CrossMark

Influence of the Coulomb exchange term on nuclear single-proton resonances

Shu-Yang Wang [1] · Zhong-Lai Zhu [1] · Zhong-Ming Niu[1]

Abstract Nuclear single-proton resonances are sensitive to the Coulomb field, while the exchange term of Coulomb field is usually neglected due to its nonlocality. By combining the complex scaling method with the relativistic mean-field model, the influence of the Coulomb exchange term on the single-proton resonances is investigated by taking Sn isotopes and $N = 82$ isotones as examples. It is found that the Coulomb exchange term reduces the single-proton resonance energy within the range of 0.4–0.6 MeV and leads to similar isotopic and isotonic trends of the resonance energy as those without the Coulomb exchange term. Moreover, the single-proton resonance width is also reduced by the Coulomb exchange term, whose influence generally decreases with the increasing neutron number and increases with the increasing proton number. However, the influence of the Coulomb exchange term cannot change the trend of the resonance width with respect to the neutron number and proton number. Furthermore, the influence of the Coulomb exchange term on the resonance width is investigated for the doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb. It is found that the Coulomb exchange term reduces the proton resonance width within 0.2 MeV, whose magnitude depends on the specific nucleus and the quantum numbers of resonant states.

✉ Zhong-Ming Niu
zmniu@ahu.edu.cn

1   School of Physics and Materials Science, Anhui University,
     Hefei 230039, China

Keywords Single-proton resonance · Complex scaling method · Coulomb energy

1 Introduction

With the development of the radioactive ion beam facilities in recent years, exotic nuclei far from the β-stability line have become an active field of research [1–3]. Resonance is an interesting phenomenon in nuclear physics. It plays important roles in understanding many exotic nuclear phenomena, such as the halo [4–7], giant halo [8, 9], deformed halo [10, 11], and collective giant resonance [12, 13]. Therefore, investigation of single-particle resonances has become a hot topic in nuclear physics.

So far, there have been many approaches to study nuclear resonances. They mainly fall into two categories: scattering theory and bound-state-like method. The scattering theory includes the R-matrix theory [14, 15], K-matrix theory [16], S-matrix method [13, 17], and Jost function approach [18, 19], which have been successfully employed to determine the resonance parameters (energy and width). Recently, the Green's function method [20–22] has also been demonstrated to be an efficient tool for describing the nuclear single-particle resonant states [23]. On the other hand, several bound-state-like methods have also been developed due to their simplicities in computation, including the real stabilization method (RSM) [24–27] and the analytic continuation in the coupling constant (ACCC) method [28–34], and the complex scaling method (CSM) [35–38].

By solving a complex eigenvalue problem, the CSM treats the bound states and resonant states on the same footing. It has been widely used to study resonances not

only in atomic nuclei [39–43], but also in the atoms and molecules [35, 44]. Recently, the relativistic mean-field (RMF) theory has attracted wide attention due to its successful applications in describing many nuclear structure phenomena [45–48] and simulating abundances of stellar nucleosynthesis [48–52]. The combination of the complex scaling method with the RMF theory (RMF-CSM) was first developed in Ref. [53] and was used to determine the energies and widths of neutron resonant states in ^{120}Sn, which agree well with the results from the *S*-matrix method, the ACCC method, and the RSM within the framework of the RMF theory (denoted by RMF-S, RMF-ACCC, and RMF-RSM, respectively). Furthermore, the RMF-CSM was employed to investigate the single-neutron resonant states of Zr isotopes and the pseudospin symmetry in the resonant states [54].

Unlike the neutron resonant states, the calculations of proton resonant states should take into account the Coulomb field among protons. In the mean-field approximation, there exist the direct (Hartree) and exchange (Fock) terms for the Coulomb field. However, the Coulomb exchange term is usually neglected, since it is very complicated and time consuming due to the nonlocality in the Fock mean field. Recent studies found that the Coulomb exchange term plays an important role in some nuclear phenomena, such as Cabibbo–Kobayash–Maskawa matrix [55] and mass differences of mirror nuclei [56]. Some approximation techniques have been developed to effectively take into account the Coulomb exchange term in the Hartree approximation. The local density approximation (LDA) is usually employed to estimate the Coulomb exchange energies in the nonrelativistic framework [57–60]. In Ref. [61], the relativistic local density approximation (RLDA) for the Coulomb exchange term was developed, and it was found that the relativistic corrections are important to reproduce the exact Coulomb exchange energies. Furthermore, guided by the RLDA, a phenomenological formula for the coupling strength of the Coulomb field was proposed in Ref. [56]. The accuracy of this method is even better than the RLDA, and the relative deviations of the Coulomb exchange energy in the calculations with phenomenological formula are less than 1 % for Ca, Ni, Sn, and Pb isotopes [56].

The proton resonant states of ^{120}Sn have been studied with the RMF-ACCC method [62], while the Coulomb exchange term is neglected in the calculations. With the RMF-CSM, the Coulomb exchange term has been taken into account using the phenomenological coupling strength of the Coulomb field [63]. However the influence of the Coulomb exchange term on resonances has not been studied for an isotopic or isotonic chain. Moreover, its influence on the resonance width is not studied as well.

