

Properties of color-flavor locked strange quark matter in an external strong magnetic field*

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The properties of color-flavor locked strange quark matter in an external strong magnetic field are investigated in a quark model with density-dependent quark masses. Parameters are determined by stability arguments. It is found that the minimum energy per baryon of the color-flavor locked (MCFL) matter decreases with increasing magnetic-field strength in a certain range, which makes MCFL matter more stable than other phases within a proper magnitude of the external magnetic field. However, if the energy of the field itself is added, the total energy per baryon will increase.

Keywords: Strange quark matter, MCFL, Mass-density-dependent model

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I. INTRODUCTION

Strange quark matter (SQM) is an interesting topic [1] not only because of its great theoretical significance, but also its many applications, e.g., in studying quantum chromodynamics (QCD) phase diagrams [2–6], properties of strangelets [7, 8], and the structure of compact stars [9, 10], etc. In 1984, Witten conjectured that quark matter with strangeness might be the true ground state of QCD [11]. Soon after Witten's conjecture, Farhi and Jaffe studied the stability of SQM with the conventional MIT bag model and found that SQM is absolutely stable around the normal nuclear density for a wide range of model parameters [12]. Since then, SQM has become a main topic in a number of meaningful works [13–20].

It has been demonstrated that SQM at high density may be in the color-flavor locked (CFL) phase where quarks with different color and flavor quantum numbers form Cooper pairs with a large binding energy [21]. It is, thus, possible that CFL matter, rather than nuclear matter, may be the ground state of strange quark matter at high density. Therefore, a compact star is suggested to include color superconducting quark matter in its inner core [22].

It is generally believed that properties of quark matter have been strongly affected in the presence of a strong magnetic field [23, 24]. A strong magnetic field widely exists on the surface of stars. The observed magnetic field strength on the surface of pulsars could be in the order of 10^{12} – 10^{13} G. And the magnetic-field strength on the so-called magnetars could be in the order of 10^{14} – 10^{15} G or even higher [25, 26]. In fact, the biggest magnetic field that can be sustained by strange stars was estimated to be as large as 1.5×10^{20} G [27]. Although the origin of the strong magnetic field is not completely clear and still under active investiga-

tions, some ways to understand its existence are available, e.g., the amplification of the relatively small magnetic field during the star's collapse with magnetic flux conservation [28], and the magneto-hydrodynamic dynamo mechanism with large magnetic fields generated by rotating the plasma of a protoneutron star [29]. Furthermore, noncentral high-energy heavy-ion collisions could generate intense magnetic fields as high as about 10^{19} G [27], corresponding to $eB_m \sim 6m_\pi^2$, where e is the fundamental electric charge and m_π is the pion mass. It is therefore necessary to study the properties of CFL in the presence of an external magnetic field.

In past years, magnetized strange quark matter (MSQM) and CFL matter have been studied with many phenomenological models, e.g., the bag model [30], the Nambu-Jona-Lasinio (NJL) model [31–36], and the mass-density dependent model [37, 38], etc. The MCFL matter has a wide range of model parameters characterized by the so-called stability window [39], and has also been studied in the NJL model [40–44], as well as in the quasiparticle model [45, 46].

As is well known, particle masses vary with environment, i.e., they depend on density or chemical potential. The equiv-particle model [1, 2] takes this effect into account by density-dependent quark masses. In recent years, this model has been extensively applied to study the properties of SQM [1, 15, 47–55]. In this paper, we extend it to investigate the properties of CFL matter when a strong magnetic field appears. It is found that MCFL matter is more stable than the other phases within a proper magnitude of the magnetic field. At a fixed density, the energy density of MCFL matter varies with the magnetic field strength. At $B_m \geq 10^{19}$ G, the energy per baryon, pressure, and quark chemical potentials get smaller because the quantum number of the corresponding Fermi momentum approaches to zero.

This paper is organized as follows. In Section II, we give the thermodynamic formulas used for the study of MCFL matter in the equiv-particle model with density-dependent quark masses. Then we present the numerical results and discussions in Section III. Section IV is a short summary.

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II. THERMODYNAMIC TREATMENT

Our starting point is the thermodynamic potential density of a free-particle system, i.e.

$$\Omega_f = \sum_i \frac{2g_i}{(2\pi)^3} \int \left(\sqrt{p^2 + m_i^2} - \mu_i \right) d^3p, \quad (1)$$

where the summation index, i , goes over u, d, s quarks and electrons, m_i is the corresponding particle mass, μ_i is the chemical potential, g_i is the degeneracy factor with a value of 3 for quarks and 1 for electrons, while the degeneracy due to spin has been denoted by a factor of 2.

In the case of CFL phase, due to the energy gap, Δ , determined by solving the gap equation, a new term should be added to the above expression. The thermodynamic potential density of CFL matter is then

$$\Omega_{\text{CFL}} = \sum_i \frac{2g_i}{(2\pi)^3} \int (\varepsilon_i - \mu_i) d^3p - \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} + B, \quad (2)$$

where $\varepsilon_i = \sqrt{p^2 + m_i^2}$ is the dispersion relation of a free particle with a mass of m_i , $\bar{\mu} = (\mu_u + \mu_d + \mu_s)/3$ is the average of the quark chemical potentials. The second term is from the pairing contribution, and the last term, B , is the famous MIT bag constant to take the vacuum energy into account.

