic fields (0–10 T), and the larger magnetic field is, the more surface muons can be captured. Finally, the proton phase space, Courant–Snyder parameters, and intensities of surface muons of different beam fractions were given with magnetic fields of 0 and 5 T. The solenoid capture system can focus proton and surface muon beams and collect $\pi^{\pm}$ and $\mu^{\pm}$ particles. It can also provide an intense energetic positron source.

Keywords Surface muon · Muon spin rotation · Spin polarization · Superconducting solenoid · G4beamline

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1 Introduction

The muon acts as a local probe, which is an independent determination of the magnetic moment and magnetic volume fraction. This young nuclear solid-state technique is called μSR technique. μSR means muon spin rotation/relaxation/resonance [1] and intends to emphasize the analogy with Nuclear Magnetic Resonance (NMR). The external magnetic fields are not necessary for μSR measurements. It is a big advantage, compared to NMR, that μSR measurement is allowed to investigate magnetic systems without perturbation. It also has a number of merits in contrast with other nuclear solid-state methods (Nuclear Quadruple Resonance, Mossbauer spectroscopy, and so on): a purely magnetic probe, interstitial probe, being particularly suitable for very weak effects, full polarization in zero field, high sensitivity, and large fluctuation time window (10^{-11}–10^{-5}s) [2]. The principle of μSR technique is described as follows: when the spin-polarized muons stop in the sample, the muon's spin can interact with the local magnetic field where muons stop; then muons decay to positrons with a mean lifetime of 2.2 μs, and these positrons are emitted preferentially along the spin direction of muons due to the parity violating decay. By measuring the spatial and temporal characteristics of the anisotropic distribution of these decay positions, the magnetic information of the sample is determined. Muon sources frequently used by μSR experiments in the world are achieved based on proton accelerators by the decay of pions. According to the positions of the pion decays, the muons can be classified into three kinds: surface μ^+, cloud $\mu^\pm$, and decay $\mu^\pm$. Surface muons are produced by the positive pions, which stop near the target surface. There are no negative surface muons because the negative pions are captured by the target nucleus promptly when π^- appears inside the target before escaping. Surface muons are monochromatic (4.12 MeV, 29.8 MeV/c), highly spin-polarized ($\sim 100\%$), and give a large impetus to μSR technique [3]. Cloud muons and decay muons are obtained by the decay of pions in the free space close to the production target and in flight in the transport channel, respectively. These two latter types of muons have a lower spin polarization than surface muons because of the production of backward muons in the pion center-of-mass frames.

Superconducting solenoids are used in the large-scale muon facilities to provide strong magnetic fields. The Muon Ionization Cooling Experiment (MICE) in FermiLab [4], COherent Muon to Electron Transition (COMET) [5] in J-PARC, and MEG experiment in PSI [6] have completed the solenoid tests for $\mu \rightarrow e + \gamma$ rare decay experiment. These researches need very high-intensity muons, but they are not interested in high spin polarization level. KEK [7] developed a four-superconducting-solenoid muon channel with a solid angle acceptance of 1 sr. μE4 beam line at PSI [8] constructed two normal-conducting solenoids with a solid angle acceptance of $\Omega \sim 135$ mrad. The acceptances of these muon channels are small because they just collect muons from one lateral side of the production target (Fig. 1a). MuSIC at RCNP of Osaka University proposed a large acceptance collection system using a superconducting solenoid capture system which achieved an intense continuous muon beam source [9, 10]. This high-intensity muon beam will be used in various fields: particle physics, nuclear physics, material science, and so on, while μSR technique in material science needs highly spin-polarized muons. MuSIC tested the flux of muons [11], but did not give the muon spin polarization, which is very important for μSR measurements. The novel solenoid capture system was also proposed at CSNS (China Spallation Neuron Source). According to the layout of the High Energy Proton Experiment Area [12], the capture system is at the downstream of the transport system, as shown in Fig. 1b. A long graphite target was put inside the superconducting solenoid in this capture system. It is different from the normal one where the target is outside the collecting system. This capture method was calculated to achieve the intense surface muon beam of two orders more than the normal one [13], but the muon spin polarization level is uncertain, a quantity which is of importance for μSR scientists. In our study, the spin polarization and production rate studies of surface muons of this novel capture system were given based on CSNS.

