Study of time-domain digital pulse shaping algorithms for nuclear signals

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Abstract With the development on high-speed integrated circuit, fast high resolution sampling ADC and digital signal processors are replacing analog shaping amplifier circuit. This paper firstly presents the numerical analysis and simulation on R-C shaping circuit model and C-R shaping circuit model. Mathematic models are established based on 1st order digital differential method and Kirchhoff Current Law in time domain, and a simulation and error evaluation experiment on an ideal digital signal are carried out with Excel VBA. A digital shaping test for a semiconductor X-ray detector in real time is also presented. Then a numerical analysis for Sallen-Key (S-K) low-pass filter circuit model is implemented based on the analysis of digital R-C and digital C-R shaping methods. By applying the 2nd order non-homogeneous differential equation, the authors implement a digital Gaussian filter model for a standard exponential-decaying signal and a nuclear pulse signal. Finally, computer simulations and experimental tests are carried out and the results show the possibility of the digital pulse processing algorithms.

Key words R-C shaping, C-R shaping, Nuclear pulse, Digital *S-K* filter, Digital Gaussian filter

1 Introduction

The acquisition and processing of nuclear signals is an important method and plays a significant role in many fundamental and application researches. Gaussian analog shaping was once used to process detector output signals in most spectroscopic systems[1,2]. With the development of high-speed integral circuit (IC), digital shaping became popular for nuclear pulse signal processing. However, differential equations only provide the analytical solution for specific signals, such as step signal[3,4]. And no specific analytical solutions are available for output signals of nuclear detectors, which are of complex statistical fluctuation. Usually, they are converted from time domain to frequency domain using Laplace transformation for various process algorithms[1]. Also, the detector output signal with important time-domain characteristics, such as pulse shape, pulse width and so on, need to be studied. In the Gaussian forming process of nuclear pulse signals, some mathematical models were established, and tested by computer simulation[5–9]. Jordanov[5] *et al*. worked on digital processing technology of real-time pulse signal shaping. Chen S G *et al*.[10,11] simulated the Gaussian shaping of standard index signal in different scale parameters by analyzing the Gaussian pulse shaping theory of exponential decay signals for appropriate wavelet and scale basis functions. Li D C *et al*.[12,13] established the Gaussian shaping model with a pattern of transfer function based on S-K filter, and simulated it with the PSIM code.

In this paper, three methods are used to shape digital pulse nuclear signals, i.e. C-R differential shaping model, R-C integral shaping model and S-K low-pass filter shaping model, based on digital series

Supported by Shanghai Synchrotron Radiation Facility project

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Received date: 2011-11-21

differential theory. Analog signals are converted into discrete digital series with small time interval using high-speed ADC. Three digital recursive models are obtained by digital differential analysis approaches, learning from the time-domain analysis approach of discrete systems[3]. The three models are digital filters to realize relevant algorithm. Concept of the R-C, C-R, and S-K digital filters are introduced first. Gaussian digital filter and digital forming are studied with digital S-K filter. The minimum sampling time interval of digital differentiation can be regulated arbitrarily, with improved simulation accuracy by reducing the sampling time interval. The simulation results are of elative errors of <0.1%[14].

2 Digital R-C shaping model

2.1 R-C circuit model and numerical analysis

R-C integral circuit is a basic analog shaping approach in nuclear signal processing process. Fig.1 shows in the insert an R-C integral circuit, with the input and output signals being V_{in} and as V_{out}, respectively. The circuit can be equivalent to a differential equation of Eq.1 based on Kirchhoff's Current Law (KCL), Which shows that the gross current out of and in a point must be zero at any time.

$$(V_{in}-V_{out})/R = \mathrm{C}\mathrm{d}V_{out}/\mathrm{d}t \tag{1}$$

Let $V_{in} = x$ and $V_{out} = y$, we have

$$RCy'+y = x \tag{2}$$

To facilitate digital pulse signal processing in time-domain, a first-order numerical differentiation method, instead of conventional numerical method, is used to solve Eq.2, which can be written as:

$$RC(y_{n+1}-y_n)/\Delta t + y_{n+1} = x_{n+1} \tag{3}$$

where Δt is the time interval from x_n to x_{n+1} or the sampling time period of ADC, and x_n and y_n are the input and output data series, respectively. Let $RC/\Delta t = k$ and similar item combination, one has

$$y_{n+1} = (ky_n+x_{n+1})/(k+1),\ y_0 = 0 \tag{4}$$

Therefore, Eq.4 is the numerical model corresponding to the R-C shaping circuit.