To consider the effect of a magnetic field, we assume a constant magnetic field with a strength of B_m along the z axis. Due to Landau diamagnetism, the single particle energy spectrum can be written as

$$\varepsilon_{i,l} = \sqrt{p_z^2 + m_i^2 + 2|q_i|B_m[l + 1/2 - \text{sgn}(q_i)S]}, \quad (3)$$

where p_z is a component of the particle momentum along the direction of the magnetic field, q_i is the electric charge of quarks when $q_u = 1/3$, $q_d = q_s = -1/3$, $q_e = -1$, $l = 0, 1, 2, \dots$ is the principal quantum number for the allowed Landau levels, and $S = \pm 1/2$ refers to spin-up and spin-down states, respectively. The sign function 'sgn' equals 1 with a positive argument and -1 with a negative argument.

For the sake of convenience, one normally sets $\nu = l + 1/2 - \text{sgn}(q_i)S$, where $\nu = 0, 1, 2, \dots$. The single particle energy becomes

$$\varepsilon_{i,\nu} = \sqrt{p_z^2 + M_{i,\nu}^2}, \quad (4)$$

where $M_{i,\nu} \equiv \sqrt{m_i^2 + 2\nu|q_i|B_m}$.

The integration over the p_x - p_y plane in the momentum space should be replaced by

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dp_x dp_y \longrightarrow 2\pi |q_i| B_m \sum_{\nu} (2 - \delta_{\nu 0}). \quad (5)$$

After this substitution, Eq. (2) becomes

$$\Omega_{\text{MCFL}} = \sum_i \sum_{\nu=0}^{\nu_{\text{max}}} \frac{f_{i,\nu}}{2} \int_0^{p_{i,\nu}} (\varepsilon_{i,\nu} - \mu_i) dp_z - \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} + B, \quad (6)$$

where we have used the notations:

$$p_{i,\nu} \equiv \sqrt{\mu_i^2 - M_{i,\nu}^2}, \quad f_{i,\nu} \equiv \frac{g_i |q_i| B_m}{4\pi^2} (2 - \delta_{\nu 0}). \quad (7)$$

The upper bound ν_{max} of the summation index ν in Eq. (6) is

$$\nu_{\text{max}} \equiv \text{int} \left(\frac{\mu_i^2 - m_i^2}{2|q_i|B_m} \right), \quad (8)$$

where the function $\text{int}(x)$ means taking the integer part of its argument x .

After carrying out the integration in Eq. (6), we have the thermodynamic potential density in the conventional bag model

$$\Omega_{\text{MCFL}} = - \sum_i \sum_{\nu=0}^{\nu_{\text{max}}} f_{i,\nu} \left[\mu_i \sqrt{\mu_i^2 - M_{i,\nu}^2} - M_{i,\nu}^2 \text{arcsch} \left(\frac{\mu_i}{M_{i,\nu}} \right) \right] - \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} + B. \quad (9)$$

To include medium effect, the quark masses should be density/chemical potential dependent. In the chemical potential dependent case, one can use the quasiparticle model, as has been done in Ref. [45]. In the density-dependent case, the actual chemical potential, μ_i , should be replaced with an effective chemical potential, μ_i^* [1], i.e.

$$\Omega_0 = \sum_i \Omega_i - \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} + B, \quad (10)$$

where $\bar{\mu}$ is now understood as the average of the effective chemical potentials, and Ω_i is connected to the effective chemical potentials by

$$\Omega_i = - \sum_{\nu=0}^{\nu_{\text{max}}} f_{i,\nu} \left[\mu_i^* \sqrt{\mu_i^{*2} - M_{i,\nu}^2} - M_{i,\nu}^2 \text{arcsch} \left(\frac{\mu_i^*}{M_{i,\nu}} \right) \right]. \quad (11)$$

All other thermodynamic quantities can be derived from Ω_0 . Specially, the particle number density is given by $n_i = -\partial \Omega_0 / \partial \mu_i^*$, giving

$$n_i = \frac{g_i |q_i| B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{\nu 0}) \sqrt{\mu_i^{*2} - M_{i,\nu}^2} + \frac{2\Delta^2 \bar{\mu}}{\pi^2}. \quad (12)$$

The energy density for the MCFL matter is then

$$E_{\text{MCFL}} = \Omega_0 + \sum_i \mu_i^* n_i. \quad (13)$$

Upon application of Eqs. (10)–(12), we have

$$E_{\text{MCFL}} = \sum_i \frac{g_i |q_i| B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{\nu 0}) \left[\mu_i^* \sqrt{\mu_i^{*2} - M_{i,\nu}^2} - M_{i,\nu}^2 \text{arcsch} \left(\frac{\mu_i^*}{M_{i,\nu}} \right) \right] - \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} + B. \quad (14)$$

Because the quark masses are density dependent, the actual chemical potential is generally not equal to its effective one. In fact, they are linked by

$$\mu_i = \mu_i^* + \mu_I, \quad (15)$$

where μ_I is due to the density dependence of the quark masses. Its explicit expression is obtained by applying the fundamental differential equality $dE_{\text{MCFL}} = \sum_i \mu_i dn_i$ with Eqs. (10)–(15), giving

$$\mu_I = \sum_j \frac{\partial \Omega_0}{\partial m_j} \frac{dm_j}{dn_i}, \quad (16)$$

$$= - \sum_{j,\nu} \frac{g_j |q_j| B_m D}{18\pi^2 n^{4/3}} (2 - \delta_{\nu 0}) m_j \text{arcsinh} \left(\frac{\mu_j^*}{M_{j,\nu}} \right). \quad (17)$$

Because electrons do not participate in strong interactions, their actual chemical potential is equal to the effective one, i.e., $\mu_e = \mu_e^*$.