CSNS provides a good platform for many disciplines, and the effective neutron flux is expected to be 2×10^{16} cm^{-2}s^{-1} [14]. The accelerators of CSNS [15, 16] can also provide an energetic proton beam of 1.6 GeV for the first muon source construction in China. The spin polarization and production rate of surface muons in this capture system were analyzed by the G4beamline [17] (version 2.16). The G4beamline is a particle tracking simulation program based on Geant4 [18, 19], and it is easy and flexible to simulate complex beamlines [20, 21].

2 Muon spin precession in a constant uniform magnetic field

Polarized muons are implanted into materials in μSR measurements, where their polarizations evolve in the local magnetic field until they decay [22]. The basic principle of μSR technique is to measure the muon spin relaxation and rotation in the local field of a sample. In this section, the dynamical evolution of a muon's spin in a constant uniform magnetic field was calculated, and the muon decay was ignored. Figure 2 shows the muon spin precession model in

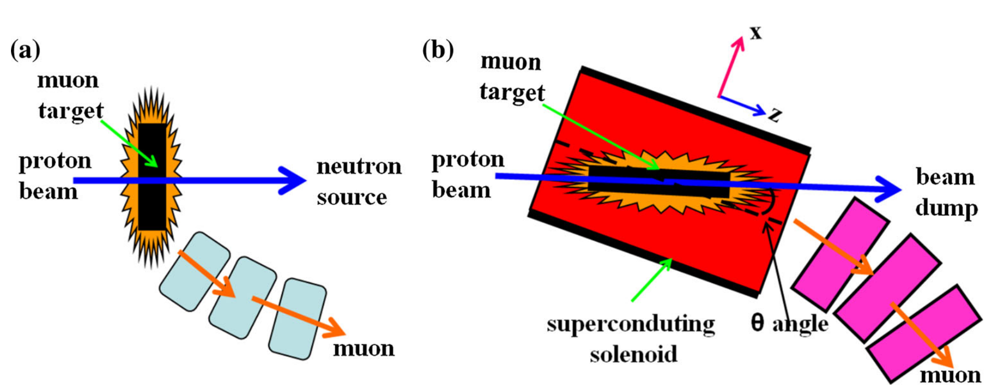

Fig. 1 The normal muon capture system (**a**) and the novel solenoid capture system proposed at CSNS (**b**)

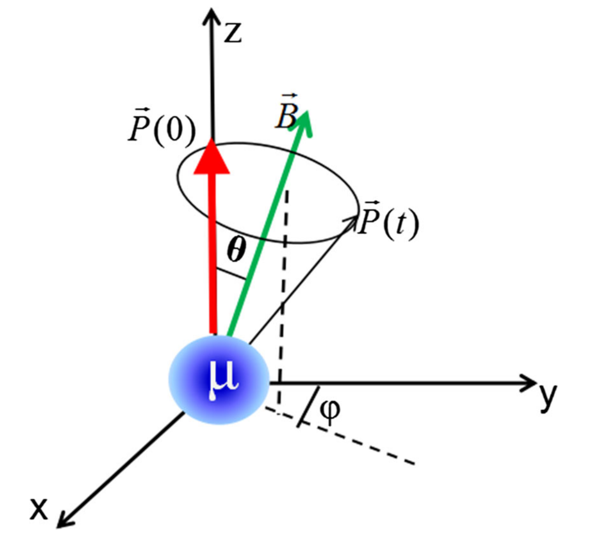

Fig. 2 Muon Spin precession in a constant field ($\vec{B}$). The initial polarization is along the *z*-axis

a constant magnetic field, the initial polarization ($P(\vec{0})$) is along the *z*-axis, and the angle between $\vec{B}$ and *z*-axis is β. The polarization evolution at time, *t*, can be derived in this model, where the Hamiltonian is expressed as $H = \vec{\mu_\mu} \cdot \vec{B} = \frac{e}{m_\mu c}\vec{S} \cdot \vec{B}$ ($\vec{S}$ is the muon spin). The derived *x*, *y*, and *z* spin polarization components are as follows:

$$P_x = \frac{1}{2}\sin(2\theta)\cos\varphi(1 - cos(\omega t)) + \sin\theta\cos\varphi\sin(\omega t),$$
$$P_y = \frac{1}{2}\sin(2\theta)\sin\varphi(1 - cos(\omega t)) - \sin\theta\sin\varphi\cos(\omega t),$$
$$P_z = \cos^2\theta + \sin^2\theta\cos(\omega t), \tag{1}$$

where $w = \frac{eB}{m_\mu c}$.

