Instability of OTSG in movable NPP by using multivariable frequency domain method

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Abstract The instability occurring in OTSG (Once-Through Steam Generator) of movable nuclear power plants is presented by a multivariable frequency domain theory. As concerning coupling interactions of OTSG tubing, it is more efficient for analyzing the instability of OTSG compared the common single variable method. A mathematical model for the system is derived from the fundamental equations by using the perturbation, Laplace-transform and the nodalization techniques. The stable boundary and parameters which influence the stability of the system are evaluated through computer simulation. Numerical examples are given in the paper and the predictions of the model agree with the experimental results well.

Key words OTSG, Flow instability, Two-phase flow, Multivariable frequency domain theory

1 Introduction

Two-phase flow instability is usually considered in many industrial systems and equipment because it may cause flow excursions or oscillations of flow and thermal parameters. The undesirable flow instability may result in sustained flow oscillations and boiling crisis, and induce boiling crisis and wall temperature oscillation that may eventually lead to tube disrepair due to thermal fatigue[1], affecting safety of the systems and equipment directly. For a movable NPP (Nuclear Power Plants) operating at low pressures (< 3MPa), the two-phase flow instability occurring in OTSG (Once- Through Steam Generator) is vulnerable. Studies on mechanism of the two-phase instability are important for designing and operating a movable NPP.

Researches in two-phase flow stability have been focused on a single channel of OTSG, while Lee *et al.* [2] reported a single-variable frequency-domain model, which neglected the reciprocal influences between heat transfer tubes and external loop. On the other hand, the coupling effect plays an important role in the OTSG under certain conditions. However, the model of three parallel channel systems obtained by Romberg[3] using electrical network theory does not include the primary and secondary side fluids.

In this paper, we propose a new method, based on the multivariable frequency-domain theory, for investigating the flow instability of a multi-channel system, such as OTSG in a movable NPP. We focus on the Density Wave Oscillation (DWO), the most common type of thermal-hydrodynamic instability. The transfer matrix of the multi-IO (Input and Output) is obtained using linear perturbation theory and Laplace transformation, and system stability is determined by Nyquist Stability Criterion of multi-variable frequency-domain control theory. The results show that the multivariable frequency-domain theory is suitable for OTSG concerning bi-influence of the tubes and it explains some instability phenomena in a multi-channel system.

2 Mathematical model

Fig.1 is a schematic diagram of the secondary side loop in OTSG. The heated section of every channel can be divided into three regions (Fig. 2): the preheat,

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two-phase and superheat regions. Because of the non-uniform distribution of heat flux in the whole channel, each region is subdivided axially into small control volumes, in which some factors are assumed to be uniform, such as heat flux profile and heat transfer coefficient. The primary side and tube wall is considered as lumped parameters. Fig.3 shows schematically a control volume, where T, w, h, and P represent the temperature, flow rate, enthalpy and pressure, respectively.

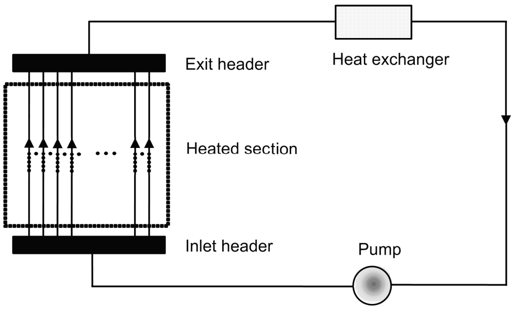

Fig.1 Schematic diagram of the OTSG.

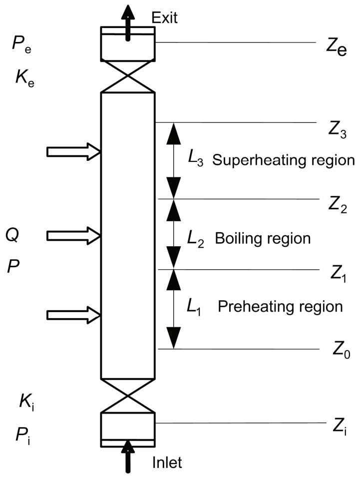

Fig. 2 Schematic diagram of a channel.

