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The study on nuclear level density of some deformed Dy radionuclides using collective model approach

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Abstract Nuclear level density (NLD) is a characteristic property of many-body quantum mechanical systems. NLDs are of special importance to make statistical calculations in reactor studies and various theoretical and experimental nuclear physics and engineering applications. In this study, we have investigated a set of particle states in distinct rotational and vibrational bands to calculate nuclear level density parameters and the NLDs of accessible states of some deformed Dy radionuclides using a collective model approach, which included different excitation bands of the observed nuclear spectra. The method used assumes equidistant spacing of collective coupled state bands of the considered nuclei. The results of the calculated NLD have been compared with the experimental and compiled data obtained by the Oslo group, shell model Monte Carlo, Hartree–Fock–Bogoliubov + combinatorial approach, Bardeen–Cooper–Schrieffer approach and are in a good agreement.

Keywords Nuclear level density · Collective model · Excitation bands

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1 Introduction

The investigation of the nuclear level density (NLD) is one of the most interesting issues in pure and applied nuclear physics since it is important when describing the properties of the structure of the excited nuclei within statistical models. The knowledge about NLD plays an important role in many nuclear structure studies and applications, such as the analysis of nuclear reactions, astrophysics, intermediate-energy heavy-ion collisions, medicine physics, spallation neutron measurements, accelerator-driven subcritical systems (ADSs), and cross-sectional calculations [1]. Hence, determining the NLD of a nucleus is a great challenge to figure out a many-body quantum mechanical system.

The NLD can be obtained from experimental measurements and theoretical calculations using different approaches and models. Most studies related to the NLD have been based on the Bethe theory [2], which assumes a single-particle nuclear model, as well as on some certain extensions and modifications [3] of this theory. The most widely used description of the NLD is based on the thermodynamic relation between entropy and the average energy of a system considered in the framework of non-interacting particles of the Fermi model. When the NLD of an atomic nucleus is known, thermodynamic quantities of the many-body quantum mechanical systems, such as entropy, temperature, and heat capacity, can be extracted. In the NLD calculation, it can be more difficult to achieve the accurate values of the NLD since several factors, such as spin, parity, angular momentum, binding energy, and cut-off factors of the nuclei, are taken into account. Recently, new approaches such as shell model Monte Carlo (SMMC) [4–7], Hartree–Fock–Bogoliubov + combinatorial approach (HFB + combinatorial) [8], Bardeen–

Cooper–Schrieffer (BCS) approach [9], back shifted Fermi gas (BSFG) model [10, 11], constant temperature (CT) model [10], and experimental methods [12–14] have been developed to provide the NLD over wide energy regions. The approach of Oslo group is for one of the experimentalists, which measure γ spectra at the Oslo cyclotron laboratory using experimental method with nuclear reactions ((^{3}He, ^{3}He), (^{3}He, ^{4}He), etc.), to obtain the accurate values of the NLD [12–14] in statistical calculations.

In the present work, we have investigated the excitation spectra of some deformed Dy radionuclides in the rare earth elements region using a simple model based on the Bethe theory, in which the collective character of the nuclear excitations is available to calculate the level density parameter. The NLD values of the deformed nuclei have been obtained with the calculated level density parameters for different excitation bands, and the obtained results have been compared by the theoretical and experimental methods and are in good agreement.

2 Statistical model for nuclear level density calculation

The present study is based on the Bethe theory, similar to the other studies [2, 3]. The Bethe theory gives the dependence of the NLD on the total angular momentum I of the nucleus. The expression used for the observable nuclear level density at any excitation energy U and momentum I can be written as [2, 3]

$$\rho(U,I) = \frac{\sqrt{\pi}\exp(2\sqrt{aU})}{12a^{1/4}U^{5/4}}\ \frac{(2I+1)\exp\left[-(2I+1)^2/2\sigma^2\right]}{2\sqrt{2\pi}\sigma^3}, \tag{1}$$

where a is the nuclear level density parameter and σ is the spin cut-off (distribution) parameter. These parameters are defined by

$$a = \frac{\pi^2}{6}g(\varepsilon_F), \tag{2}$$

$$\sigma^2 = g(\varepsilon_F)\,\langle m^2\rangle\, t, \tag{3}$$

here, $g(\varepsilon_F)$, $\langle m^2\rangle$, and t are the density of single-particle levels at the Fermi energy ε_F, the mean square magnetic quantum number for single-particle states, and the nuclear thermodynamic temperature of an excited nucleus in the Fermi gas model, respectively. These parameters are expressed as follows:

$$g(\varepsilon_F) = \frac{3}{2}\frac{A}{\varepsilon_F}, \quad \langle m^2\rangle = 0.146A^{2/3}, \quad t^2 = \frac{U}{a}, \tag{4}$$

where A is the mass number of a nucleus.