Now, we assume that the external magnetic field is 1 T along the *y*-axis ((0, 1, 0) T) and the initial spin is along the minus *z*-axis, then Eq. 1 can be simplified as the following:

$$P_x = sin(\gamma_\mu Bt), P_y = 0, P_z = -cos(\gamma_\mu Bt), \tag{2}$$

where $\gamma_\mu = 135.534$ MHz/T. The *x* and *z* components of spin polarization are sine/cosine functions with a period of 7.38 ns ($T = 2\pi/\omega = 2\pi/(\gamma_\mu B)$), and the *y* component keeps constant at 0. The above simple situation was simulated by the G4beamline (2.16), which is a useful simulation tool in the muon beam simulations [23, 24]. G4beamline 2.16 can work well with the muon spin using the spinTracking command [21]. The physics list used in the simulations in this paper is QGSP_BERT package, which uses Geant4 Bertini cascade for primary protons, neutrons, pions, and Kaons below 10 GeV [25]. Compared to QGSP which uses the low energy parameterized (LEP)

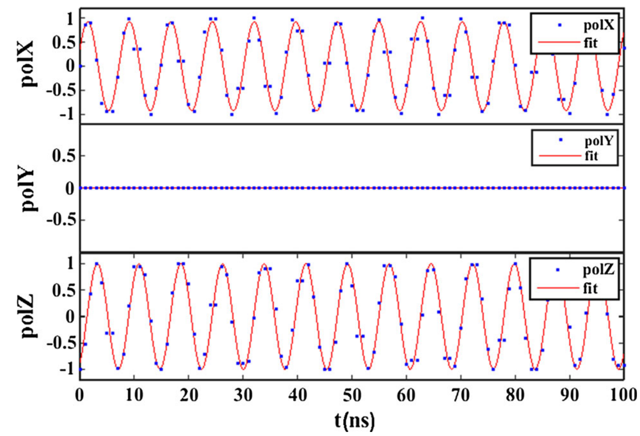

Fig. 3 *x,y,z* components of the spin polarization (polX, polY, polZ) in a uniform magnetic field of 1 T in *y*-axis

model for all particles, QGSP_BERT has improved agreement to experimental data [21]. Figure 3 gives the simulated spin polarization evolution in 100 ns of a muon with momentum of 29.8 MeV/c and an initial spin of (0, 0, -1). In *x* and *z* axes, the spin polarization motions are sine and cosine functions, respectively, the spin polarization in y-axis is zero. These results are the same with Eq. (2). The simulated period of the polarization motion periods in the *x* and *z*-axis is 7.65 $\pm$ 0.03 and 7.66 $\pm$ 0.02 ns, respectively, which almost agree with the values derived from Eq. (2).

3 The novel solenoid capture system based on CSNS

The muon beam source will be constructed at the High Energy Proton Experimental Area (HEPEA) of CSNS. In this area, 4% of the proton beam extracted from the Rapid Cycling Synchrotron (RCS) is used to bombard the target nucleus to produce pions [26]. Typical proton–nucleon reactions to produce pions have single and double pion production processes, which are described in Refs. [27, 28]. The double pion process has a larger possibility of obtaining pions. The threshold energy for the single pion process is 280 MeV, and the production cross section reaches a peak at the proton energy of 800 MeV. For the double pion process, the threshold is about 650 MeV; this double pion reaction reaches to the top when the proton energy is 1.5 GeV and it keeps the same with the increase of the proton energy. The power of the CSNS protons used for our muon source is 4 kW with energy of 1.6 GeV, which is advantageous to obtain more muons. The repeat frequency rate of the pulsed proton beam is 25 Hz. One Hz of the proton beam with the intensity of 1.56×10^{13} protons will be used to drive the muon source. The space and angular dispersion distributions of the proton beam in the simulations are both double Gaussian distributions with $\sigma_x = \sigma_y = 5.732$ mm and $\sigma_{Xp} = \sigma_{Yp} = 14.13$ mrad, respectively. The QGSP_BERT package of the G4beamline is chosen as the physics lists, same as Sect. 2. The inner radius of the capture solenoid is 450 mm, and the length is 1000 mm. The graphite target is cylindrical with the radius of 20 mm. The centroid of the target coincides with that of the capture solenoid. While their axes can have an angle (θ) (Fig. 1b), according to the layout of HEPEA, because the proton beam has a 44.8° bending with respect to the following muon channel [12].