Therefore, in this work, we will apply the RMF-CSM to calculate the resonance parameters of Sn isotopes and $N = 82$ isotones. The phenomenological formula will be used to evaluate the influence of the Coulomb exchange term on the resonance parameters. Furthermore, the influence of the Coulomb exchange term on single-proton resonance widths will be investigated for the doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb.

2 Numerical details

The basic ansatz of the RMF theory is a Lagrangian density where nucleons are described as Dirac particles that interact via the exchange of mesons (the scalar, σ, the vector, ω, and isovector vector, ρ) and the photon. From the Lagrangian density, the Dirac equation of the nucleon can be obtained using the classical variation principle. For spherical nuclei, meson fields and densities depend only on the radial coordinate r and the Dirac equation is simplified to the radial Dirac equation, which is

$$\begin{pmatrix} V + S + M & -\dfrac{\mathrm{d}}{\mathrm{d}r} - \dfrac{1}{r} + \dfrac{\kappa}{r} \\ \dfrac{\mathrm{d}}{\mathrm{d}r} + \dfrac{1}{r} + \dfrac{\kappa}{r} & V - S - M \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \varepsilon \begin{pmatrix} f \\ g \end{pmatrix}, \qquad (1)$$

where V and S are the vector and scalar potentials, and M is the nucleon mass. In this work, those parameters of effective interaction in V and S are taken from the NL3 parameter set [64]. To investigate the influence of the Coulomb exchange term, the effective charge factor in Ref. [56],

$$\eta(Z, A) = 1 - aZ^b + cA^d, \qquad (2)$$

is employed with $a = 0.366958$, $b = -0.645775$, $c = 0.030379$, and $d = -0.398341$, which is the factor used to multiply the coupling strength of the Coulomb field.

In the CSM, the Hamiltonian, H, and wave function, ψ, are transformed by an unbounded nonunitary scaling operator, $U(\theta)$, with a real parameter, θ, i.e.,

$$H \rightarrow H_\theta = U(\theta)HU(\theta)^{-1}, \quad \psi \rightarrow \psi_\theta = U(\theta)\psi. \qquad (3)$$

The Dirac equation is then transformed to the complex-scaled Dirac equation,

$$H_\theta \psi_\theta = \varepsilon \psi_\theta. \qquad (4)$$

The complex-scaled Dirac Eq. (4) is solved by expansion in the harmonic oscillator basis as in Ref. [53]. The eigenvalues of H_θ representing continuous spectrum rotate with θ, while eigenvalues representing bound states or resonant states do not change with θ as long as θ is large enough to expose them from the continuous spectrum. The latter are

associated with resonance complex energies $E_r - iE_i = E_r - i\Gamma/2$, where E_r is resonance energy and Γ is the width. For simplicity, the resonance energy and width calculated including the Coulomb exchange term are denoted by E_r^* and Γ^*, respectively.

In the actual calculation, there is no resonance energy that is completely independent of θ due to the numerical approximation. The best estimate for the resonance energy is given by the value of θ for which the rate of change with respect to θ is minimal, i.e, the densest place in the θ trajectory. Figure 1 shows the θ trajectory of single-proton resonant state $1h_{9/2}$ for ^{120}Sn calculated using the RMF-CSM. The densest place corresponds to $\theta = 6.0°$ and its energy and width are 7.135 and 0.0037 MeV, respectively. The result agrees with the results of the RMF-ACCC and RMF-rS methods (r stands for relativistic and S stands for scattering), whose resonance parameters (energy and width) are (7.13, 0.017 MeV) and (7.132, 0.003 MeV), respectively [62]. This indicates the reliability of the CSM to calculate the parameters of single-proton resonant states.

3 Results and discussions

To understand the influence of the phenomenal Coulomb exchange term on the resonance parameters, the total proton potential, $\Sigma(r)$, and the Coulomb potential, $V_{coul}(r)$, are shown in Fig. 2 for the cases without and with this phenomenal Coulomb exchange term. From this figure, it is clear that both potentials $\Sigma(r)$ and $V_{coul}(r)$ are reduced when the phenomenal Coulomb exchange term is included, which just reflects the attractive character of the realistic Coulomb exchange term. This will induce the reduction of the resonance parameters including the energy and width of single-proton resonant state. In the following, we will investigate the influence of the phenomenal Coulomb
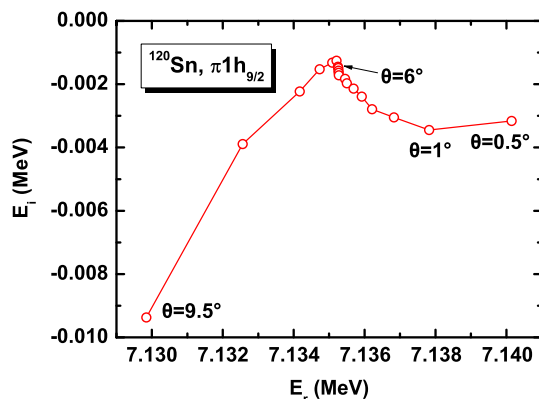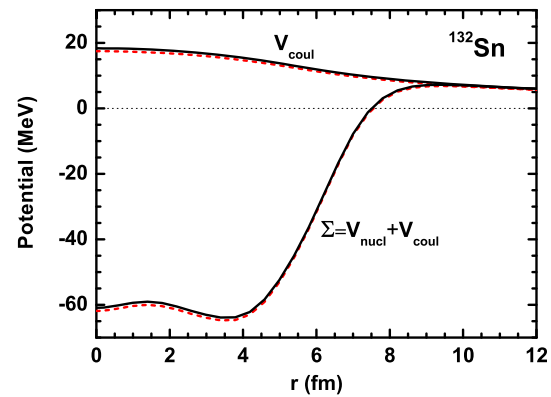