Experimental observations do not determine the different orientation of nuclear angular momentum I. Therefore, it is useful to obtain the observable level density, which has the form [1],

$$\sum \rho(U,I) = \frac{\pi^2}{12}\frac{\exp(2\sqrt{aU})}{a^{1/4}U^{5/4}}\frac{1}{\sqrt{2\pi}\sigma}. \tag{5}$$

Hence, Eqs. (2)–(4) are substituted into Eq. (5) to find the observable level density as

$$\rho(U) = \frac{a}{12\sqrt{2}\times 0.298\,A^{1/3}(aU)^{3/2}}\exp(2\sqrt{aU}). \tag{6}$$

The Bethe theory does not take into account the collective effects of the nuclear particles, such as the fermions in the excitation of the nuclei. However, when the measured magnetic and quadrupole moments are investigated, their magnitudes are observed to deviate considerably from the ones calculated using the single-particle shell model in which the closed shells forming the nuclear core play no part. In other words, the excited states and the magnetic and quadrupole moments are the results of the collective motion of many nucleons, not just of those nucleons that are outside the closed shell [1]. The collective motion of the nucleons may be described as a vibrational motion about the equilibrium position and a rotational motion that maintains the deformed shape of the nucleus. Almost all bodies of data on the estimated level density parameters of the deformed nuclei have been well identified on a base of collective-rotational and collective-vibrational bands, such as ground-state band, β-band, γ-band, and octupole band. We have discussed exhaustively the collective excitation modes in previous studies [1, 15–17].

3 Collective model approach

The NLD depends on the excitation energy, U, taking into account different excitation modes, such as rotational, vibrational, and other mix bands, and can be expressed in the following form:

$$\begin{aligned}\rho(U) &= \sum_i a_i\, \rho_i(U)\\ &= a_{\mathrm{GS}}\,\rho_{\mathrm{GS}}(U) + a_\beta\,\rho_\beta(U) + a_{\mathrm{oct}}\,\rho_{\mathrm{oct}}(U) + \cdots\end{aligned} \tag{7}$$

where $\rho(U)$ is the partial energy level density at the excitation, U, for i th excitation mode (ground state, beta, octupole bands, etc.), and a_i is the weighting coefficient satisfying the condition $\sum_i a_i = 1$. In this study, we used a simple expression for the energy level density, which takes into account the collective excitation modes [1, 15–17]. Here, in our determination of the nuclear level density, because of the excitation bands, the "equidistant" condition between energy levels, which is the important properties of observed energy spectrum of considered nuclei, should be satisfied. These properties can approximately be verified for the energies of the coupled state bands in deformed nucleus by considering the ratios given by

$$R_1 : R_2 : R_3 : R_4 : \; \cdots \; = 1 : r : 2r : 3r : \cdots, \tag{8}$$

where, R_1, R_2, R_3, R_4,… are the ratios of sequential level energies to the appropriate energy unit of a corresponding band. When the above relation is satisfied in our study, the nuclear level density formula introduced depending on the excitation energy, U, and energy unit, ε_{o}, for i th excitation band can be represented as [1, 15–17]

$$\rho_i(U, \varepsilon_{\mathrm{o}i}) \cong \frac{\pi^2 a_{\mathrm{o}i}}{24\sqrt{3}(a_{\mathrm{o}i}U)^{3/2}} \exp(2\sqrt{a_{\mathrm{o}i}U}), \tag{9}$$

which is fairly simple and contains only one parameter $a_{\mathrm{o}i}$ defined as,

$$a_{\mathrm{o}i} = \frac{\pi^2}{6\;\varepsilon_{\mathrm{o}i}} \tag{10}$$

Table 1 Calculated and compiled values of the nuclear level density parameters for some deformed Dy radionuclides

