Analysis of prompt supercritical process with heat transfer and temperature feedback

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Abstract The prompt supercritical process of a nuclear reactor with temperature feedback and initial power as well as heat transfer with a big step reactivity ($\rho_0>\beta$) is analyzed in this paper. Considering the effect of heat transfer on temperature of the reactor, a new model is set up. For any initial power, the variations of output power and reactivity with time are obtained by numerical method. The effects of the big inserted step reactivity and initial power on the prompt supercritical process are analyzed and discussed. It was found that the effect of heat transfer on the output power and reactivity can be neglected under any initial power, and the output power obtained by the adiabatic model is basically in accordance with that by the model of this paper, and the analytical solution can be adopted. The results provide a theoretical base for safety analysis and operation management of a power reactor.

Key words Neutron kinetics, Prompt supercritical process, Point reactor, Temperature feedback, Heat transfer

1 Introduction

The output power will increase sharply, and the core will be damaged, if a reactivity accident occurs, and analysis of power response to especially big reactivity is of great importance. Studies on prompt and delayed supercritical process with temperature feedback were carried out in recent years [1-11], but all the studies are mainly based on N-F model [1-3], in which the neutron transient occurs in very short time under the reactivity, hence the adoption of the adiabatic model to relate the power and temperature. However, can the adiabatic model, with heat transfer all along, be suitable for ordinary operation state? In this paper, we propose a new physical model and mathematical description for the question. Some results of significance have been obtained.

2 Physical model and analysis

The effect of extraneous neutron source can be neglected for a reactor operated in the critical state with steady output power, and one group of point reactor neutron kinetics equation is

$$\mathrm{d}n(t)/\mathrm{d}t = [\rho(t)-\beta]\, n(t)/l + \lambda C(t) \qquad (1)$$

$$\mathrm{d}C(t)/\mathrm{d}t = \beta\, n(t)/l - \lambda C(t) \qquad (2)$$

where $n(t)$ is the neutron density at time t, $\rho(t)$ is the reactivity, β is total fraction of the delayed neutrons, l is the mean generation time, λ is decay constant of the delayed neutron precursor fission products, and $C(t)$ is average density of the delayed neutron precursors. When both sides of Eqs. (1) and (2) are multiplied with the density/power ratio, $n(t)$ is the reactor power.

Assuming that the reactor has a negative temperature coefficient of reactivity α ($\alpha>0$) on introduction of the big step reactivity ρ_0 ($>\beta$), the reactivity of reactor with temperature feedback is

$$\rho=\rho_0-\alpha[T(t)-T_0] \qquad (3)$$

where $T(t)$ is the reactor temperature at time t, and T_0 is initial temperature of the reactor.

With the big reactivity ρ_0 entering into the reactor, heat can be transferred out of the reactor by the heat exchanger between first and second loops and by the heat leak, then the relation between power and

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temperature should be

$$dT/dt = K_c[n(t) - Q_0] \tag{4}$$

where K_c is the reciprocal of thermal capacity of reactor, and Q_0 is the heat being transferred out per unit time.

It is assumed that Q_0 is determined and $Q_0 \le n_0$. From the derivation of Eq.(3) with respect to t and using Eq.(4) one gets:

$$d\rho/dt = -\alpha K_c(n - Q_0) \tag{5}$$

or: $$d^2\rho/dt^2 = -\alpha K_c\, dn/dt \tag{6}$$

At t=0, when a big step reactivity $\rho_0(>\beta)$ goes into the reactor, a prompt supercritical state will occur, and the power increases to high level so sharply that at $t \ge 0$ the contribution of delayed neutron precursors is negligible[1-3], and Eq. (1) can be simplified as: ,

$$dn(t)/dt = [\rho(t) - \beta]\, n(t)/l \tag{7}$$

Combining Eqs.(5–7), one has the second order differential equation about reactivity:

$$d^2\rho/dt^2 - [(\rho-\beta)/l](d\rho/dt - \alpha K_c Q_0) = 0 \tag{8}$$

From Eq.(7), the corresponding ρ_{max} to maximum power n_{max} is:

$$\rho_{max} = \beta \tag{9}$$

Although it contains only one variable, Eq.(8) cannot be solved by an analytical solution but a numerical solution, from which transient characteristic of the prompt supercritical process can be obtained by substituting ρ into Eq.(7). Using Eq.(9), the time to reach maximum power can be obtained, too.