Fig. 2 (Color online) The influence of the phenomenal Coulomb exchange term on the total proton potential, $\Sigma(r)$, and the Coulomb potential, $V_{coul}(r)$. The *solid* and *dashed lines* denote the potentials without and with the phenomenal Coulomb exchange term, respectively

exchange term on the energies and widths of single-proton resonant states, taking the Sn isotopes and $N = 82$ isotones as examples.

With the RMF-CSM, the energies and widths of single-proton resonant states of Sn isotopes are calculated by phenomenologically including the Coulomb exchange term with Eq. (2). The corresponding results are shown in Fig. 3. Clearly, the energies and widths decrease with increasing neutron number. Moreover, the energy decreases more
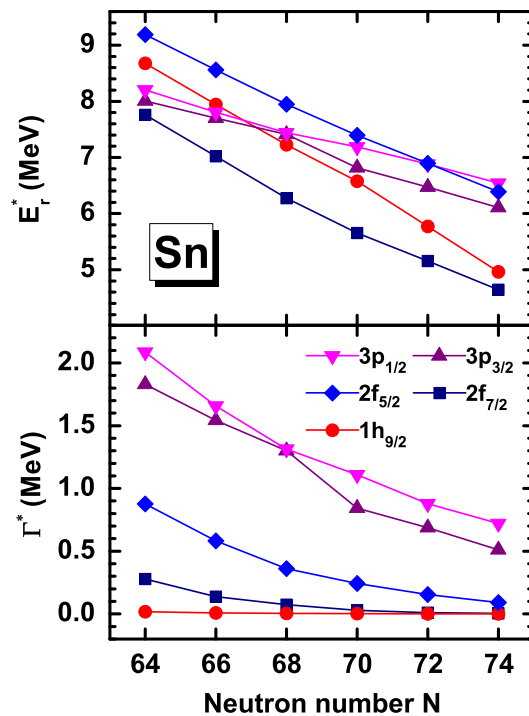

Fig. 1 (Color online) The θ trajectory of single-proton resonant state $1h_{9/2}$ for ^{120}Sn calculated with the RMF-CSM

Fig. 3 (Color online) Energies, E_r^*, and widths, Γ^*, of single-proton resonant states for Sn isotopes calculated by the RMF-CSM with the phenomenal Coulomb exchange term

rapidly for the state with larger orbital angular momentum, l, and hence leads to the crossing phenomenon appearing in the resonance levels. For the spin doublets $(2f_{7/2}, 2f_{5/2})$ and $(3p_{3/2}, 3p_{1/2})$, variation of energy with neutron number is similar due to the same centrifugal potential, while their energy differences are attributed to the spin-orbit potential. This isotopic trend for the resonance energy and width is similar to those without the Coulomb exchange term [63].