Nucleus	BSFG model a (MeV^{-1}) [11]	Behkami-Soltani a (MeV^{-1}) [20]	Rohr a (MeV^{-1}) [21]	Calculated a_0 (MeV^{-1})	Excitation band
^{156}Dy	–	–	–	11.9276	GS band
				12.4887	γ-Vibration band
				21.8635	Octupole band
				10.7374	β-Vibration band
^{157}Dy	19.46	23.95	19.797	26.837	3/2[521]-GS band
				17.5656	5/2[512] band
				20.039	3/2[532] band
				20.85	5/2[523] band
^{158}Dy	–	–	–	16.6124	GS band
				12.9625	K^π=4$^+$ band
				16.72	γ-Vibration band
				17.2884	β-Vibration band
^{159}Dy	17.62	21.553	19.939	12.044	3/2[521]-GS band
				19.1806	5/2[523] band
				22.097	5/2[512] band
^{160}Dy	–	–	20.01	18.93	GS band
				23.5357	β-Vibration band
				19.814	γ-Vibration band
				22.181	Octupole band
^{161}Dy	18.06	22.357	20.081	16.366	GS band
				18.707	5/2[512] band
				21.227	5/2[523] band
^{162}Dy	18.08	21.417	20.152	20.372	GS band
				21.9752	γ-Vibration band
				18.27	$K^\pi = 5^-$ band
				26.565	octupole band
^{163}Dy	17.31	21.375	20.223	9.8199	5/2[523]-GS band
				30.6897	3/2[521] band
				18.29	5/2[512] band
^{164}Dy	17.75	21.065	20.294	22.39	GS band
				24.7591	γ-Vibration band
				16.6693	$K^\pi = 4^-$ band
				26.33	octupole band
^{165}Dy	16.9	21.495	20.365	19.704	7/2[633]-GS band
				29.7316	3/2[521] band
				21.197	5/2[512] band

and represents a collective level density parameter corresponding to the ith band with the unit energy ε_{oi}. The unit energies are calculated from the nuclear spectra data of each nucleus such as $\varepsilon_{0GS} = E(2^+)$, $\varepsilon_{0\beta} = E(2^+) - E(0^+)$, and $\varepsilon_{0oct} = E(3^-) - E(1^-)$ for ground-state, β, and octupole bands, respectively. Similarly, the other excitation bands can be included. Using the collective excitation modes and unit energies, the level density parameters are determined for each band by Eq. (10).

Although the NLD expressions of Eqs. (6) and (9) have similar dependence on the energy, they have been obtained from different approaches. Equation (6) obtained from the Bethe theory is based on a single-particle nuclear model, whereas Eq. (9) is extracted from symmetry properties of the nuclear spectra data expressed by Eq. (8). The different level density parameters a_{oi}, which are defined by Eq. (10), for each collective-rotational and collective-vibrational mode can easily be calculated from the nuclear spectra data of deformed Dy radionuclides reported in Refs. [18, 19].

4 Results and discussions

In the present study, level density parameters of some deformed Dy radionuclides in the region of a large deformed even–even and odd-A nuclei have been calculated using different excitation bands of the observed nuclear spectra. As shown in Table 1, the calculated and compiled values of the nuclear level density parameters for some deformed Dy radionuclides have been compared with the BSFG model [11], Behkami-Soltani [20], and Rohr [21]. In Fig. 1, we illustrate the comparison of our implementations of nuclear level density parameters, a_0, with the level density parameters a, which are compiled by Refs. [11, 20, 21], versus the mass number of some deformed Dy radionuclides.

In Table 1 and Fig. 1, it is clear that the present values of the level density parameters, calculated by Eq. (10), for the considered radionuclides are well consistent with those of the compiled values for s-wave neutron resonance data. As seen in Table 1 and Fig. 1, the dominant band in the population of deformed even–even radionuclides excitations at the neutron binding energy generally seems to be a ground-state collective-rotational band. On the other hand, the mixed bands (negative and positive parity bands) are dominant bands in order to identify the level density parameters for the odd-A deformed radionuclides studied. We can say that, in the calculation of the NLD depending on nuclear excitation energy for such radionuclides, the existence of all possible nuclear collective excitation modes should be considered.