3 Analysis and discussion

3.1 Case 1

When the initial power is very small, namely $n_0 \approx 0$, the heat being transferred out of the reactor per unit time will be small and negligible, namely $Q_0 \approx 0$, Eq.(8) can be simplified as

$$d^2\rho/dt^2 - [(\rho-\beta)/l]d\rho/dt = 0 \tag{10}$$

Solving Eq.(10) with the initial condition: $\rho=\rho_0$, $n=n_0$ and $d\rho/dt=0$ at t=0, one has [4]:

$$d\rho/dt = [(\rho-\beta)^2 - (\rho_0-\beta)^2]/(2l) \tag{11}$$

Combining Eqs.(11) and (5), one has

$$n = [(\rho_0-\beta)^2 - (\rho-\beta)^2]/(2\alpha K_c l) \tag{12}$$

From Eqs.(9) and (12), the maximum power can be :

$$n_{max} = (\rho_0-\beta)^2/(2\alpha K_c l) \tag{13}$$

3.2 Case 2

Taking the initial power into consideration but neglecting the heat transfer, namely $n_0>0$ and $Q_0 \approx 0$, the initial conditions for Eq.(10) are $\rho=\rho_0$ and $d\rho/dt = -\alpha K_c n_0$ at t=0, and we have:

$$d\rho/dt = [(\rho-\beta)^2 - (\rho_0-\beta)^2]/(2l) - \alpha K_c n_0 \tag{14}$$

Substituting Eq.(14) into Eq.(5), we have:

$$n = [(\rho_0-\beta)^2 - (\rho-\beta)^2]/(2\alpha K_c l) + n_0 \tag{15}$$

With Eq.(14) and the initial conditions, we can know that the reactivity varies with time as:

$$\rho = [A+\beta-(A-\beta)(A-\rho_0+\beta)e^{At/l}/(A+\rho_0-\beta)]/ \\ [1+(A-\rho_0+\beta)\, e^{At/l}/(A+\rho_0-\beta)] \tag{16}$$

where $A = [(\rho_0-\beta)^2 + 2l\alpha K_c n_0]^{1/2}$.

Combining Eqs.(16) and (15), we can know that the power varies with time as:

$$n = \{(\rho_0-\beta)^2 - A^2[A+\rho_0-\beta-(A-\rho_0+\beta)e^{At/l}]^2/ \\ [A+\rho_0-\beta+(A-\rho_0+\beta)e^{At/l}]^2\}/2\alpha K_c l + n_0 \tag{17}$$

From Eq.(17), t_{max}, the time to reach the maximum power, is obtained:

$$t_{max} = (l/A)\ln[(A+\rho_0-\beta)/(A-\rho_0+\beta)] \tag{18}$$

and the maximum power is:

$$n_{max} = (\rho_0-\beta)^2/(2\alpha K_c l) + n_0 \tag{19}$$

The corresponding reactivity ρ_{max} at the maximum power is obtained by Eq.(9).

Eqs.(11–13) and Eqs.(16–19) are results of Ref.[4] and Ref. [9], respectively. The two cases indicate that the model and the equations in this paper are correct.

3.3 Case 3

When the initial power is not zero and there is the heat transfer, the numerical method of implicit difference is introduced. For a certain kind of PWR fueled with U^{235}, the parameters adopted are: β=0.0065, l=0.0001s, λ=0.07741s^{-1}, K_c=0.05 K·MW^{-1}·s^{-1}, α=5×10^{-5} K^{-1}. As the heat transferred from the core per unit time equals to the output power under steady operation condition, the maximum heat transfer is $Q_{0max}=n_0$ by neglecting other heat losses.

The variation of power is assumed to be faster than that of heat transfer, so $Q_0<n_0$. The power and

reactivity obtained from Eqs.(7) and (8) with the step reactivity of ρ_0=1.1β and/or 1.3β, are shown in Figs.1–5.

