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Performance evaluation of the backprojection filtered (BPF) algorithm in circular fan-beam and cone-beam CT

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Abstract: In this article we introduce an exact backprojection filtered (BPF) type reconstruction algorithm for cone-beam scans based on Zou and Pan's work. The algorithm can reconstruct images using only the projection data passing through the parallel PI-line segments in reduced scans. Computer simulations and practical experiments are carried out to evaluate this algorithm. The BPF algorithm has a higher computational efficiency than the famous FDK algorithm. The BPF algorithm is evaluated using the practical CT projection data on a 450 keV X-ray CT system with a flat-panel detector (FPD). From the practical experiments, we get the spatial resolution of this CT system. The algorithm could achieve the spatial resolution of 2.4 lp/mm and satisfies the practical applications in industrial CT inspection.

Key words Cone-beam, PI-line, Backprojection filtered (BPF) algorithm, Hilbert transform

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1 Introduction

In the last 3 years, some novel cone-beam scans and new exact reconstruction algorithms have been developed. Cone-beam circular scan is a configuration widely used in clinical and industrial computed tomography (CT), because it can make use of X-ray more effectively and shorten the scanning time. Feldkamp *et al.*[1] adopted the conventional equispatial fan-beam algorithm for cone-beam reconstruction with a circular-scanning locus, which is the most famous approximate cone-beam reconstruction and has been widely used till now. Since its publication, the Feldkamp algorithm has been extended in various scanning ways for approximate reconstruction. To improve the reconstructed image quality in circular scans, some improved Feldkamp algorithms were developed, such as P-FDK, T-FDK, HT-FDK and S-FDK methods. [2,3]

In 2004, Zou and Pan proposed a new exact backprojection filtered (BPF) type reconstruction algorithm for cone-beam helical CT, in which only the theoretically minimum helical projection data were used. [4,5] Soon, the BPF reconstruction algorithm was developed to fan-beam and cone-beam circular scans.[6-10] In this article, we introduce the exact BPF type reconstruction algorithm to cone-beam scans based on Zou and Pan's reconstruction algorithm. We make a performance evaluation of the BPF algorithm with computer simulations and practical experiments on 450 keV CT system with a flat-panel detector (FPD).

2 Mathematical background and notations

Let us start with general notations used in this ar-

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ticle. In the fixed coordinate system (x,y,z), for a circular path the X-ray source location can be expressed as

$$\vec{r}_0(\lambda)=R(\cos\lambda,\sin\lambda) \tag{1}$$

where R denotes the distance between the X-ray source and the origin. $\lambda\in[\lambda_{\min},\lambda_{\max}]$ stands for the rotation angle, while $\lambda_{\min}$ and $\lambda_{\max}$ correspond to the starting and ending points of the rotation angle, respectively.

Let $f(\vec{r})$ denotes the 3-D object, where $\vec{r}=(x,y,z)^T$ denotes the spatial distribution of the object. Generally, we assume that a compact support of the object is confined within a sphere with radius R_0, which has the center at the origin. Therefore, it can be described as

$$f(\vec{r})\begin{cases}\neq 0 & x^2+y^2+z^2\le R_0^2\\ =0 & x^2+y^2+z^2>R_0^2\end{cases} \tag{2}$$

We also assume that $f(\vec{r})$ is continuous, and the cone-beam projection from any point on the trajectory can be expressed as

$$P(\vec{r}_0(\lambda),\hat{e})=\int_0^\infty f(\vec{r}_0(\lambda)+t\hat{e})\,\mathrm{d}t \tag{3}$$

where, the unit vector $\hat{e}$ indicates the projection direction of the X-ray. For a given point $\vec{r}$ in the object $f(\vec{r})$ and an X-ray source position $\vec{r}_0(\lambda)$, $\hat{e}$ is the unit direction vector from $\vec{r}_0(\lambda)$ to $\vec{r}$. We define a rotation-coordinate system (u,v) which is fixed on the FPD and always vertical to $\vec{r}_0(\lambda)$. In this coordinate system, u and v denote the horizontal and vertical coordinate, respectively. For any point $\vec{r}=(x,y,z)^T$ in the object, the cone-beam projection data passing through it could be denoted as $P(u,v,\lambda)$ on the FPD.

3 The BPF type reconstruction algorithm for cone-beam scans

In this section we simplify the BPF algorithm to ease implement in circular cone-beam CT. In the algorithm, we choose a set of parallel PI-lines covering the whole object. And we fix the PI-lines parallel to the *x*-axis. It is impossible to exactly reconstruct the whole images for cone-beam circular scans because of the loss of the projection data. In the non-middle planes, we choose the virtual PI-lines parallel with the PI-lines in the middle plane.[6, 11] The simplified BPF type reconstruction algorithm for cone-beam CT consists of four steps given below.

Step 1. Differentiation of the cone-beam projection data.

The differentiation of the projection data with respect to λ can be written as

$$\begin{aligned}G(u,v,\lambda)&=\frac{\mathrm{d}}{\mathrm{d}\lambda}P(u,v,\lambda)\big|_{\lambda\text{ fixed}}\\&=\left(\frac{\partial}{\partial\lambda}+\frac{u^2+D^2}{D}\times\frac{\partial}{\partial u}\right)\times P(u,v,\lambda)\end{aligned} \tag{4}$$

where $P(u,v,\lambda)$ is the projection data on the detectors; u and v are the horizontal and vertical physical coordinates, respectively, on the rotation detector coordinates.

Step 2. Determine the PI-line passing through the point $\vec{r}=(x,y,z)$.