3.1 Polarization and production of surface muons in different solenoid magnetic fields

A pion can decay into a muon and a neutrino. The spin of the rest pion at the target surface is zero, so the surface muon spin direction is opposite to its momentum because of the left-handed helicity of the neutrino ($H = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}\cdot\vec{p}|} = -1$). In the normal muon and pion capture system, the spin polarization of surface muons can reach nearly 100% because these muons are collected at one direction of the production target. In our novel superconducting solenoid capture system based on CSNS, more muons can be collected in the solenoid due to the magnetic fields. The magnetic field perpendicular to the muon momentum can influence the spin direction as described in Sect. 2. The axial (z-axis) and radial (r) magnetic fields of 1 T in the simulations applied in the capture solenoid are shown in Fig. 4. In order to study the magnetic field effect to surface muons, the central solenoid magnetic field is varied from 0 to 10 T. The spin polarization and production of the surface muon (27–29.8 MeV/c, angular dispersion <500 mrad) are calculated with different target lengths and angles (θ). Θ is the angle between the target (proton beam) and the solenoid. The following Eq. (3) gives the method that we used to calculate the spin polarization, where p_i is the spin value projected on the minus z-axis, and N_i means the number of muons with p_i.

$$Pol = \frac{\sum p_i \cdot N_i}{\sum N_i} \times 100\%. \quad (3)$$

At first, we just recorded muons with the momentum of 27–29.8 MeV/c, but the spin polarization was about 60%, because these surface muons contained the cloud muons which should be removed. There are two practical methods to distinguish surface muons from other muons in the G4beamline: (1) use the "newparticlentuple" command to record the newly produced muons with momentum of 27–29.8 MeV/c outside of the target, then wipe the muons, which have the same event ID, with the newly produced muons from these collected at the exit of solenoid; (2) use the "beamlossntuple" command to record the information of surface muons, then sweep out the surface muons collected at the exit of the solenoid with the same event ID as muons obtained from rest pions. The event in Geant4 shows the process of a particle from its production to the decay of all its secondary particles. The top two figures of Fig. 5 give the spin polarization (Fig. 5a) and production rate (Fig. 5b) of surface muons collected by the solenoid with different magnetic fields and angles. The spin polarization keeps almost the same and varies within 5% with different magnetic fields increasing from 0 to 10 T and angles from 0° to 40°. The surface muon production rate is higher when the magnetic field is larger than 4 T and θ is larger than 20°. The bottom two figures of Fig. 5 show the spin polarization (Fig. 5c) and production rate (Fig. 5d) of surface muons with different target lengths and angles. The spin polarization has a slight drop with the increase of the target length, but the drop is less than 5%, which will not affect μSR measurements. To achieve high-intensity surface muons, the target length should be larger than 350 mm, the capture solenoid magnetic field should be larger than 4 T, and the best θ is larger than 20°.

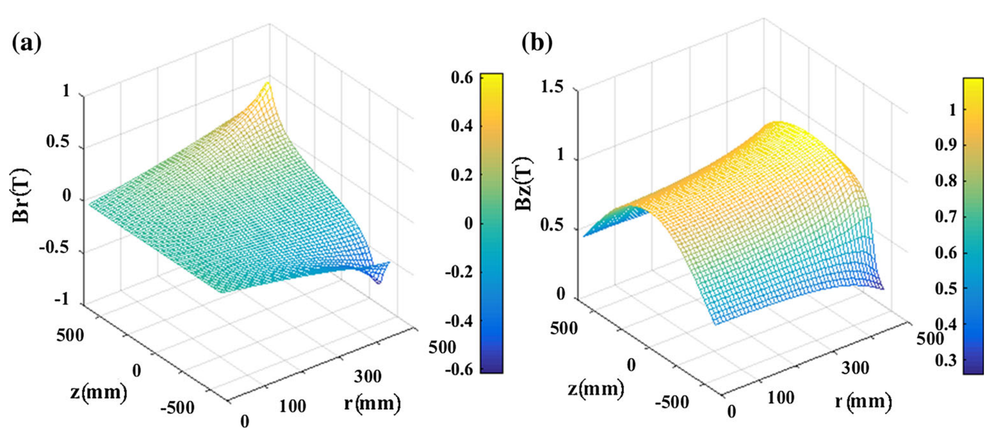

Fig. 4 The axial magnetic field distributions (**a**) and radial magnetic field distributions (**b**) in the capture solenoid of 1 T