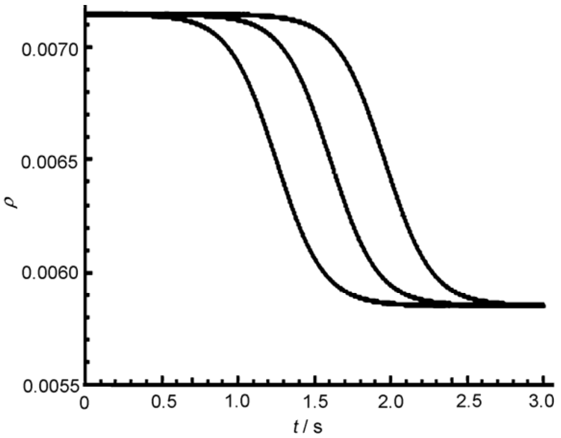

Fig.1 Variation of the reactivity at (from the left to right) n_0=1, 0.1, and 0.01 MW, with ρ_0=1.1β.

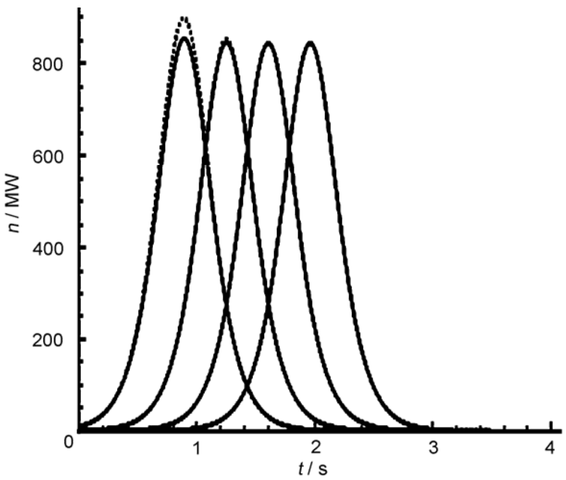

Fig.2 Variation of the power at (from the left to right) n_0=10, 1, 0.1, and 0.01 MW, with ρ_0=1.1β. Dashed line, Q_0=n_0; Solid lines, Q_0=0.

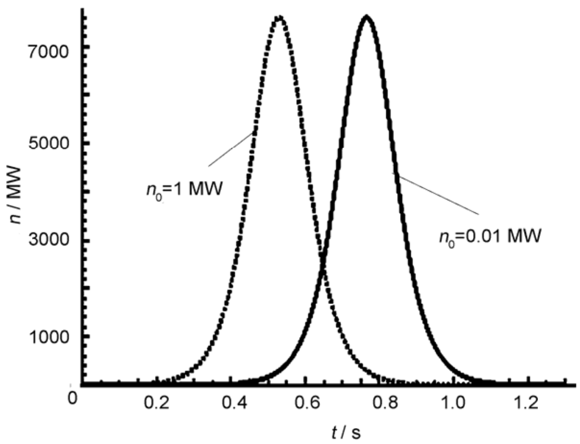

Fig.3 Effect of the heat transfer scale on the power at n_0=1 and 0.01 MW.

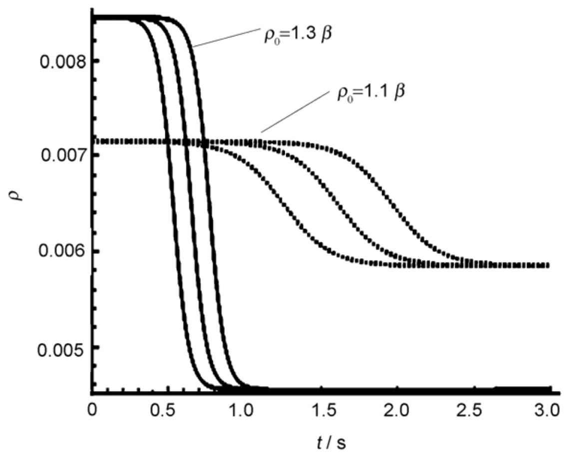

Fig.4 Effect of the initial power on the power at Q_0=n_0/2, and (from the left to right) n_0= 1, 0.1, and 0.01 MW for both groups of the curves.