For better understanding the influence of the Coulomb exchange term on the resonance parameters, the differences of resonance energies and widths calculated with and without the Coulomb exchange term are shown in Fig. 4 for Sn isotopes. It is found that the single-proton resonance energies and widths of Sn isotopes are reduced when the Coulomb exchange term is phenomenologically taken into account. The difference of the resonance energy caused by the Coulomb exchange term only slightly increases with the increasing neutron number, which is within the range of 0.4–0.6 MeV, although the resonance energy spans from 4.643 ($2f_{7/2}$ in ^{124}Sn) to 9.187 MeV ($2f_{5/2}$ in ^{114}Sn). Therefore, the isotopic trend of the resonance energy remains unchanged whether the Coulomb exchange term is taken into account. The difference of the resonance width generally decreases with the increasing neutron number, while its magnitude is relatively small and cannot change the isotopic trend of the resonance width.
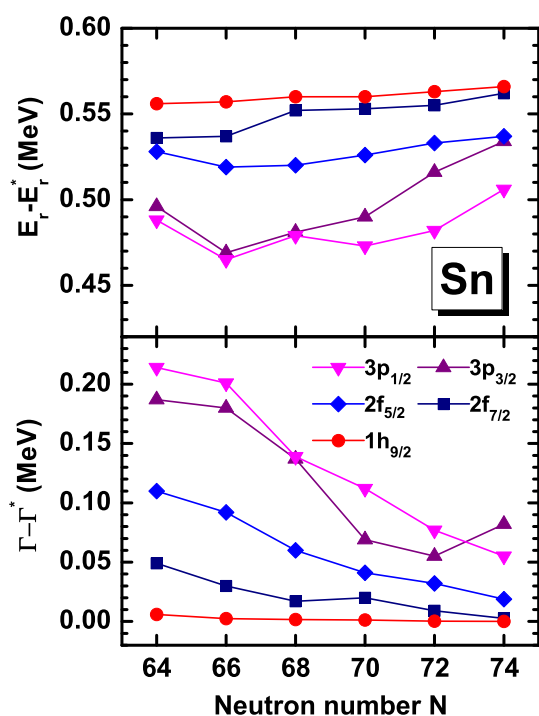
To study the influence of the Coulomb exchange term on dependence of resonance parameters on the charge number, the resonance parameters of $N = 82$ isotones are calculated by including the Coulomb exchange term phenomenologically. The corresponding results are shown in Fig. 5. The Coulomb repulsive force plays an important role in determining the resonance parameters and it is determined by the proton number in nucleus. Therefore, the resonance energy increases with the increasing proton number for $N = 82$ isotones and its change is in nearly the same magnitude for various resonant states with different l. For the width of resonant state, it also increases with the increasing proton number, while its change is remarkable for broad resonant states, such as $3p_{1/2}$ and $3p_{3/2}$. As the case in Sn isotopes, the change trend of the resonance energy and width for $N = 82$ isotones is also similar to that without the Coulomb exchange term [63].

In Fig. 6, the differences of single-proton resonance energies and widths calculated with and without the Coulomb exchange term are shown for $N = 82$ isotones. It is found that the resonance energies and widths of $N = 82$ isotones are reduced when the Coulomb exchange term is taken into account phenomenologically. In general, the difference of the resonance energy caused by the Coulomb exchange term slightly increases with the increasing proton number, and it is still within the range of 0.4–0.6 MeV as
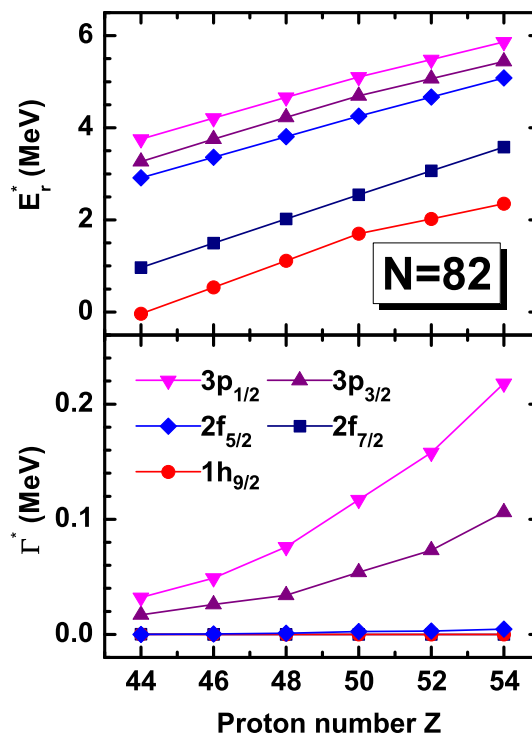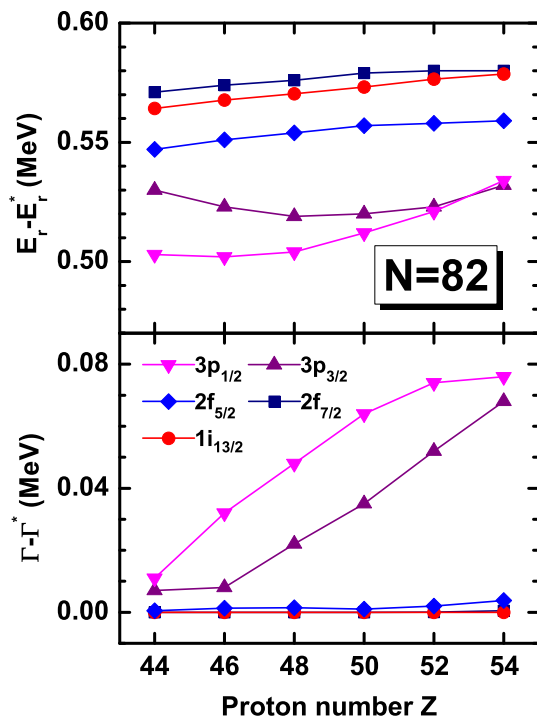

Fig. 4 (Color online) The differences of single-proton resonance energies ($E_r - E_r^*$) and widths ($\Gamma - \Gamma^*$) for Sn isotopes by phenomenally including the Coulomb exchange term