2.2 Simulations

A simulation of Eq.4 is done with VBA language in Excel. A standard negative exponential signal is used to simulate the nuclear signal, with x=2000e$^{(200-t)/50}$ and x=0. According to the characteristics of real nuclear pulse, the shaping time of input series should be less than 200 *ns*. To simulate the detector output, time constant of the input signal is set at 2500 ns.

Fig.1 shows the input and output signals, at shaping parameter k=50 and 100, with Δt =50 ns, which is the sampling rate of ADC in the system. Output 1 (k=50) looks like the output of real R-C circuit where R=5 kΩ and C=1 nF. Output 2 (k=100) is equivalent to the output of an R-C circuit where R=2.5 kΩ and C=1 nF. According to the simulating results, we can see that the width and amplitude of the digital shaping pulse change with the shaping parameter k.

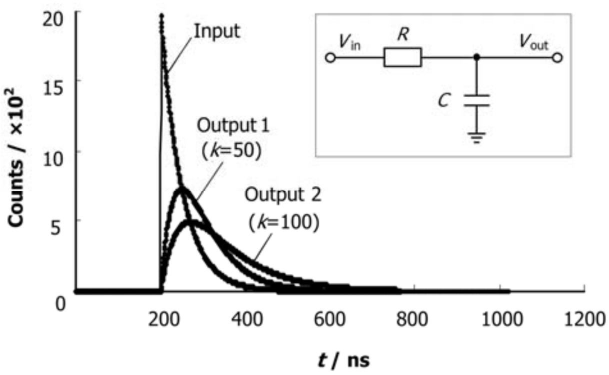

Fig.1 Simulations for standard negative exponential signal with R-C shaping model.

3 Digital C-R shaping model

3.1 C-R circuit model and numerical analysis

C-R differential shaping circuit is a basic analog shaping approach in nuclear signal processing. Fig.2 shows in the insert a C-R shaping circuit, which can be equivalent to Eq.5 based on KCL.

$$\mathrm{d}V_{in}/\mathrm{d}t - \mathrm{d}V_{out}/\mathrm{d}t = V_{out}/RC \tag{5}$$

Eq.5 can be rewritten as Eq.6 based on the same analysis method in Section 2.1

$$\begin{cases} y_{n+1} = (y_n + x_{n+1} - x_n)/(1+k) \\ y_0 = 0, k = \Delta t/(RC) \end{cases} \tag{6}$$

So, Eq.6 is the numerical model corresponding to the C-R shaping circuit.

3.2 Simulations

To verify effectiveness of the C-R model, a standard negative exponential signal the same as Section 2.2 was used to simulate the nuclear signal. Fig.2 shows the simulation results of numerical C-R shaping model, with k=0.02 and 0.01.

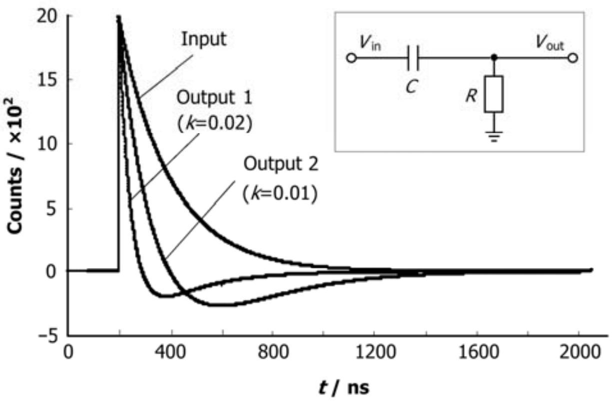

Fig.2 The simulations for standard negative exponential signal with C-R shaping model

Because C-R differential shaping circuit is also used for extraction of nuclear pulse signal from the output of nuclear detector with carrier signal, a simulation for basic step signal with this model is also carried out, which is shown in Fig.3. at Δt = 50 ns, the attenuation constant is 2500 ns at k=0.02, and 5000 ns at k=0.01.