After the level density parameters for each considered excitation bands of Dy radionuclides have been compiled, the NLDs have been calculated for those parameters by Eq. (9). Excitation energy dependence of the calculated NLD and those of the compiled level density values based on SMMC taken from [6, 7], HFB + combinatorial approach taken from [8], BCS approach taken from [9], and the experimental values taken from Oslo group [13, 14] for $^{160-163}$Dy radionuclides are given in Fig. 2. As shown in Fig. 2, the present values of the calculated NLDs have been compared with experimental and compiled values, and they have been found to be in good agreement. A slight discrepancy exists in Fig. 2, which may be caused by the

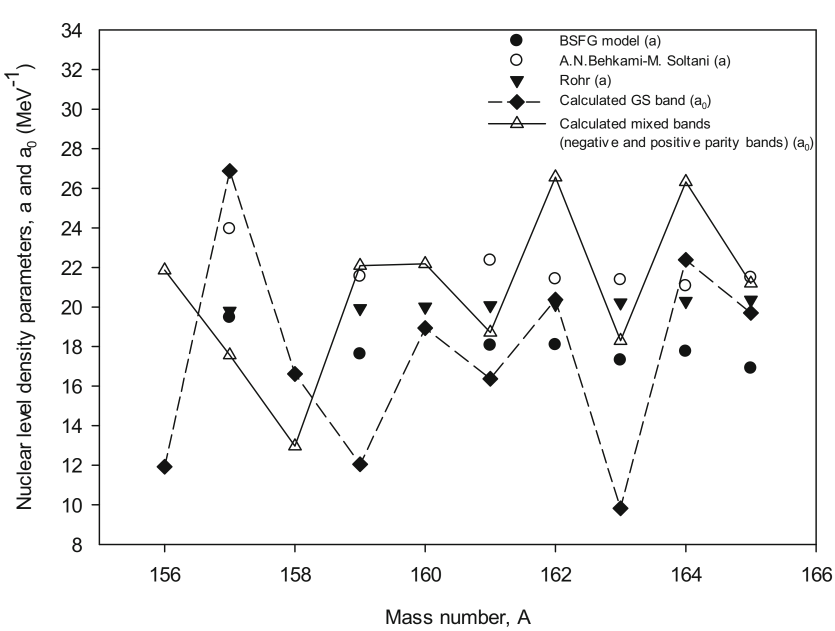

Fig. 1 Mass dependence of the calculated nuclear level density parameters, a_0, and those of the compiled values, a, for some deformed Dy radionuclides