Fig. 5 (Color online) Energies E_r^* and widths Γ^* of single-proton resonant states for $N = 82$ isotones calculated by the RMF-CSM with the phenomenal Coulomb exchange term

Fig. 6 (Color online) The differences of single-proton resonance energies $(E_r - E_r^*)$ and widths $(\Gamma - \Gamma^*)$ for $N = 82$ isotones by phenomenally including the Coulomb exchange term

the case in Sn isotopes. Therefore, the isotonic trend of the resonance energy remains unchanged. The difference of the resonance width increases with the increasing proton number, and the change in width even reaches about 0.07 MeV for the broad resonant states $3p_{1/2}$ and $3p_{3/2}$ of ^{136}Xe. However, its magnitude is still not large enough to change the trend of the resonance width for $N = 82$ isotones.

Through the above research, it is found that the Coulomb exchange term plays an important role not only in the resonance energy, but also in resonance width. In Ref. [63],
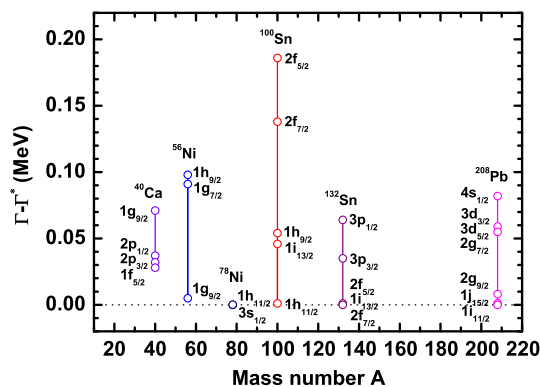
the influence of the Coulomb exchange term on single-proton resonance energy has been studied in different nuclear regions, taking the doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb as examples. So it is interesting to further investigate the influence of the Coulomb exchange term on the single-proton resonance width. In Fig. 7, the differences of single-proton resonance widths for doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb are shown. It is clear that the Coulomb exchange term reduces the resonance width, whose reduction is within 0.2 MeV. In addition, the change of the width is larger for the spin unaligned state than that for the spin aligned state, such as the spin doublets $(3p_{1/2}, 3p_{3/2})$ of ^{40}Ca. In an isotope, the state with the same quantum number always decreases with the increasing neutron number, such as $2f_{5/2}$ in ^{100,132}Sn. These indicate that the contribution of the Coulomb exchange term to the proton resonance width is related to the specific nucleus and the quantum numbers of the state. Furthermore, the relative deviations of resonant widths are presented in Fig. 8. It is found that the relative deviations of the resonant widths for the states with high angular momentum are generally large, though their absolute deviations are very small. This can be understood since their resonant widths are generally very small.

4 Summary

In summary, the influence of the Coulomb exchange term on the single-proton resonances is investigated with the complex scaling method based on the relativistic mean-field model. Taking Sn isotopes and $N = 82$ isotones as examples, it is found that the Coulomb exchange term reduces the single-proton resonance energy, and the dependence of this influence on neutron number and proton number is so weak that energy reduction is within the range of 0.4–0.6 MeV. Therefore, the isotopic and isotonic trends
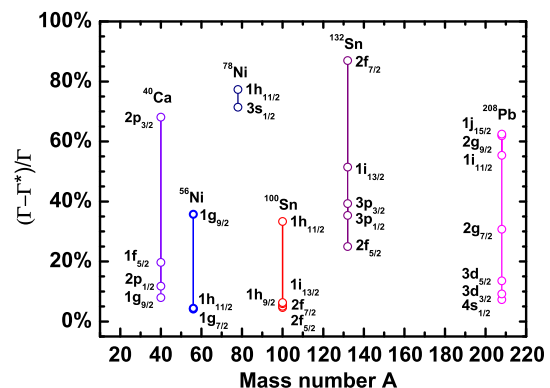

Fig. 7 (Color online) The difference of single-proton resonance widths for doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb by phenomenally including the Coulomb exchange term

Fig. 8 (Color online) The same as Fig. 7 but for the relative deviations of resonant widths $(\Gamma - \Gamma^*)/\Gamma$

of the resonance energy are similar to those without the Coulomb exchange term. Moreover, the Coulomb exchange term also reduce the single-proton resonance width, and the reduction is generally larger for the border single-proton resonance. For Sn isotopes, the influence of the Coulomb exchange term on the resonance width generally decreases with the increasing neutron number, while it increases with the increasing proton number for $N = 82$ isotones. However, the influence of the Coulomb exchange term on resonance width is still not large enough to change the trend of resonance width with respect to the neutron number and proton number. Furthermore, the influence of the Coulomb exchange term on resonance width is further investigated in a wide range of nuclear region, taking the doubly magic nuclei ^{40}Ca, ^{56,78}Ni, ^{100,132}Sn, and ^{208}Pb as examples. It is found that the Coulomb exchange term reduces the proton resonance width within 0.2 MeV, whose magnitude depends on the specific nucleus and the quantum numbers of resonant states.