Due to the external magnetic field, the longitudinal pressure and transverse pressure become different, i.e.

$$P_{\parallel} = -\Omega_0 + \sum_i \mu_I n_i \quad (18)$$

and

$$P_{\perp} = -\Omega_0 + \sum_i \mu_I n_i - M_f B_m, \quad (19)$$

where $P_{\parallel}$ is the total parallel pressure and $P_{\perp}$ is the transverse pressure. The system magnetization is given by

$$M_f = - \frac{\partial \Omega_0}{\partial B_m} = - \sum_i \left(\frac{\Omega_i}{B_m} + \frac{\partial \Omega_i}{\partial M_{i,\nu}} \frac{dM_{i,\nu}}{dB_m} \right). \quad (20)$$

Upon application of Eqs. (10), (11), and (20), we have the following explicit expressions

$$\begin{aligned} P_{\parallel} = & - \sum_i \frac{g_i |q_i| B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \\ & \times \left[-\mu_i^* \sqrt{\mu_i^{*2} - M_{i,\nu}^2} + \left(M_{i,\nu}^2 + \frac{2}{3} \frac{D}{n^{1/3}} m_i \right) \right. \\ & \left. \times \ln \frac{\sqrt{\mu_i^{*2} - M_{i,\nu}^2} + \mu_i^*}{M_{i,\nu}} \right] + \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} - B, \end{aligned} \quad (21)$$

and

$$\begin{aligned} P_{\perp} = & \sum_i \frac{g_i |q_i| B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \\ & \times \left(-\frac{2}{3} \frac{D}{n^{1/3}} m_i + 2 |q_i| \nu B_m \right) \ln \frac{\sqrt{\mu_i^{*2} - M_{i,\nu}^2} + \mu_i^*}{M_{i,\nu}} \\ & + \frac{3\Delta^2 \bar{\mu}^2}{\pi^2} - B. \end{aligned} \quad (22)$$

III. THE PROPERTIES OF MCFL MATTER

In the equiv-particle model, the quark mass can be divided into two parts as

$$m_i = m_{i0} + m_I, \quad (23)$$

where m_{i0} is the quark's current mass, and m_I represents the effect due to the interaction between quarks. In principle, the density dependence of m_I should be determined from QCD. As mentioned before, however, there is no way to exactly solve QCD presently. Therefore, the density dependence is normally given phenomenologically. It can be shown that the following parametrization is reasonable,

$$m_i = m_{i0} + \frac{D}{n^{1/3}}, \quad (24)$$

where D is a fixed constant determined by stability argument, n is the total baryon number density, and the exponent of the baryon number density was derived based on the in-medium chiral condensates and linear confinement at zero temperature. Such a form satisfies $\lim_{n \rightarrow 0} m_I = \infty$ and $\lim_{n \rightarrow \infty} m_I = 0$, which are the requirements of quark confinement and asymptotic freedom.

Because weak equilibrium is always reached in SQM, relevant chemical potentials satisfy

$$\mu_d = \mu_s, \quad (25)$$

and

$$\mu_u + \mu_e = \mu_s. \quad (26)$$

Therefore, the effective chemical potentials also meet the corresponding relations

$$\mu_u^* + \mu_e = \mu_d^*, \quad (27)$$

and

$$\mu_d^* = \mu_s^*. \quad (28)$$

We also have the baryon number density

$$n = \frac{1}{3} (n_u + n_d + n_s), \quad (29)$$

and the charge density

$$Q = \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e. \quad (30)$$

The charge-neutrality condition requires $Q = 0$.

For a given total baryon number density of n , we can obtain the respective μ_u^* , μ_d^* , μ_s^* , and μ_e by solving Eqs. (27)–(30) with the help of Eq. (12). The number densities n_u , n_d , n_s , and n_e can then be obtained. The energy density is calculated by Eq. (14), while the pressures are, respectively, obtained from Eqs. (21) and (22) for different values of B , D , and B_m .