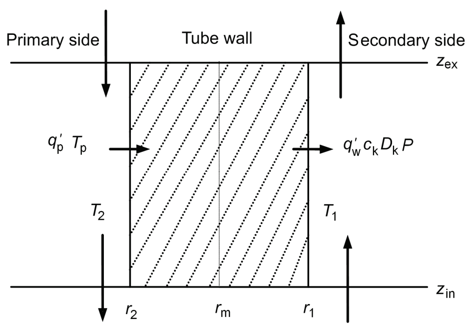

Fig.3 Schematic diagram of a control volume.

2.1 Fundamental equations

The dynamics of two-phase flow is analyzed by the equations based on the following assumptions, (1) subcooled boiling can be neglected; (2) thermo-dynamic equilibrium is established between the two phases; (3) the drift-flux model is used to describe the flow in the two-phase region; (4) compressibility of liquid in a single phase region can be neglected; and (5) densities for the saturated liquid and vapor in the two-phase region are constant

Then, the conservation equations are expressed as follows [2, 3].

2.1.1 Single-phase region

For the one-dimensional incompressible flow in a single phase region, the conservation equations for mass, energy and momentum are Eq. (1), (2) and (3), respectively.

$$\partial u/\partial Z=0 \tag{1}$$

$$\partial h/\partial t+u\partial h/\partial Z=q_w''/(\rho_L A_c) \tag{2}$$

$$-(1/\rho_L)\partial P/\partial Z=\partial u/\partial t+u\partial u/\partial Z+fu^2/(2D)+g \tag{3}$$

where, u, h and P are velocity, enthalpy and pressure, respectively; P_w, A_c, D and ρ_L are respectively wet perimeter, flow area, hydraulic diameter, and liquid density; and g, f and q_w'' are gravity acceleration, friction factor and wall heat flux, respectively, for a given geometry.

2.1.2 Two-phase region

To account for the relative motion between the phases, the drift-flux formulation, derived by Zuber [4], is used. For mass conservation of the vapor phase,

$$\partial(\alpha\rho_g)/\partial t + \partial(\alpha\rho_g u_g)/\partial z = \Gamma_g \tag{4}$$

where Γ_g is the production rate of gas phase. For mass conservation of the mixture,

$$\partial\rho_m/\partial t + \partial(\rho_m u_m)/\partial z = 0 \tag{5}$$

For energy conservation of the mixture,

$$\rho_m(\partial h_m/\partial t + u_m\partial h_m/\partial z) = q_w''P_w/A_c-\partial P/\partial t-\partial[(\rho_L-\rho_m)V'^2_{gi}h_g/(\rho_m\Delta\rho)]/\partial z \tag{6}$$

For momentum conservation of the mixture:

$$\rho_m(\partial u_m/\partial t + u_m\partial u_m/\partial z) = -\partial P/\partial z -\partial\{(\rho_L-\rho_g)\rho_L\rho_g V'^2_{gi}/[(\rho_m-\rho_g)\rho_m]\}/\partial Z-\partial t/\partial z-\rho_m g \tag{7}$$

where, subscripts L, g and m denote liquid phase, gas phase, and mixture, respectively.

2.1.3 Dynamics of heat wall

The heat conduction equation of heater wall can be written for cylindrical geometry as,

$$(1/r)\partial(r\partial T_t/\partial r)/\partial r = (1/k)\partial T_t/\partial t \qquad r_a \le r \le r_b \tag{8}$$

where, T_t and k are temperature and thermal diffusivity of the tube, respectively. The heat flux q_w'' in Eqs.(2) and (6) can be related to dynamics of the tube wall by,

$$q''_w = \lambda\partial T_t/\partial r|_{r=ra} = \alpha_w(T_t - T_w) \tag{9}$$

where, λ is conductivity of the tube wall, α_w is the heat transfer coefficient, and T_w is temperature in the wall surface.

2.2 Transfer matrix equations

All the equations for every control volume are perturbed with respect to time, linear and Laplace transform into frequency-domain to yield transfer function relationships between various parameters. After rearranging, the transfer matrix equations of the control volumes can be obtained. The multiple coupling relations in the matrix equations is shown in Fig.4, where $\boldsymbol{G}_1(s)$, $\boldsymbol{G}_2(s)$ and $\boldsymbol{G}_3(s)$ are all 2×2 transfer function matrices, $\boldsymbol{G}_4(s)$ is a 4×4 transfer function matrix, q'' is the heat flux through the tube wall, f_1 and f_2 are parameters of representing the feedback effects of heat transfer coefficient and fluid temperature in both sides. Subscripts 1, 2 and w denote primary side, secondary side and tube wall, respectively.