References

1. B. Hongy, Prospects of nuclear physics research using rare isotope beams at RAON in Korea. Nucl. Sci. Tech. **26**, S20505 (2015). doi:10.13538/j.1001-8042/nst.26.S20505

2. B.H. Sun, YuA Litvinov, I. Tanihata, Y.H. Zhang, Toward precision mass measurements of neutron-rich nuclei relevant to r-process nucleosynthesis. Front. Phys. **10**, 102102 (2015). doi:10.1007/s11467-015-0503-z

3. C.L. Guo, G.L. Zhang, W.W. Qu, S. Terashima, I. Tanihata, X.Y. Le, Simulation of (p, d) reaction on RIBLL2 for study of tensor force. Nucl. Sci. Tech. **26**, 040501 (2015). doi:10.13538/j.1001-8042/nst.26.040501

4. I. Tanihata et al., Measurements of interaction cross sections and nuclear radii in the light p-shell region. Phys. Rev. Lett. **55**, 2676–2679 (1985). doi:10.1103/PhysRevLett.55.2676

5. J. Meng, P. Ring, Relativistic Hartree–Bogoliubov description of the neutron halo in 11Li. Phys. Rev. Lett. **77**, 3963–3966 (1996). doi:10.1103/PhysRevLett.77.3963

6. W. Pöschl, D. Vretenar, G.A. Lalazissis et al., Relativistic Hartree–Bogoliubov theory with finite range pairing forces in coordinate space: neutron halo in light nuclei. Phys. Rev. Lett. **79**, 3841–3844 (1997). doi:10.1103/PhysRevLett.79.3841

7. N. Sandulescu, N. Van Giai, R.J. Liotta, Resonant continuum in the Hartree–Fock-BCS approximation. Phys. Rev. C **61**, 061301(R) (2000). doi:10.1103/PhysRevC.61.061301

8. J. Meng, P. Ring, Giant halo at the neutron drip line. Phys. Rev. Lett. **80**, 460–463 (1998). doi:10.1103/PhysRevLett.80.460

9. Y. Zhang, M. Matsuo, J. Meng, Pair correlation of giant halo nuclei in continuum Skyrme–Hartree–Fock–Bogoliubov theory. Phys. Rev. C **86**, 054318 (2012). doi:10.1103/PhysRevC.86.054318

10. I. Hamamoto, Interpretation of Coulomb breakup of Ne31 in terms of deformation. Phys. Rev. C **81**, 021304(R) (2010). doi:10.1103/PhysRevC.81.021304

11. S.G. Zhou, J. Meng, P. Ring, E.G. Zhao, Neutron halo in deformed nuclei. Phys. Rev. C **82**, 011301 (2010). doi:10.1103/PhysRevC.82.011301

12. P. Curutchet, T. Vertse, R.J. Liotta, Resonant random phase approximation. Phys. Rev. C **39**, 1020–1031 (1989). doi:10.1103/PhysRevC.39.1020

13. L.G. Cao, Z.Y. Ma, Exploration of resonant continuum and giant resonance in the relativistic approach. Phys. Rev. C **66**, 024311 (2002). doi:10.1103/PhysRevC.66.024311

14. E.P. Wigner, L. Eisenbud, Higher angular momenta and long rang interaction in resonance reactions. Phys. Rev. **72**, 29–41 (1947). doi:10.1103/PhysRev.72.29

15. G.M. Hale, R.E. Brown, N. Jarmie, Pole structure of the $J^\pi = 3/2^+$ resonance in He5. Phys. Rev. Lett. **59**, 763–766 (1987). doi:10.1103/PhysRevLett.59.763

16. J. Humblet, B.W. Filippone, S.E. Koonin, Level matrix, 16N β decay, and the 12C(α, γ)16O reaction. Phys. Rev. C **44**, 2530–2535 (1991). doi:10.1103/PhysRevC.44.2530

17. J.R. Taylor, *Scattering Theory, The Quantum Theory on Non-relativistic Collisions* (Wiley, New York, 1972)

18. B.N. Lu, E.G. Zhao, S.G. Zhou, Pseudospin symmetry in single particle resonant states. Phys. Rev. Lett. **109**, 072501 (2012). doi:10.1103/PhysRevLett.109.072501

19. B.N. Lu, E.G. Zhao, S.G. Zhou, Pseudospin symmetry in single-particle resonances in spherical square wells. Phys. Rev. C **88**, 024323 (2013). doi:10.1103/PhysRevC.88.024323

20. E.N. Economou, *Green's Fucntion in Quantum Physics* (Springer, Berlin, 2006)

21. S.T. Belyaev, A.V. Smirnov, S.V. Tolokonnikov et al., Sov. J. Nucl. Phys. **45**, 783 (1987)

22. Y. Zhang, M. Matsuo, J. Meng, Persistent contribution of unbound quasiparticles to the pair correlation in the continuum Skyrme–Hartree–Fock–Bogoliubov approach. Phys. Rev. C **83**, 054301 (2011). doi:10.1103/PhysRevC.83.054301