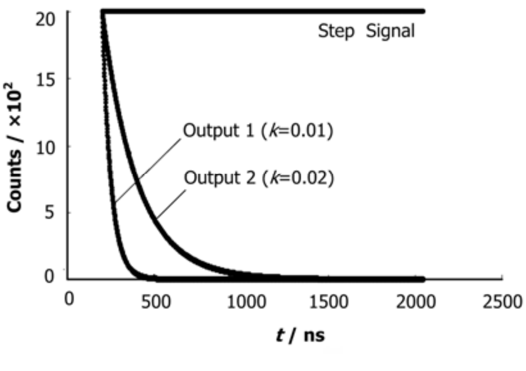

Fig.3 The simulation for step signal with C-R shaping model.

In Eq.5, dV_{in}/dt is zero when the input signal is a basic step signal. Let $y=V_{out}$, we will have Eq.11 by the following deductions,

$$-dy/dt = y/(RC) \tag{7}$$

$$dy/y = -dt/(RC) \tag{8}$$

$$\ln(y) = -t/(RC) + I_0 \tag{9}$$

$$y(t) = e^{I_0 - t/(RC)} \tag{10}$$

$$y(t) = Ke^{-t/(RC)} \tag{11}$$

Therefore, we know that C-R shaping output of the step signal is a negative exponential signal, the same as the conclusion in Ref.[2], and K is determined by initial value of V_i and RC is the shaping time constant of output signal. for error evaluation of the C-R shaping model (Eq.6), a comparative calculation between C-R numerical model (Eq.6) and theoretical model (Eq.11) is done. Table 1 shows the results at k=0.005 (Δt=25 ns). The error is less than 0.1%. We think that the error is induced by Δt, which is very small itself. At k=0.01 (Δt=50 ns), the error is less than 0.3%, while it is less than 0.01% at k=0.002 (Δt=1 ns). So, with improved ADC's sampling rate, this algorithm can replace analog circuit with perfect performances.

Table 1 Comparison between theoretical values and C-R shaping model output

C-R model	Theoretical
1990.050	1990.025
1980.149	1980.100
1970.298	1970.224
1960.495	1960.397
1950.741	1950.620
1941.036	1940.891
1931.379	1931.211
1921.770	1921.579
1912.209	1911.995
1902.696	1902.459
1893.230	1892.970
1883.811	1883.529
1874.438	1874.135
1865.113	1864.788
1855.834	1855.487
1846.601	1846.233
1837.414	1837.025
1828.272	1827.862
1819.176	1818.746
1810.126	1809.675

The C-R differential process of continuous step signals ia simulated with the same method (Fig.4). Such carrier signals are widely seen in preamplifier output of semiconductor detectors. From the results, this C-R numerical model is an important processing method of nuclear signals.

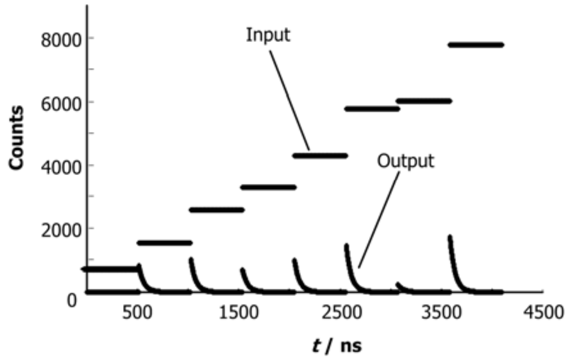

Fig.4 The simulation for continuous step signals with C-R shaping model(k=0.02, Δt=50 ns).

4 Digital Gaussian shaping model

4.1 Sallen-Key filter

S-K filter is a common active-filter circuit developed by R.P. Sallen and E.L. Key. There are two typical circuits of S-K filter, i.e. high-pass and low-pass filters. For positive feedback control, it possesses a large quality factor. Using it to the shape nuclear signals, Gaussian waveform can be obtained by less orders. The schematic circuit of S-K filter is sketched in Fig.5.