In Fig. 1, we give the energy per baryon of SQM, MSQM, CFL matter, and MCFL matter, respectively, as a function of

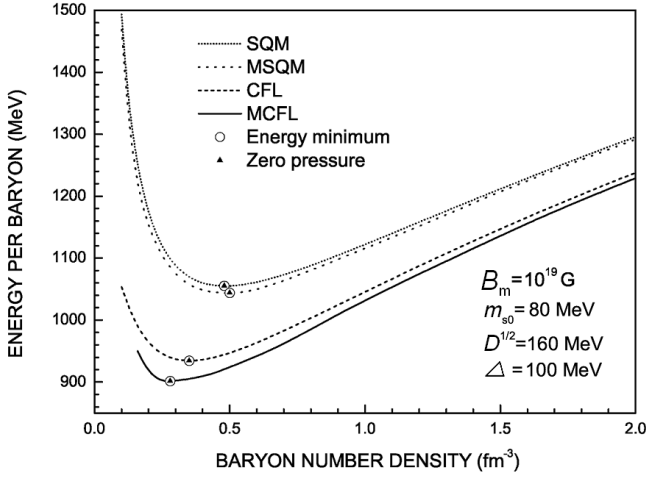


Fig. 1. Comparison of the energy per baryon versus the baryon number density for SQM, MSQM, CFL matter, and MCFL matter, respectively. The pressure at the energy minimum is exactly zero for each quark matter phase.

the baryon number density. Because the current mass of u/d quarks are very small, we simply take $m_{u0} = m_{d0} = 0$. But for the current mass of the strange quark, we take $m_{s0} = 80$ MeV. The electron does not participate in the strong interaction, its mass is very tiny, and also ignored. For convenience of comparison with previous works, we take the pairing parameter to be $\Delta = 100$ MeV. The bag constant, B , and the confinement parameter, D , are taken to be $B^{1/4} = 140$ MeV and $D^{1/2} = 120$ MeV. In our calculation, we assume the magnetic field to be constant with its direction along the z axis. Because the system will become unstable when the magnetic field strength is higher than 10^{20} G, as discussed by Chakrabarty [23], we take the magnetic field strength to be $B_m = 10^{19}$ G. From top to bottom in Fig. 1, there are three features. Firstly, the energy minimum (the solid triangle) corresponds exactly to the zero pressure (open circle) for each case. In fact, the exact coincidence of the lowest energy state and zero pressure is a basic requirement of the fundamental thermodynamics, as pointed out in Ref. [15], and derived in detail in Ref. [1]. Secondly, the energy per baryon of CFL matter and MCFL matter is lower than that of SQM and MSQM. So we can see that the quark pairing effect greatly increases the stability chances of SQM. Thirdly, the energy per baryon of MSQM and MCFL matter is lower than that of SQM and CFL matter, respectively. We can see that the external magnetic field in a proper magnitude lowers the energy per baryon through the rearrangement of the Landau energy level of magnetized quark matter. Generally, we have the inequality relation of the energy per baryon as

$$\left. \frac{E}{n} \right|_{\text{MCFL}} < \left. \frac{E}{n} \right|_{\text{CFL}} < \left. \frac{E}{n} \right|_{\text{MSQM}} < \left. \frac{E}{n} \right|_{\text{SQM}}. \quad (31)$$

In Fig. 2, we give the minimum energy per baryon of MCFL phase as a function of the magnetic field strength. When the magnetic field strength, B_m , is small, it is obvious that the energy is nearly constant. The energy per baryon

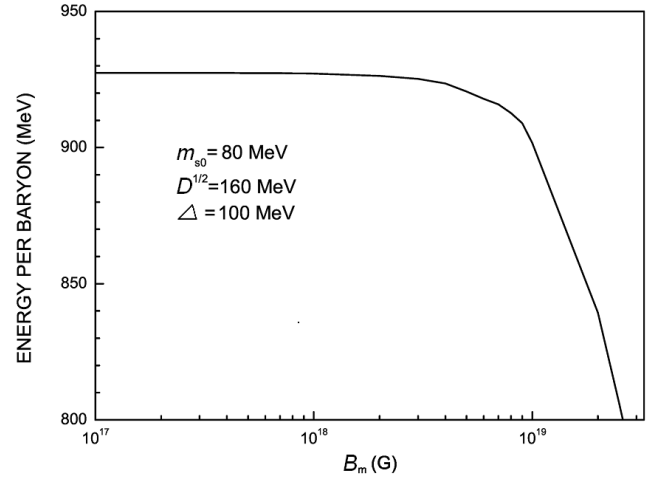


Fig. 2. The minimum energy per baryon of MCFL matter as a function of the magnetic field strength. Relevant parameters are indicated in the figure.

starts to decrease obviously as a function of the magnetic field between 10^{18} G and 10^{19} G. When the magnetic field strength exceeds 10^{19} G, the energy per baryon decreases quickly. Therefore, an external magnetic field with proper strength lowers the energy per baryon. In this regard, one should note that the energy from the external magnetic field was not added. Otherwise, the total energy per baryon will increase.

There are different views on whether or not the energy contribution from the magnetic field should be included. If one would like to include the field contribution, one should know how the quark matter produce the magnetic field. As mentioned in the introduction, the origin of the the strong magnetic field is presently not very clear, although some ways to understand it are available. Therefore, we treat the magnetic field as an externally forced field.