23. T.T. Sun, S.Q. Zhang, Y. Zhang et al., Greens function method for single-particle resonant states in relativistic mean field theory. Phys. Rev. C **90**, 054321 (2014). doi:10.1103/PhysRevC.90.054321

24. A.U. Hazi, H.S. Taylor, Stabilization method of calculating resonance energies: model problem. Phys. Rev. A **1**, 1109–1120 (1970). doi:10.1103/PhysRevA.1.1109

25. V.A. Mandelshtam, T.R. Ravuri, H.S. Taylor, Calculation of the density of resonance states using the stabilization method. Phys. Rev. Lett. **70**, 1932–1935 (1993). doi:10.1103/PhysRevLett.70.1932

26. A.T. Kruppa, K. Arai, Resonances and the continuum level density. Phys. Rev. A **59**, 3556–3561 (1999). doi:10.1103/PhysRevA.59.3556

27. L. Zhang, S.G. Zhou, J. Meng et al., Real stabilization method for nuclear single-particle resonances. Phys. Rev. C **77**, 014312 (2008). doi:10.1103/PhysRevC.77.014312

28. V.I. Kukulin, V.M. Krasnopl'sky, J. Horáček, *Theory of Resonances, Principles and Applications* (Kluwer, Dordrecht, The Netherlands, 1989)

29. N. Tanaka, Y. Suzuki, K. Varga, Exploration of resonances by analytic continuation in the coupling constant. Phys. Rev. C **56**, 562–565 (1997). doi:10.1103/PhysRevC.56.562

30. N. Tanaka, Y. Suzuki, K. Varga et al., Unbound states by analytic continuation in the coupling constant. Phys. Rev. C **59**, 1391–1399 (1999). doi:10.1103/PhysRevC.59.1391

31. G. Cattapan, E. Maglione, From bound states to resonances: analytic continuation of the wave function. Phys. Rev. C **61**, 067301 (2000). doi:10.1103/PhysRevC.61.067301

32. S.C. Yang, J. Meng, S.G. Zhou, Exploration of unbound states by analytical continuation in the coupling constant method within

relativistic mean field theory. Chin. Phys. Lett. **18**, 196–198 (2001). doi:10.1088/0256-307X/18/2/314

33. S.S. Zhang, J. Meng, S.G. Zhou et al., Analytic continuation of single-particle resonance energy and wave function in relativistic mean field theory. Phys. Rev. C **70**, 034308 (2004). doi:10.1103/PhysRevC.70.034308

34. S.S. Zhang, M.S. Smith, G. Arbanas et al., Structures of exotic 131,133Sn isotopes and effect on r-process nucleosynthesis. Phys. Rev. C **86**, 032802 (2012). doi:10.1103/PhysRevC.86.032802

35. Y.K. Ho, The method of complex coordinate rotation and its applications to atomic collision processes. Phys. Rep. **99**, 1–68 (1983). doi:10.1016/0370-1573(83)90112-6

36. J.Y. Guo, J. Wang, B.M. Yao et al., The relativistic development of basis expansion method with complex scaling for the description of bound and resonant states. Int. J. Mod. Phys. E **19**, 1357–1370 (2010). doi:10.1142/S0218301310015795

37. J.S. Feng, Z. Liu, J.Y. Guo, Bound and resonant states of the hulthen potential investigated by using the complex scaling method with the oscillator basis. Chin. Phys. Lett. **27**, 110304 (2010). doi:10.1088/0256-307X/27/11/110304

38. Q. Liu, Z.M. Niu, J.Y. Guo, Resonant states and pseudospin symmetry in the Dirac–Morse potential. Phys. Rev. A **87**, 052122 (2013). doi:10.1103/PhysRevA.87.052122

39. A.T. Kruppa, P.H. Heenen, H. Flocard et al., Particle-unstable nuclei in the Hartree–Fock theory. Phys. Rev. Lett. **79**, 2217–2220 (1997). doi:10.1103/PhysRevLett.79.2217

40. K. Arai, Resonance states of C12 in a microscopic cluster model. Phys. Rev. C **74**, 064311 (2006). doi:10.1103/PhysRevC.74.064311

41. Q. Liu, J.Y. Guo, Z.M. Niu et al., Resonant states of deformed nuclei in the complex scaling method. Phys. Rev. C **86**, 054312 (2012). doi:10.1103/PhysRevC.86.054312

42. M. Shi, Q. Liu, Z.M. Niu et al., Relativistic extension of the complex scaling method for resonant states in deformed nuclei. Phys. Rev. C **90**, 034319 (2014). doi:10.1103/PhysRevC.90.034319

43. T. Myo, Y. Kikuchi, K. Kato, Five-body resonances of 8C using the complex scaling method. Phys. Rev. C **85**, 034338 (2012). doi:10.1103/PhysRevC.85.034338

44. N. Moiseyev, Quantum theory of resonances: calculating energies, widths and cross-sections by complex scaling. Phys. Rep. **302**, 212–293 (1998). doi:10.1016/S0370-1573(98)00002-7