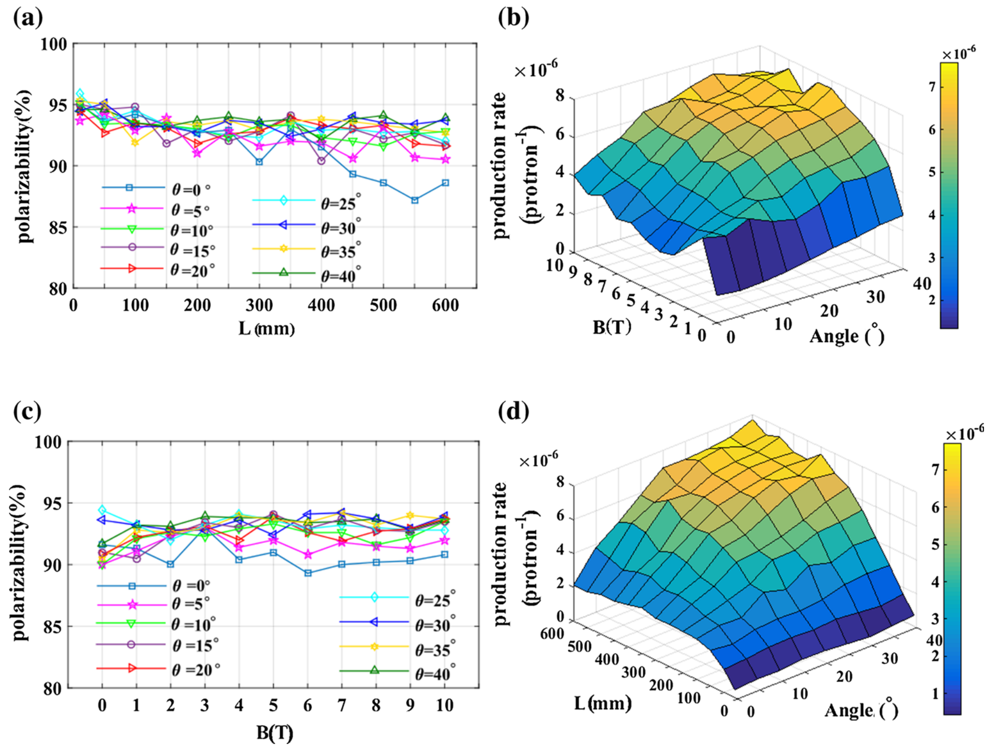

Fig. 5 Spin polarization and production rate with different magnetic fields and angles (**a**, **b**) and different target lengths and angles (**c**, **d**). The number of the primary proton event is 10^8

3.2 Impacts of the 5 T solenoid magnetic field to the proton beam and secondary particles

After the solenoid parameter analyses in Sect. 3.1, the solenoid magnetic field was fixed at 5 T, the angle between the solenoid and the target was 20°, and the target length was 400 mm for the surface muon capture system. The impacts of the solenoid capture system of 5 T on the proton beam and secondary particles were also studied. The track of the proton beam is very important for the shield of the superconducting solenoid and the placement of the beam dump. In our novel capture system design, the proton beam goes directly to the beam dump after bombarding the muon production target (see Fig. 1b), while in the normal muon capture system of PSI [29] and J-PARC [30] the proton will be reused for the production of neutrons (see Fig. 1a) and that is why they often use thin targets. Figure 6 gives the phase space distributions ($x - x'$ and $y - y'$) of protons at the exit of the solenoid with 0 and 5 T, where x and y mean horizontal and vertical positions and x' and y' represent the corresponding angular dispersions. From these figures, we can see that the proton beam goes straight through the solenoid (mean x =178.6 mm ($\approx \tan 20° \times$ 1000/2 mm) and mean y = 0 in Fig. 6a, b) with $B = 0$. The proton is deflected from the center in the vertical position (mean y =57.7 mm, about 6.7°) by the solenoid magnetic field of $B = 5$ T (Fig. 6c, d), while x keeps the same. The 4-D volume of the proton space distributions ($\epsilon_x \times \epsilon_y$) of $B = 0$ and $B = 5$ T are 1.38 $\times 10^6$ and 4.64 $\times$ $10^5 \pi^2 \cdot$ mm$^2 \cdot$ mrad^2 with 90% beam percentage, respectively. The solenoid magnetic field can make the proton beam emittance to be smaller, which is good for the capture solenoid shield and the beam dump. The magnetic field can also bend the proton beam from the x-axis center.