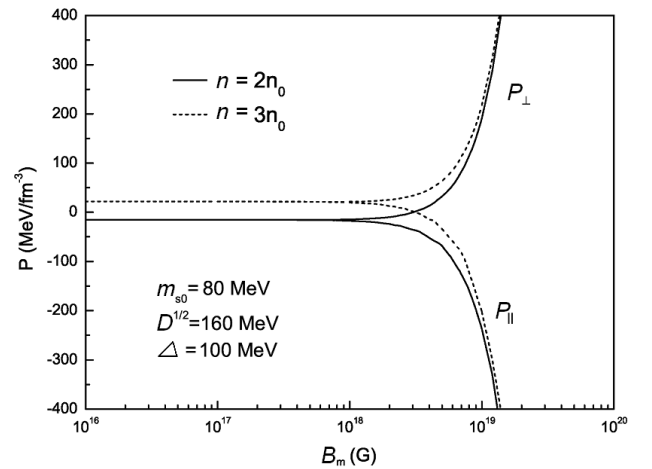


Fig. 3. The pressure in MCFL matter as a function of the magnetic field strength at two (the solid line) and three (the short-dot line) times the nuclear saturation density $n_0 = 0.165 \text{ fm}^{-3}$.

As is well known, the space becomes anisotropic when an external magnetic field is presented. To compare the magnitude of the longitudinal and transverse pressures, we plot, in Fig. 3, the $P_{\parallel}$ and $P_{\perp}$ at the given densities of $n = 2n_0$ (the solid line) and $n = 3n_0$ (the short-dot line) where $n_0 = 0.165 \text{ fm}^{-3}$ is the nuclear saturation density, as functions of the magnetic field strength. The difference between $P_{\parallel}$ and $P_{\perp}$ reflects the breaking of the rotational symmetry by the magnetic field. We can see that the pressure stays as constant when the magnetic field strength is lower than 10^{18} G . When the magnetic field strength is larger than 10^{18} G , the pressure anisotropy starts to become noticeable: the parallel pressure, $P_{\parallel}$, increases far beyond the constant value, while the vertical pressure, $P_{\perp}$, decrease from the constant value.

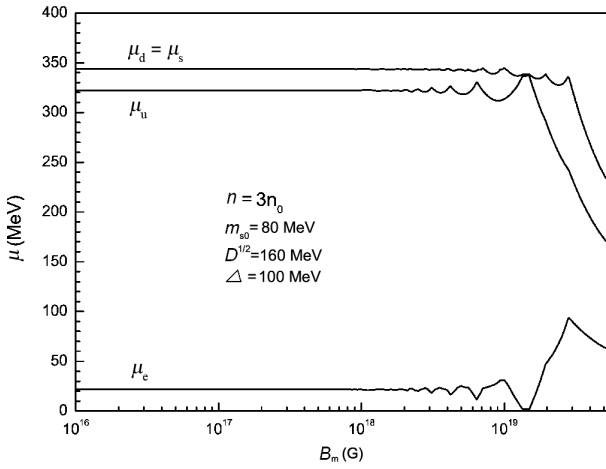


Fig. 4. The chemical potential of quarks in MCFL matter as a function of the magnetic field strength for $n = 3n_0$.

In Fig. 4, the chemical potentials are shown as functions of the magnetic field strength for $n = 3n_0$. When the magnetic

field strength, B_m , is small, all the chemical potentials, μ_u , μ_d , μ_s , and μ_c , are approximately constant. The chemical potentials oscillate when the magnetic field strength is in the range of 10^{18} G to 10^{19} G . When the magnetic field strength exceeds a critical value, about 10^{19} G , the energy decreases fast. At the range of $B_m \geq 10^{19} \text{ G}$, the chemical potentials decrease with the magnetic field. This is also the reason why the pressure oscillates and decreases when increasing the magnetic field.

IV. CONCLUSION

We have extended the equiv-particle model with density-dependent quark masses to the investigation of MCFL matter in an external strong magnetic field. The exact zero pressure at the energy minimum demonstrates the self-consistency of our treatment. The stability property of MCFL matter is calculated and compared with SQM, MSQM, and CFL matter. For a proper magnitude of the external magnetic field, the MCFL phase is more stable than the other phases of quark matter. The impact of the external strong magnetic field on the properties of MCFL matter is changed by the magnetic-field strength. When $B_m \leq 10^{18} \text{ G}$, the magnetic field affects the properties of the system only slightly. When $10^{18} \text{ G} \leq B_m \leq 10^{19} \text{ G}$, Landau oscillation appears in the chemical potentials, and the effect becomes obvious. When $B_m \geq 10^{19} \text{ G}$, the maximum Landau level ν_{max} only takes the lowest value, and accordingly, the effects on the chemical potential, energy density, pressure are all dramatically large. Importantly in this case, the minimum energy per baryon gets smaller.

Naturally, the present study is limited in many aspects while the quark matter field is rapidly developing [56]. Therefore, further investigations are needed.