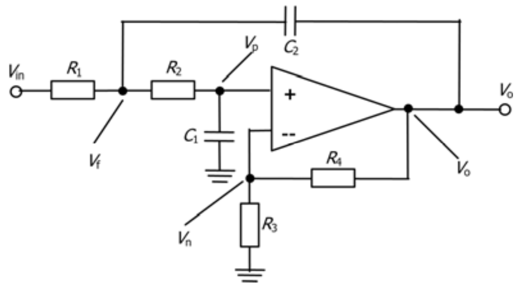

Fig.5 Schematic circuit of S-K filter.

4.2 The digital Gaussian shaping model

Four nodes are marked in Fig.5. The voltage of each node is marked as V_f, V_p, V_n and V_{out}. According to Kirchhoff 's Current Law, the voltage transmission can be described by Eqs.(12−15).

$$(V_{in}-V_f)/R_1 = (V_f-V_p)/R_2 + C_2\mathrm{d}(V_f-V_{out})/\mathrm{d}t \qquad (12)$$

$$(V_f-V_p)/R_2= C_1\,\mathrm{d}V_p/\mathrm{d}t \qquad (13)$$

$$V_n=V_pR_3/(R_3+R_4) \qquad (14)$$

$$V_n=V_p \qquad (15)$$

In Fig.5, the resistors and capacitors are of the same value. Taking Eqs.(14,15), we have Eqs.(16−18).

$$(V_{in}-V_f)/R = (V_f-V_p)/R + C\,\mathrm{d}(V_f-V_{out})/\mathrm{d}t \qquad (16)$$

$$(V_f-V_p)/R= C\,\mathrm{d}V_p/\mathrm{d}t \qquad (17)$$

$$V_p=V_{out}/2 \qquad (18)$$

Taking Eq.18 into Eqs.16 and 17 respectively, a synthetic equation can be obtained

$$2V_{in}=(RC)^2\mathrm{d}(\mathrm{d}V_{out}/\mathrm{d}t)/\mathrm{d}t +RC\,\mathrm{d}V_{out}/\mathrm{d}t + V_{out} \qquad (19)$$

Eq.19 can be rewritten as

$$(RC)^2y'' +RC\ y' + y = 2x \qquad (20)$$

Eq.20 shows that the Gaussian shaping model based on S-K filter can be summarized as a function $y=f(x)$, which is a non-homogeneous quadratic differential equation. When the equation is solved, the theoretical relationship between the input index signal and the output Gaussian signal can be obtained. Taking the statistical fluctuations of nuclear pulse signal into account, numerical differentiation algorithm is applied.

Eq.20 can be rewritten as

$$(RC)^2[(y_{n+1}-y_n)/\Delta t-(y_n-y_{n-1})/\Delta t]/\Delta t +$$

$$RC(y_{n+1}-y_n)/\Delta t + y_{n-1} = 2x \qquad (21)$$

Defining $RC/\Delta t = k$ and similar items combination, Eq.10 can be rewritten as

$$y_n=[(k+2k^2)y_{n-1}-k^2y_{n-2}+2x_n]/(1+k+k^2),\ \ n\geq 0 \qquad (22)$$

$$y_n=x_n=0,\ \ n<0 \qquad (23)$$

The input signal at a certain time is defined as x_n from Eq.22. The output Gaussian signal is defined as y_n, which is a general numerical recursive root if the time $n\geq 0$, while $y_n = 0$ if $n<0$. Then, digital Gaussian shaping of nuclear signal can be implemented by recursive-calling of Eq.22. Different type of the output signal can be acquired at different k, the shaping parameter of the output Gaussian signal.

5 Application of digital Gauss filter

A computer simulation platform is developed with VBA language in Microsoft Office Excel 2003. Numerical model and its input signal are implemented through Macro programming. When the filter k value is nearly equal to the noise forming time of input signal, numerical filter function of digital Gauss filter are functioned, keeping basic characteristics of the original input signal.