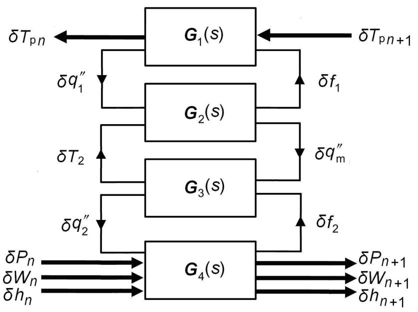

Fig. 4 Feedback relations in a control volume.

The transfer matrix equations of control volumes of preheat and superheat regions can be obtained similarly. The transfer matrix equations of every control volume of every channel, together with all the boundary condition equations can be manipulated into transfer matrix equations as,

$$\delta\boldsymbol{W} = \boldsymbol{Q}(s)\delta\Delta\boldsymbol{P} \tag{10}$$

$$\delta\Delta\boldsymbol{P} = \delta\Delta\boldsymbol{R} - \boldsymbol{F}(s)\delta\boldsymbol{W} \tag{11}$$

where, $\delta\boldsymbol{W}$, $\delta\Delta\boldsymbol{P}$, and $\delta\boldsymbol{R}$ are the perturbation vectors of flow rate at the inlet of every channel, the external input perturbations, the pressure drop in preheat region, respectively; $\boldsymbol{Q}(s)$ and $\boldsymbol{F}(s)$ are the system matrices. A standard feedback configuration of multivariable system is shown in Fig.5.

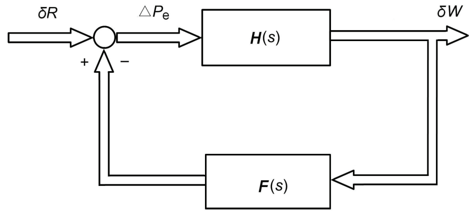

Fig. 5 Transfer function matrix model of OTSG.

3 Stability analysis method

The closed-loop characteristic polynomial $\rho_c(s)$ of the system can be expressed as [5]

$$\rho_c(s) = \rho_o(s)\det\boldsymbol{D}(s) \tag{12}$$

where, $\rho_o(s)$ is the open-loop characteristic polynomial, $\boldsymbol{D}(s)$ is the return-difference matrix, and its determinant can be expressed as

$$\det\boldsymbol{D}(s)–\det[\boldsymbol{I}_{k+}\boldsymbol{Q}(s)\boldsymbol{F}(s)] \tag{13}$$

where, $\boldsymbol{I}_k$ is a unit matrix of order K and K is the number of the channels. The determinant in Eq.(13) can be easily calculated because the $\boldsymbol{Q}(s)$ is a diagonal matrix and $\boldsymbol{F}(s)$ is a symmetric matrix.

Based on the Nyquist stability theorem of multivariable system [6], the operating state of the flow system is stable if and only if

$$N_d = P_o \tag{14}$$

where, N_d is the anticlockwise encirclement of the origin by the Nyquist locus of $\det\boldsymbol{D}(s)$ as s traverses the standard Nyquist contour in clockwise direction, P_o is the total number of right-half plane poles of $\boldsymbol{Q}(s)\boldsymbol{F}(s)$. It can be shown that P_o is equal to zero.

According to the generated Nyquist stability criterion [7], another form of the stability criterion can be obtained

$$\sum_{i=1}^{K} N_i = P_0 \tag{15}$$

Eq.(15) can be stated in the following way. The system in Fig.5 will be closed-loop stable if, and only if, the net sum of anticlockwise encirclements of the critical point $(-1, j_0)$ by the set of characteristic loci of

$\boldsymbol{Q}(s)\boldsymbol{F}(s)$ or $\boldsymbol{F}(s)\boldsymbol{Q}(s)$ is equal to the number of right-half plane poles for the open-loop system.

A computer program was developed for the calculation, which is based on the mathematical model and the stability analysis method described above. The computer code can be used to predict the stability behavior of OTSG and other multi-channel boiling flow system, and to study effects of changes in system parameters and the coupling interactions of the system.