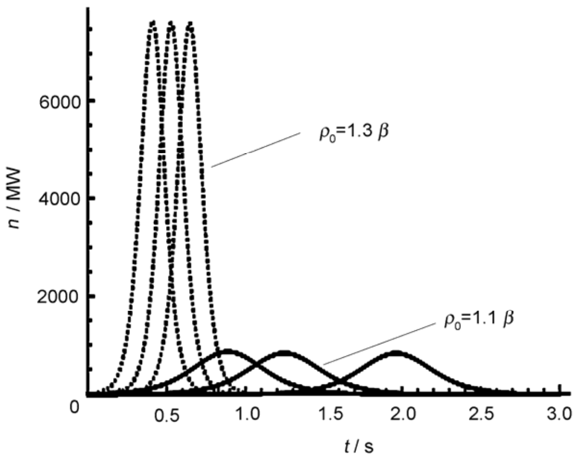

Fig.5 Effect of the initial reactivity on the power at (from the left to right) n_0 =1, 0.1, and 0.01 MW for both groups of the curves.

Fig.1 shows that after introduction of the big step reactivity into the reactor, the reactivity ρ varies basically in the same manner at different initial powers, with or without heat transfer. From Fig.2, the initial power does not seem to affect the maximum power $n_{\max}$, but the time to reach $n_{\max}$, decreases markedly with increasing initial power. In addition the power variation under the heat transfer model is basically the same as that under the adiabatic model. This indicates that the results of adiabatic model are credible and acceptable. From the effect of heat transfer scale on the power variation shown in Fig.3, the heat transfer almost has no effect on the prompt supercritical process, which differs from the variation law of delayed supercritical process in which the heat transfer increases with the power.

Fig.4 shows that a bigger ρ_0 leads to faster variation of ρ. From Fig.5, the peak power n_{max} is related to not only the initial power n_0 and heat transfer Q_0 (Fig.3) but also the step reactivity ρ_0, i.e. the larger ρ_0, the larger the ratio (n_{max}/n_0) of the maximum power to initial power and the smaller the peak power width. However the initial power has no effect on n_{max}/n_0 and the peak power width. All these results are just the same as those without heat transfer Q_0 [9].

4 Conclusion

The transient of prompt supercritical is much faster than that of delayed supercritical and usually is too fast for the heat to be transferred. The prompt supercritical process with the big step reactivity ($\rho_0>\beta$) introduced into the reactor, the temperature feedback and heat transfer is analyzed with a new model. The analysis indicates that the relation of power and temperature obtained with adiabatic model is in accordance with the fact in most cases. Therefore Eqs.(16−19) can be adopted to analyze the power and reactivity response of prompt supercritical process with heat transfer.

References

1 Zhang F B. Operating physics of nuclear reactor. Beijing: Atomic Energy Press (in Chinese), 2002: 31-236.

2 CAI Zhangsheng, Nuclear power reactor neutron dynamics. Beijing: National Defence Industry Press (in Chinese), 2005: 97-109.

3 CAI Zhangsheng, CAI Zhiming, CHEN Lisheng. J Naval Univ Eng (in Chinese), 2000, **12**(3): 25-27.

4 CAI Zhangsheng, CAI Zhiming, CHEN Lisheng. Nucl Power Eng (in Chinese), 2001, **22**(5): 390-393.

5 Aboanber A E, Hamada Y M. Ann Nucl Energy, 2003, 30: 1111-1122.

6 Aboanber A E. Prog Nucl Energy, 2004, **44**(4): 347-368.

7 Nahla A A. Prog Nucl Energy, 2009, 51: 124-128.

8 Sathiyasheela T. Ann Nucl Energy, 2009, 36: 246-250.

9 CHEN Wenzhen, KUANG Bo, GUO Lifeng, *et al*. Nucl Eng Design, 2006, **236**: 1326-1329

10 CHEN Wenzhen, GUO Lifeng, ZHU Bo, *et al*. Prog Nucl Energy, 2007, **49**(4): 290-302.

11 LI Haofeng, CHEN Wenzhen, LUO Lei, *et al*. Ann Nucl Energy, 2009, **36**: 427-432.