Emittances (ϵ), Courant–Snyder (CS) parameters and intensities of surface muons with different beam fractions collected by the solenoid with $B = 0$ and $B = 5$ T are calculated as shown in Table 1. Emittances and Courant–Snyder parameters (α, β, γ) can be well used to describe the coupled x–y transverse beam dynamics [31]. Comparing these parameters with and without magnetic fields, we found that the magnetic field could change the phase space distributions. The emittances of surface muons (ϵ_x, ϵ_y) collected by the solenoid field were smaller, and the estimated intensities of surface muons with different beam fractions (last two columns in Table 1) captured by the 5 T magnetic field were larger than that without the magnetic field.

The momentum distributions of the secondary particles produced by protons bombarding the graphite target with $B = 0$ and $B = 5$ T are shown in Fig. 7: positrons, positive pions, negative pions, positive muons, and negative muons. The intensity of positrons collected by the solenoid magnetic field was higher than that without the magnetic field, and these positions are energetic ($\sim$MeV) compared with the normal positron source produced by Na-22 and can be used as a new positron source for positrons annihilation

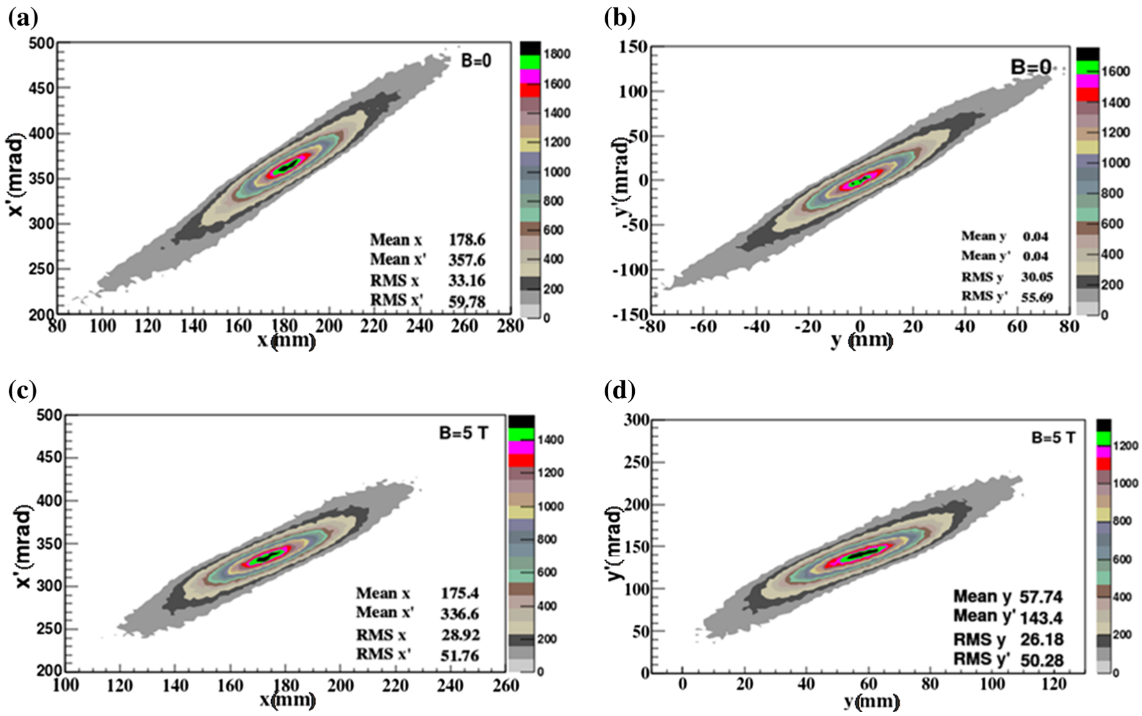

Fig. 6 Phase space distributions of proton beam at the exit of the solenoid with magnetic field $B = 0$ (**a**, **b**) and $B = 5$ T (**c**, **d**). The number of the primary proton event is 10^6