4 Results and discussion

For the OTSG, we assume that the inlet or exit pressure for each channel are equal to the common inlet or exit headers pressure, and the inlet enthalpy is held constant. Therefore, the characteristic equation can be obtained from the Eq.(13)

$$\boldsymbol{F}_1(s)–k_f\boldsymbol{F}_1(s) = 0 \tag{16}$$

where

$$k_f = K\ k_P \tag{17}$$

$$k_P = -\ \delta\Delta \boldsymbol{P}_t/\delta \boldsymbol{W}_t \tag{18}$$

$$F_1(\mathbf{s}) = \prod_{i=1}^{K}[1+\mathrm{Y}_i(\mathbf{s})/\mathrm{X}_i(\mathbf{s})] \tag{19}$$

and

$$F_2(\mathrm{s}) = \prod_{i=1}^{K}\left\{[1/X_i(\mathrm{s})]\cdot\prod_{n\neq i}^{K}[1+Y_n(\mathrm{s})/X_n(\mathrm{s})]\right\}\Big/K \tag{20}$$

and $X_k(s)$, $Y_k(s)$, $k=\{1,2,\cdots,K\}$, are transfer functions of pressure drop perturbation versus flow rate perturbation of the k^{th} channel, and they can be calculated from the transfer function matrices of control volumes of the channels.

The parameter k_f is related to the number of the channel K and the slop of the external characteristic curve k_p. Therefore, k_f represents the effects of coupling interactions between channels and other parts of the loop. For the reason, k_f may be called "coupling factor". There are three special cases of Eq. (13) with regard to k_f.

1) $k_f = 0$

In this case, the coupling effects are negligible and Eq. (13) can be simplified to

$$\boldsymbol{F}_1(s)= 0 \tag{21}$$

Eq.(21) is equivalent to

$$\begin{cases} 1 + Y_1(s)/X_1(s) =0 \\ 1 + Y_2(s)/X_2(s) =0 \\ \quad\vdots \\ 1 + Y_k(s)/X_k(s) =0 \end{cases} \tag{22}$$

The system is stable if and only if the Nyquist plots of every equation above don't encircle the critical point($-1, j_0$), that is

$$\sum_{i=1}^{K}\mathrm{N}_i = \mathrm{P}_0 \tag{23}$$

where N_i is the clockwise encirclements of the origin by the Nyquist plot of $Y_i(s)/X_i(s)$. In this case the problem of multi-channel can be splited to that of single channel. The stability criterion of Eq. (23) can easily be gotten from Eq. (15) because ***Q(s)*** and ***F(s)*** are both diagonal matrices when the coupling effects are not present.

2) $k_f = \infty$

When the external characteristic curve is very hard, the value of k_f becomes very large (Fig.6). In this case, Eq. (16) can be simplified to the form

$$\boldsymbol{F}_2(s)=0 \tag{24}$$

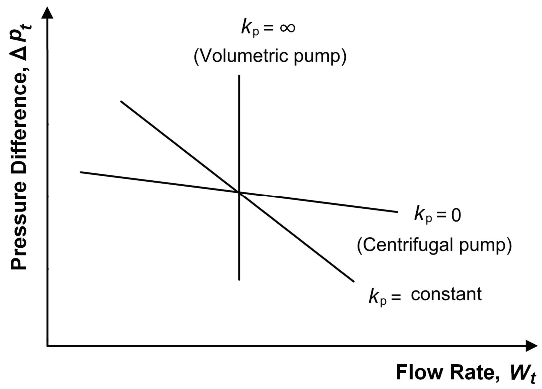

Fig.6 External characteristic curve.

Combining Eqs.(20) and (24), and let K=3, the characteristic equation becomes

$$H_1(s)H_2(s)+H_2(s)H_3(s)+H_1(s)H_3(s)=0 \tag{25}$$

$$\mathrm{H}_n(\mathrm{s}) = \mathrm{X}_n(\mathrm{s}) + \mathrm{Y}_n(\mathrm{s}), \qquad n = 1,2,3 \tag{26}$$

Eq.(25) is in agreement with the results obtained by Romberg who used the electrical network theory[3]. In practice, this case means that total flow rate is approximately constant while the branch flow rate in different channels exhibit oscillations with phase differences. The phenomena were observed in experiments [8,9].

3) k_f = constant.

When the external curve in vicinity of the operating point may be approximately replaced by a straight line (Fig.6), the k_f take a constant value. If we further assume that every channel is identical, the Eq. (16) can be simplified as following

$$1+[Y(s)-k_{\rm f}]/X(s)=0 \tag{27}$$

where $X(s)=X_1(s)=\cdots=X_k(s)$, $Y(s)=Y_1(s)=\cdots=Y_k(s)$ and $k_{\rm f}=Kk_{\rm p}$ = constant > 0. Therefore, the stability criterion can be described as that if the Nyquist plots of Eq.(27) don't encircles the critical point the flow in the system is stable.