- [1] Xia C J, Peng G X, Chen S W, *et al.* Thermodynamic consistency, quark mass scaling, and properties of strange matter. *Phys Rev D*, 2014, **89**: 105027. DOI: [10.1103/PhysRevD.89.105027](https://doi.org/10.1103/PhysRevD.89.105027)
- [2] Peng G X, Li A and Lomberto U. Deconfinement phase transition in hybrid neutron stars from the Brueckner theory with three-body forces and a quark model with chiral mass scaling. *Phys Rev C*, 2008, **77**: 065807. DOI: [10.1103/PhysRevC.77.065807](https://doi.org/10.1103/PhysRevC.77.065807)
- [3] Wu C, Qian W L, Ma Y G, *et al.* Quark deconfinement phase transition in nuclear matter for improved quark mass density-dependent model. *Europhys Lett*, 2012, **98**: 21001. DOI: [10.1209/0295-5075/98/21001](https://doi.org/10.1209/0295-5075/98/21001)
- [4] Wen X J. Equation of state in hybrid stars and the stability window of quark matter. *Physica A*, 2013, **392**: 4388–4399. DOI: [10.1016/j.physa.2013.05.050](https://doi.org/10.1016/j.physa.2013.05.050)
- [5] Shao G Y, Colonna M, Di Toro M, *et al.* Isoscalar-vector interaction and its influence on the hadron-quark phase transition. *Nucl Sci Tech*, 2013, **24**: 050523. DOI: [10.13538/j.1001-8042/nst.2013.05.023](https://doi.org/10.13538/j.1001-8042/nst.2013.05.023)
- [6] Liu F M. Explore QCD phase transition with thermal photons. *Nucl Sci Tech*, 2013, **24**: 050524. DOI: [10.13538/j.1001-8042/nst.2013.05.024](https://doi.org/10.13538/j.1001-8042/nst.2013.05.024)
- [7] Peng G X, Wen X J and Chen Y D. New solutions for the color-flavor locked strangelets. *Phys Lett B*, 2006, **633**: 314–318. DOI: [10.1016/j.physletb.2005.11.081](https://doi.org/10.1016/j.physletb.2005.11.081)
- [8] Madsen J and Larsen J M. Strangelets as cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff. *Phys Rev Lett*, 2003, **90**: 121102. DOI: [10.1103/PhysRevLett.90.121102](https://doi.org/10.1103/PhysRevLett.90.121102)
- [9] Zheng X P, Zhou X and Yang S H. Novel non-equilibrium phase transition caused by non-linear hadronic-quark phase structure. *Phys Lett B*, 2014, **729**: 79–84. DOI: [10.1016/j.physletb.2014.01.004](https://doi.org/10.1016/j.physletb.2014.01.004)
- [10] Dexheimer V, Torres J R and Menezes D P. Stability windows for proto-quark stars. *Eur Phys J C*, 2013, **73**: 1–11. DOI: [10.1140/epjc/s10052-013-2569-5](https://doi.org/10.1140/epjc/s10052-013-2569-5)
- [11] Witten E. Cosmic separation of phases. *Phys Rev D*, 1984, **30**: 272–298. DOI: [10.1103/PhysRevD.30.272](https://doi.org/10.1103/PhysRevD.30.272)
- [12] Farhi E and Jaffe R L. Strange matter. *Phys Rev D*, 1984, **30**: 2379–2390. DOI: [10.1016/0375-9474\(86\)90583-X](https://doi.org/10.1016/0375-9474(86)90583-X)