Table 1 The emittances, C-S parameters and intensities of surface muons at the exit of the solenoid with $B = 0$ and $B = 5$ T. The number of the primary proton event is 10^9

Fraction /% × %	ϵ_x/π(mm mrad)		α_x		β_x		ϵ_y/π(mm mrad)		α_y		β_y		Intensity/ $\times 10^7$ s^{-1}	
	0	5 T	0	5 T	0	5 T	0	5 T	0	5 T	0	5 T	0	5 T
10 × 10	172.9	69.6	−1.24	0	0.91	0.11	209.6	109.9	−2.21	0	1.28	0.09	1.34	5.42
30 × 30	318.3	119.5	−1.24	0	2.49	0.12	462.5	190.0	−2.27	0	1.99	0.07	3.94	16.1
50 × 50	470.6	184.9	−2.05	0	1.32	0.20	708.5	249.6	−7.67	0	5.00	0.16	6.31	27.3
90 × 90	899.7	298.7	−2.15	0	1.51	0.29	1308.4	397.9	−6.00	0	3.81	0.30	11.8	49.1

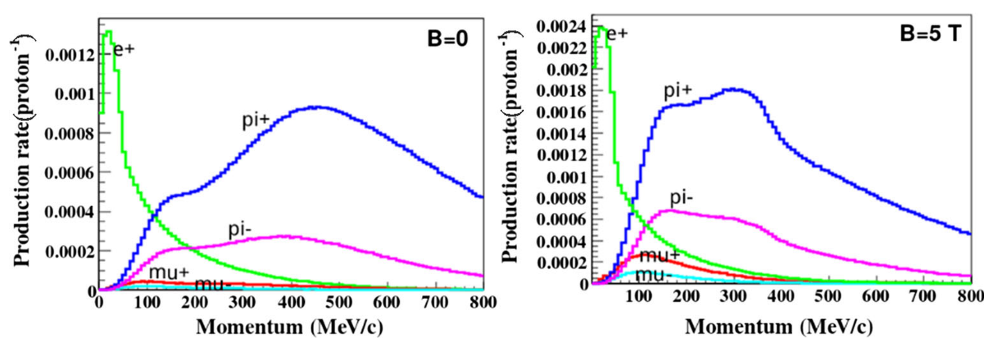

Fig. 7 Momentum distributions of the secondary particles at the solenoid captures system with $B = 0$ (*left*) and $B = 5$ T (*right*). The number of the primary proton event is 10^8

spectroscopy (PAS) through an appropriate channel [32]. The solenoid magnetic field could capture more positive and negative pions simultaneously and therefore could achieve more decay positive and negative muons for other applications.

4 Conclusion

A novel surface muon capture system using superconducting solenoids was proposed at CSNS. The muon spin evolution in a transverse uniform magnetic field with respect to the muon momentum was studied by the G4beamline. When the muon precesses in the magnetic field of (0, 1, 0) T, the muon spin direction could also be changed. Our solenoid capture system can collect more muons than the normal collection system through the effect of the magnetic field, and the spin polarization is also studied since it is very important for μSR measurements. The spin polarization and production rate of surface muons collected by the solenoid were given with different solenoid magnetic fields, θ angles and target lengths. The simulated results showed that the spin polarization of these surface muons was still high. To achieve higher-intensity surface muons, the target length should better be larger than 350 mm, the capture solenoid magnetic field should be larger than 4 T, and the optimal θ angle is larger than 20°. The proton phase space, Courant–Snyder parameters of surface muons and the other secondary particles' momenta were given at $B = 0$ and 5 T, where the target length is 400 mm and θ angle is chosen as 20°. The simulation results showed that this capture system can focus the protons and surface muons and can capture more positrons and positive and negative pions at the exit of the solenoid.

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