45. P. Ring, Relativistic mean field theory in finite nuclei. Prog. Part. Nucl. Phys. **37**, 193–263 (1996). doi:10.1016/0146-6410(96)00054-3

46. D. Vretenar, A.V. Afanasjev, G.A. Lalazissis et al., Relativistic Hartree–Bogoliubov theory: static and dynamic aspects of exotic nuclear structure. Phys. Rep. **409**, 101–259 (2005). doi:10.1016/j.physrep.2004.10.001

47. J. Meng, H. Toki, S.G. Zhou et al., Relativistic continuum Hartree Bogoliubov theory for ground-state properties of exotic nuclei. Prog. Part. Nucl. Phys. **57**, 470–563 (2006). doi:10.1016/j.ppnp.2005.06.001

48. J. Meng (Eds.), International Review of Nuclear Physics - Vol. 10, Relativistic Density Functional for Nuclear Structure, World Scientific (2016)

49. B.H. Sun, F. Montes, L.S. Geng et al., Application of the relativistic mean-field mass model to the r-process and the influence of mass uncertainties. Phys. Rev. C **78**, 025806 (2008). doi:10.1103/PhysRevC.78.025806

50. Z.M. Niu, B.H. Sun, J. Meng, Influence of nuclear physics inputs and astrophysical conditions on the Th/U chronometer. Phys. Rev. C **80**, 065806 (2009). doi:10.1103/PhysRevC.80.065806

51. X.D. Xu, B.H. Sun, Z.M. Niu et al., Reexamining the temperature and neutron density conditions for r-process nucleosynthesis with augmented nuclear mass models. Phys. Rev. C **87**, 015805 (2013). doi:10.1103/PhysRevC.87.015805

52. Z.M. Niu, Y.F. Niu, H.Z. Liang et al., Beta-decay half-lives of neutron-rich nuclei and matter flow in the r-process. Phys. Lett. B **723**, 172–176 (2013). doi:10.1016/j.physletb.2013.04.048

53. J.Y. Guo, X.Z. Fang, P. Jiao et al., Application of the complex scaling method in relativistic mean-field theory. Phys. Rev. C **82**, 034318 (2010). doi:10.1103/PhysRevC.82.034318

54. Y. Liu, S.W. Chen, J.Y. Guo, Research on the single-particle resonant states by the complex scaling method. Acta Phys. Sin. **61**, 112101 (2012). doi:10.7498/aps.61.112101

55. H.Z. Liang, N. Van Giai, J. Meng, Isospin corrections for superallowed Fermi beta decay in self-consistent relativistic random-phase approximation approaches. Phys. Rev. C **79**, 064316 (2009). doi:10.1103/PhysRevC.79.064316

56. Z.M. Niu, Q. Liu, Y.F. Niu et al., Nuclear effective charge factor originating from covariant density functional theory. Phys. Rev. C **87**, 037301 (2013). doi:10.1103/PhysRevC.87.037301

57. C. Titin-Schnaider, P. Quentin, Coulomb exchange contribution in nuclear Hartree–Fock calculations. Phys. Lett. B **49**, 397–400 (1974). doi:10.1016/0370-2693(74)90617-0

58. J. Skalski, Self-consistent calculations of the exact Coulomb exchange effects in spherical nuclei. Phys. Rev. C **63**, 024312 (2001). doi:10.1103/PhysRevC.63.024312

59. J. Le Bloas, M.H. Koh, P. Quentin et al., Exact Coulomb exchange calculations in the Skyrme–Hartree–Fock-BCS framework and tests of the Slater approximation. Phys. Rev. C **84**, 014310 (2011). doi:10.1103/PhysRevC.84.014310

60. M. Anguiano, J.L. Egido, L.M. Robledo, Coulomb exchange and pairing contributions in nuclear Hartree–Fock–Bogoliubov calculations with the Gogny force. Nucl. Phys. A **683**, 227–254 (2001). doi:10.1016/S0375-9474(00)00445-0

61. H.Q. Gu, H.Z. Liang, W.H. Long et al., Slater approximation for Coulomb exchange effects in nuclear covariant density functional theory. Phys. Rev. C **87**, 041301(R) (2013). doi:10.1103/PhysRevC.87.041301

62. S.S. Zhang, W. Zhang, S.G. Zhou et al., Relativistic wave functions for single-proton resonant states. Eur. Phys. J. A **32**, 43–49 (2007). doi:10.1140/epja/i2006-10299-9

63. Z.L. Zhu, Z.M. Niu, D.P. Li et al., Probing single-proton resonances in nuclei by the complex-scaling method. Phy. Rev. C **89**, 034307 (2014). doi:10.1103/PhysRevC.89.034307

64. G.A. Lalazissis, J. König, P. Ring, New parametrization for the Lagrangian density of relativistic mean field theory. Phys. Rev. C **55**, 540–543 (1997). doi:10.1103/PhysRevC.55.540