- [13] Peng G X, Chiang H C, Ning P Z, *et al.* Charge and critical density of strange quark matter. *Phys Rev C*, 1999, **59**: 3452–3454. DOI: [10.1103/PhysRevC.59.3452](https://doi.org/10.1103/PhysRevC.59.3452)
- [14] Peng G X, Chiang H C, Yang J J, *et al.* Mass formulas and thermodynamic treatment in the mass-density-dependent model of strange quark matter. *Phys Rev C*, 1999, **61**: 015201. DOI: [10.1103/PhysRevC.61.015201](https://doi.org/10.1103/PhysRevC.61.015201)
- [15] Peng G X, Chiang H C, Zou B S, *et al.* Thermodynamics, strange quark matter, and strange stars. *Phys Rev C*, 2000, **62**: 025801. DOI: [10.1103/PhysRevC.62.025801](https://doi.org/10.1103/PhysRevC.62.025801)
- [16] Weber F. Strange quark matter and compact stars. *Prog Part Nucl Phys*, 2005, **54**: 193–288. DOI: [10.1103/PhysRevC.62.025801](https://doi.org/10.1103/PhysRevC.62.025801)
- [17] Schertler K, Greiner C and Thoma M. Medium effects in strange quark matter and strange stars. *Nucl Phys A*, 1997, **616**: 659–679. DOI: [10.1016/S0375-9474\(97\)00014-6](https://doi.org/10.1016/S0375-9474(97)00014-6)
- [18] Buballa M and Oertel M. Strange quark matter with dynamically generated quark masses. *Phys Lett B*, 1999, **457**: 261–267. DOI: [10.1016/S0370-2693\(99\)00533-X](https://doi.org/10.1016/S0370-2693(99)00533-X)
- [19] Bao T, Liu G Z, Zhao E G, *et al.* Self-consistently thermodynamic treatment for strange quark matter in the effective mass bag model. *Eur Phys J A*, 2008, **38**: 287–293. DOI: [10.1140/epja/i2008-10682-6](https://doi.org/10.1140/epja/i2008-10682-6)
- [20] Perez-Garcia M A, Silk J and Stone J R. Dark matter, neutron stars, and strange quark matter. *Phys Rev Lett*, 2010, **105**: 141101. DOI: [10.1103/PhysRevLett.105.141101](https://doi.org/10.1103/PhysRevLett.105.141101)
- [21] Rajagopal K and Wilczek F. Enforced electrical neutrality of the color-flavor locked phase. *Phys Rev Lett*, 2001, **86**: 3492–3495. DOI: [10.1103/PhysRevLett.86.3492](https://doi.org/10.1103/PhysRevLett.86.3492)
- [22] Alford M, Rajagopal K, Reddy S, *et al.* Minimal color-flavor-locked-nuclear interface. *Phys Rev D*, 2001, **64**: 074017. DOI: [10.1103/PhysRevD.64.074017](https://doi.org/10.1103/PhysRevD.64.074017)
- [23] Chakrabarty S. Quark matter in a strong magnetic field. *Phys Rev D*, 1996, **54**: 1306–1316. DOI: [10.1103/PhysRevD.54.1306](https://doi.org/10.1103/PhysRevD.54.1306)
- [24] Chu P C and Chen L W. Quark stars in strong magnetic fields. *Phys Rev D*, 2014, **90**: 063013. DOI: [10.1103/PhysRevD.90.063013](https://doi.org/10.1103/PhysRevD.90.063013)
- [25] Duncan R C and Thompson C. Formation of very strongly magnetized neutron stars-implications for gamma-ray bursts. *Astrophys J*, 1992, **392**: L9–L13. DOI: [10.1086/186413](https://doi.org/10.1086/186413)
- [26] Kouveliotou C, Desai U, Cline T, *et al.* A 2.2 second period in the 1984 August 5 gamma-ray burst. *Astrophys J*, 1988, **330**: L101–L105. DOI: [10.1086/185214](https://doi.org/10.1086/185214)
- [27] Ferrer E J, Incera V de La, Keith J P, *et al.* Equation of state of a dense and magnetized fermion system. *Phys Rev C*, 2010, **82**: 065802. DOI: [10.1103/PhysRevC.82.065802](https://doi.org/10.1103/PhysRevC.82.065802)
- [28] Tatsumi T, Maruyama T, Nakano E, *et al.* Ferromagnetism in quark matter and origin of the magnetic field in compact stars. *Nucl Phys A*, 2006, **774**: 827–830. DOI: [10.1016/j.nuclphysa.2006.06.145](https://doi.org/10.1016/j.nuclphysa.2006.06.145)
- [29] Vink J and Kuiper L. Supernova remnant energetics and magnetars: no evidence in favour of millisecond proto-neutron stars. *Mon Not R Astron Soc Lett*, 2006, **370**: L14–L18. DOI: [10.1111/j.1745-3933.2006.00178.x](https://doi.org/10.1111/j.1745-3933.2006.00178.x)
- [30] Felipe R G, Cuesta H M, Martinez A P, *et al.* Quantum instability of magnetized stellar objects. *Chinese J Astron Ast*, 2005, **5**: 399–411. DOI: [10.1088/1009-9271/5/4/007](https://doi.org/10.1088/1009-9271/5/4/007)
- [31] Mizher A J, Chernodub M and Fraga E S. Phase diagram of hot QCD in an external magnetic field: possible splitting of deconfinement and chiral transitions. *Phys Rev D*, 2010, **82**: 105016. DOI: [10.1103/PhysRevD.82.105016](https://doi.org/10.1103/PhysRevD.82.105016)
- [32] Frolov I, Zhukovsky V C and Klimenko K. Chiral density waves in quark matter within the Nambu–Jona-Lasinio model in an external magnetic field. *Phys Rev D*, 2010, **82**: 076002. DOI: [10.1103/PhysRevD.82.076002](https://doi.org/10.1103/PhysRevD.82.076002)
- [33] Fayazbakhsh S and Sadooghi N. Phase diagram of hot magnetized two-flavor color-superconducting quark matter. *Phys Rev D*, 2011, **83**: 025026. DOI: [10.1103/PhysRevD.83.025026](https://doi.org/10.1103/PhysRevD.83.025026)
- [34] Rabhi A and Providencia C M C. Quark matter under strong magnetic fields in chiral models. *Phys Rev C*, 2011, **83**: 055801. DOI: [10.1103/PhysRevC.83.055801](https://doi.org/10.1103/PhysRevC.83.055801)
- [35] Torres J R and Menezes D P. Quark matter equation of state and stellar properties. *Europhys Lett*, 2013, **101**: 42003. DOI: [10.1209/0295-5075/101/42003](https://doi.org/10.1209/0295-5075/101/42003)
- [36] Avancini S, Menezes D and Providencia C. Finite temperature quark matter under strong magnetic fields. *Phys Rev C*, 2011, **83**: 065805. DOI: [10.1103/PhysRevC.83.065805](https://doi.org/10.1103/PhysRevC.83.065805)
- [37] Chen Y D, Peng G X and Wen X J. Color-flavor locked strange quark matter in a mass density-dependent model. *High Energ Phys Nuc*, 2007, **31**: 269–273.
- [38] Hou J X, Peng G X, Xia C J, *et al.* Magnetized strange quark matter in a mass-density-dependent model. *Chinese Phys C*, 2014, **37**: 010201. DOI: [10.1088/1674-1137/39/1/015101](https://doi.org/10.1088/1674-1137/39/1/015101)
- [39] Paulucci L and Horvath J. Color-flavor locked strange matter and strangelets at finite temperature. *Phys Rev C*, 2008, **78**: 064907. DOI: [10.1103/PhysRevC.78.064907](https://doi.org/10.1103/PhysRevC.78.064907)
- [40] Ferrer E J, V de La Incera and Manuel C. Magnetic color-flavor locking phase in high-density QCD. *Phys Rev Lett*, 2005, **95**: 152002. DOI: [10.1103/PhysRevLett.95.152002](https://doi.org/10.1103/PhysRevLett.95.152002)
- [41] Ferrer E J, V de La Incera and Manuel C. Color-superconducting gap in the presence of a magnetic field. *Nucl Phys B*, 2006, **747**: 88–112. DOI: [10.1016/j.nuclphysb.2006.04.013](https://doi.org/10.1016/j.nuclphysb.2006.04.013)
- [42] Feng B, Huang R and Jia Y. Gauge amplitude identities by on-shell recursion relation in S-matrix program. *Phys Lett B*, 2011, **695**: 350–353. DOI: [10.1016/j.physletb.2010.11.011](https://doi.org/10.1016/j.physletb.2010.11.011)
- [43] Noronha J L and Shovkovy I A. Color-flavor locked superconductor in a magnetic field. *Phys Rev D*, 2007, **76**: 105030. DOI: [10.1103/PhysRevD.76.105030](https://doi.org/10.1103/PhysRevD.76.105030)
- [44] Fukushima K and Warringa H J. Color superconducting matter in a magnetic field. *Phys Rev Lett*, 2008, **100**: 032007. DOI: [10.1103/PhysRevLett.100.032007](https://doi.org/10.1103/PhysRevLett.100.032007)
- [45] Wen X J. Color-flavor locked strange quark matter in a strong magnetic field. *Phys Rev D*, 2013, **88**: 034031. DOI: [10.1103/PhysRevD.88.034031](https://doi.org/10.1103/PhysRevD.88.034031)
- [46] Luo L J, Yan J C, Zong H S, *et al.* A thermodynamically consistent quasi-particle model without density-dependent infinity of the vacuum zero-point energy. *Eur Phys J C*, 2013, **73**: 1–9. DOI: [10.1016/j.physletb.2012.03.079](https://doi.org/10.1016/j.physletb.2012.03.079)
- [47] Lugones G and Horvath J E. Quark-diquark matter equation of state and compact star structure. *Int J Mod Phys D*, 2003, **12**: 495–507. DOI: [10.1142/S0218271803002755](https://doi.org/10.1142/S0218271803002755)
- [48] Zheng X P, Liu X W, Kang M, *et al.* Bulk viscosity of strange quark matter in a density-dependent quark mass model and dissipation of the r mode in strange stars. *Phys Rev C*, 2004, **70**: 015803. DOI: [10.1103/PhysRevC.70.015803](https://doi.org/10.1103/PhysRevC.70.015803)
- [49] Wen X J, Zhong X H, Peng G X, *et al.* Thermodynamics with density and temperature dependent particle masses and properties of bulk strange quark matter and strangelets. *Phys Rev C*, 2005, **72**: 015204. DOI: [10.1103/PhysRevC.72.015204](https://doi.org/10.1103/PhysRevC.72.015204)
- [50] Modarres M and Gholizade H. Strange quark matter in the framework of one gluon exchange and density and temperature dependent particle mass models. *Int J Mod Phys E*, 2008, **17**: 1335–1355. DOI: [10.1142/S0218301308010453](https://doi.org/10.1142/S0218301308010453)
- [51] Li A, Peng G X and Lu J X. Strange star candidates re-