5.1 Digital Gaussian filtering

Digital S-K filter is used in the real-time signal processing, and the input signal is from a random noise signal (range 100) superimposed on an original exponential-decaying signal (range 2000, τ=100) .The random noise signal is from the random number generator function. Fig.6 shows that the digital S-K filter has the advantages in filtering the signal noise. The calculation of the digital S-K filter is easy and the parameters can be adjusted easily, hence the convenience in the real-time signal processing.

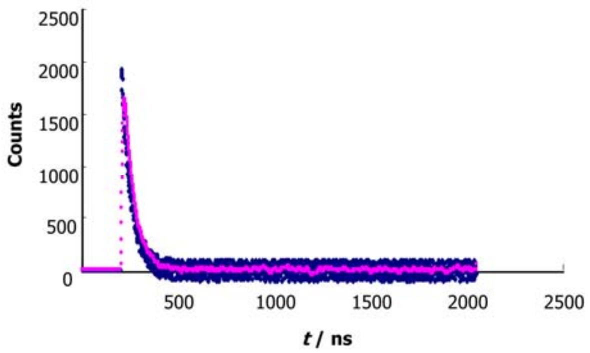

Fig.6 Digital Gaussian filtering.

5.2 Digital Gaussian shaping

In a nuclear spectra analysis system, the output signal of the detector should be first processed in noise filter, and the exponential-decaying signal becomes the Gaussian pulse signal with higher SNR. Fig.7 shows the Gaussian shaping waveforms for an exponential signal at different shaping parameters. The attenuation constants of exponential-decaying signal is 100 (τ=100). The noise signal superimposes on the input signal. When the s/n of input signal is 10 db, the s/n of output signal exceeds 40 db in three parameters.

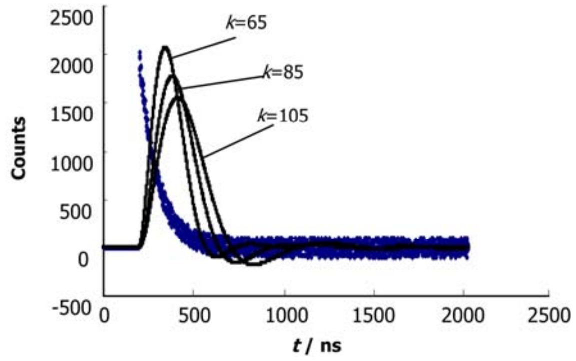

Fig.7 Gaussian shaping at different shaping parameters.

5.3 Nuclear signal and its digital Gaussian shaping

Fig.8 shows the numerical Gaussian shaping for a real nuclear signal at different shaping parameters. In this experiment, the input pulse signal was obtained from a SI-PIN X-Ray detector (Moxtek XPIN-BT). This pulse was sampled using a 12-bit ADC(AD9235) operating at 20 MHz，after a linear amplifier (AD844). The digital output of ADC was transmitted to the PC for implementation of the recursive shaping algorithm.

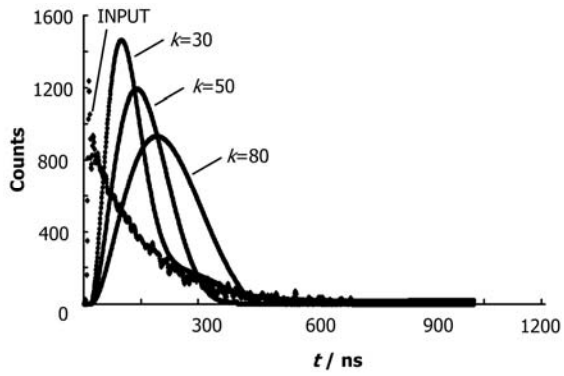

Fig.8 Gaussian shaping waveforms for nuclear signal at different shaping parameters.

6 Conclusion

In this paper, R-C and C-R numerical models are established through numerical recursive solution of 1st order differential equation in time-domain. They are simulated and tested, with the error estimated by using step signal as the input. With the same analysis method, a novel digital Gaussian shaping model is also established based on S-K low-pass filter. The algorithm is verified by simulation and test.

The algorithms can be used for analyzing superimposed nuclear pulse because they have no restriction on the input signal series. Further studies on superimposed nuclear pulse signal processing using the algorithms will be carried out.

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