Table 1 Geometry and operating parameters.

Tube inner diameter, $D_{\rm inner}$ /m	0.010
Heated length, L /m	1.5
Tube outer diameter, $D_{\rm outer}$ /m	0.013
Pressure, P /Pa	3×10^6
Temperature of subcooled inlet, $T_{\rm sub}$ /°C	0.0–200.0
Mass flow rate, M /kg·m^{-2}·s^{-1}	220
Inlet orifice loss coefficient, $k_{\rm i}$	0–600.0
Channel power level, Q /kW	40–70

As a numerical example, we take the operating conditions of the experimental set-up in Table 1 for OTSG[11], and predict the stable and unstable boundary by using the computer code, and the results are compared with experimental data[10], as shown in Fig.7, which is the stability map in the $\Delta h_{\rm sub}/h_{\rm lg}$ and $Q/(w{\cdot}h_{\rm lg})$ parameters plane, where $\Delta h_{\rm sub}$, $h_{\rm lg}$, Q, and w are the inlet subcooling (in enthalpy unit), the latent heat, the heater power and the flow rate, respectively. As the heat flux increases, the enthalpy rate increases quickly in the upper part (stable region) of the two parameter plan but increases slowly in the low part (unstable region). The predicted stable-unstable curve agrees with the experimental results.

The inlet subcooling at various coupling factor simulated by the computer code is shown in Fig. 8. It can be seen that the effects of inlet subcooling and coupling factor on stability of the system, where $k_{\rm g}$ is a measure of the stability margin, and it takes the absolute value of the distance between the critical point and cross point at which the locus cross the real axis. As $k_{\rm f}$ increases, the coupling factor $k_{\rm g}$ decreases first and then increases with inlet subcooling of fluid. The results show that when the inlet increasing is low, an increase of the parameter decreases the stability, while the stability increases with the inlet subcooling when it is high.

Many experimental results show the similar phenomena[11]. Furthermore, Fig.8 also shows that changes in coupling factor affect the stability of the system. Many interesting results can be obtained by using the computer program and will be reported in other paper.

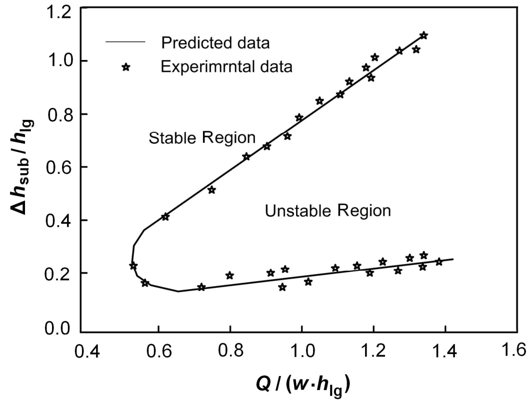

Fig.7 Predicted stable-unstable boundary.

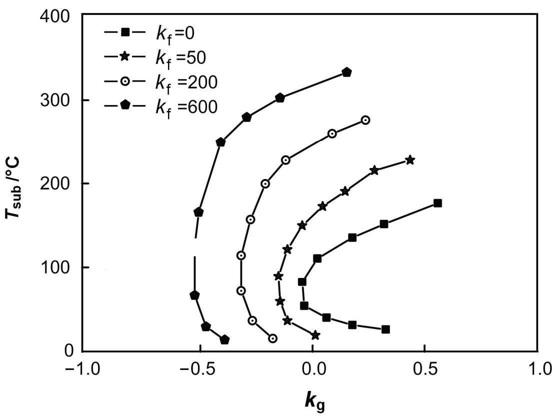

Fig. 8 Effects of the inlet subcooling and coupling factor $k_{\rm g}$.

5 Conclusions

The instability of OTSG and parallel multi-channel system are analyzed by the multivariable method. A transfer function matrix model is obtained according to fundamental equations. A general stability criterion of the OTSG in movable nuclear power plants is obtained by Nyquist stability of multivariable system. The computer program developed from the mathematical model and the analysis method are used to predict the stability boundary and change of system parameters, which is in agreement with the experimental results. The present method enable the various coupling effects in multi-channel system can be conveniently included in the model.

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