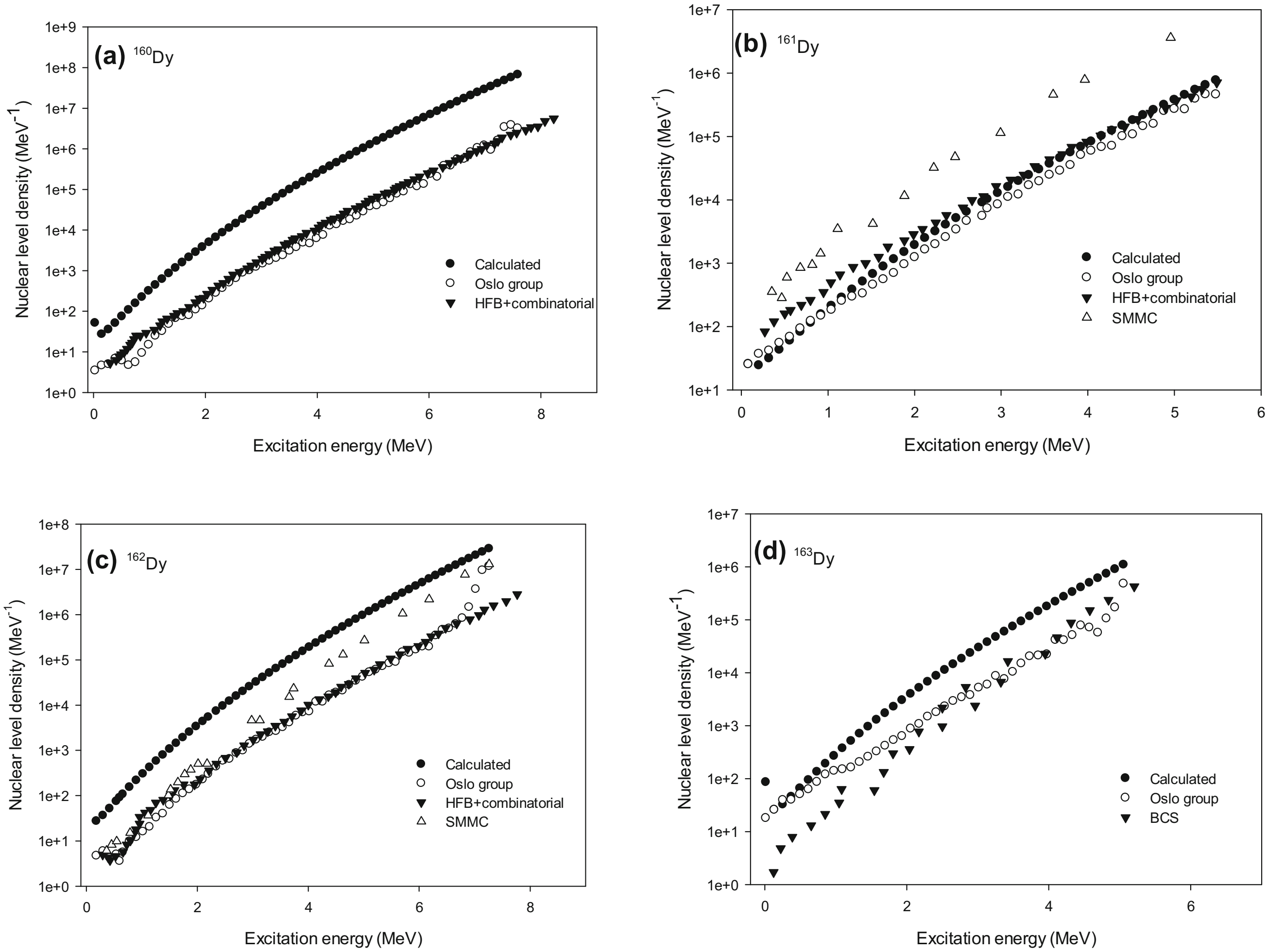

Fig. 2 Excitation energy dependence of the calculated nuclear level density for $^{160-163}$Dy radionuclides (**a**–**d**). The calculated results are compared with experimental and compiled level densities by Oslo group, SMMC, HFB + combinatorial, and BCS approaches

effects of adjustable parameters, including spin, parity, angular momentum, spin cut-off, etc. between theoretical and experimental statistical calculations for many-body particle systems. In this manner, analytical expressions, which contain several parameters adjusted on scarce experimental data, are generally preferred [22].

5 Conclusion

We have calculated the level density parameters and the NLDs of some deformed Dy radionuclides using experimental excitation energy and taking into account characteristic properties of the collective excitation modes. On the base of the presented discussion, we can conclude that the nuclear level density parameters can be identified using the collective excitation modes taking into consideration the equidistant character of these modes. A dominant band alone is not enough to exactly obtain the NLD of the deformed nuclei. Hence, we should take into account the effects of all of the collective excitation modes. In order to take into account the contributions of different excitation mechanisms to the formation of the final nuclear level density, we have made use the assumption related to independence and individuality of different nuclear excitation mechanisms, which is reliable at least for low-lying level spectra. The level energies for different bands must also be thought as independent from each other. Therefore, they must be treated individually since the level energy in each case is not composed of energies from different excitation modes. In such a case, the total level density function taking into account the nuclear levels of a different character is considered. The contribution of different excitation modes, as represented by weighting coefficients and appeared in Eq. (7), needs to be investigated both theoretically and experimentally.

As a main conclusion of the present work, it should be highlighted that the collective model approach includes nuclear collective excitation modes, which is quite meaningful in order to obtain the level density parameters of different nuclei and calculate the NLDs of the atomic nuclei. Furthermore, the adjustable parameters, which are being studied for our subsequent works, may cause a slight discrepancy between theoretical and experimental results, and it should be taken into account the statistical calculations of the many-body particle systems.

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