- vised within a quark model with chiral mass scaling. *Res Astron Astrophys*, 2011, **11**: 482–490. DOI: [10.1088/1674-4527/11/4/010](https://doi.org/10.1088/1674-4527/11/4/010)
- [52] Chen S W and Peng G X. Color-flavor locked strangelets in a new quark model with linear confinement and coulomb-type interactions. *Commun Theor Phys*, 2012, **57**: 1037–1045. DOI: [10.1088/0253-6102/57/6/16](https://doi.org/10.1088/0253-6102/57/6/16)
- [53] Chen S W, Gao L and Peng G X. One-gluon-exchange effect on the properties of quark matter and strange stars. *Chinese Phys C*, 2012, **36**: 947–953. DOI: [10.1088/1674-1137/36/10/005](https://doi.org/10.1088/1674-1137/36/10/005)
- [54] Torres J R and Menezes D P. Quark matter equation of state and stellar properties. *Europhys Lett*, 2013, **101**: 42003. DOI: [10.1209/0295-5075/101/42003](https://doi.org/10.1209/0295-5075/101/42003)
- [55] Chu P C and Chen L W. Quark matter symmetry energy and quark Stars. *Astrophys J*, 2014, **780**: 135. DOI: [10.1088/0004-637X/780/2/135](https://doi.org/10.1088/0004-637X/780/2/135)
- [56] Xu J F, Peng G X, Liu F, *et al.* Strange matter and strange stars in a thermodynamically self-consistent perturbation model with running coupling and running strange quark mass. *Phys Rev D*, 2015, **92**: 025025. DOI: [10.1103/PhysRevD.92.025025](https://doi.org/10.1103/PhysRevD.92.025025)