A PI-line parallel to the *x*-axis is chosen as shown in Fig.1 and, this PI-line passing the point (x,y,z) can be expressed by (x,λ_1,λ_2), where λ_1 and λ_2 are determined as

$$\begin{cases}\lambda_1=\arcsin\left(\dfrac{y}{R}\right),\quad \lambda_2=\pi-\arcsin\left(\dfrac{y}{R}\right) & y\ge 0 \quad (5)\\ \lambda_1=\pi-\arcsin\left(\dfrac{y}{R}\right),\lambda_2=2\pi+\arcsin\left(\dfrac{y}{R}\right) & y<0 \quad (6)\end{cases}$$

where R denotes the radius of the circular source trajectory.

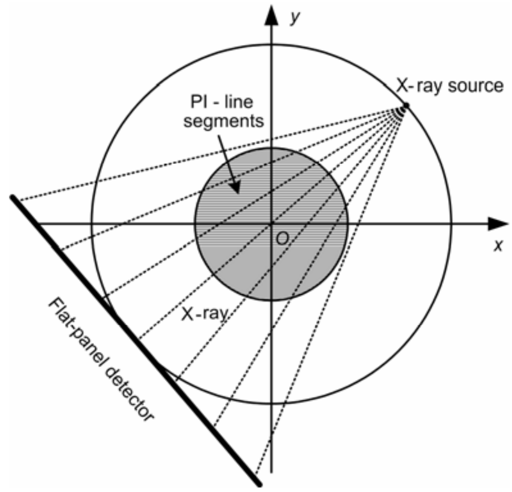

Fig.1 Parallel PI-lines and geometrical relationship in the coordinate system of the middle plane for cone-beam projections on a flat-panel detector.

Step 3. Weighted backprojection on the PI-line segment.

This step mainly gets the object image on PI-line segment, and the weighted backprojection is calculated for pixels on the fixed PI-line segment in Step 2.

$$g_{\pi}(\vec{r}) = \int_{\lambda_1}^{\lambda_2} \frac{G(u,v,\lambda)}{\|\vec{r} - \vec{r}_0(\lambda)\|} \mathrm{d}\lambda \quad (7)$$

Here, $\vec{r}$ is the point (x, y, z) and it can also be expressed by $(x, \lambda_1, \lambda_2)$ as in Step 2. To reconstruct the image at $\vec{r}$, the angular range of backprojection in Eq. (7) is from λ_1 to λ_2 instead of 2π in the FDK algorithm. This is the reason that the BPF algorithm has a higher computational efficiency than the FDK algorithm.

Step 4. Inverse Hilbert transform and get the reconstruction image.

The object image on the PI-line segment can be calculated as

$$f(x,y,z) = \frac{1}{2\pi} \times \frac{1}{\sqrt{(x_2 - x)\times(x - x_1)}} \times \left[\int_{x_1}^{x_2} \frac{\sqrt{(x_2 - x')\times(x' - x_1)}}{\pi(x - x')} \times g_{\pi}(x', y, z)\,\mathrm{d}x' + C \right] \quad (8)$$

where x_1 and x_2 denote the two end points of the PI-line segment; C is the double integral of $f(0, \lambda_1, \lambda_2)$:

$$C = 2P(\lambda_1, \lambda_2) \quad (9)$$

Finally, we obtain the 3-D object $f(x, y, z)$ using the BPF type reconstruction algorithm from the cone-beam projection. Because we choose a set of PI-lines parallel to x-axis covering the object, we can directly obtain the targeted images in Cartesian coordinates and need not re-sample the reconstructed images.

4 Experiments

In this section, we design computer-simulation studies and practical experiments to evaluate the above BPF algorithm in both circular fan-beam and cone-beam scans. All the computer simulations are done on our workstation with an AMD 64-bit 1800+ CPU and 4G RAM. Matlab 7.0.4 is chosen to implement the algorithms. For the first computer simulation we use 3D Shepp-Logan phantom,[3] and the geometrical configuration in the experiment is listed in Table 1. In the second computer simulation, we use a cylindrical model to respectively calculate the point-spread functions of the FDK algorithm and the above BPF algorithm in the middle plane.

Table 1 Geometrical configuration in the computer simulation experiment

Scanning configuration parameters	Values
Trajectory radius / mm	15
Object radius / mm	1
Source-to-detector distance / mm	30
Projections per circle	640
Flat-panel detector size / mm × mm	5 × 5
Detector unit number	256 × 256
Reconstructed image dimensions	256 × 256 × 256

In the practical experiments, we use our 450 keV CT system to evaluate the spatial resolution of the above BPF algorithm in this system. A linepair faceplate is used from 1.6—3.0 lp (linepair)/mm. In this CT system, a bi-focus X-ray tube with energy up to 450 keV is chosen as the X-ray source. The detector is a FPD. The physical size of each pixel on the detector is 0.127 mm × 0.127 mm. In the experiments, the sampling rate of the view angles is chosen to be $\pi/180$.

5 Results and performance evaluation

The computer simulation and practical experimental results are presented in this section. In Fig. 2, reconstructed images of the 3D Shepp--Logan phantom are shown with the geometrical configuration in Table 1. Figs 2(a) and (b) show two different slice images at $z = 0$ mm and $y = -0.25$ mm with a display window [0.98, 1.06], which are both reconstructed by the BPF algorithm. The images are also reconstructed using FDK algorithm with the same projection data. Figs 2(c) shows two profiles reconstructed by BPF and FDK algorithm, respectively. The profiles are along the white line in Fig.2(b). The solid and dotted curve is for the BPF and FDK reconstruction, respectively, while the dash-dot curve is for the original. These results suggest that the BPF algorithm seems more sensitive than FDK in terms of the interpolating errors. To reconstruct a slice with 256 × 256 pixels, the BPF algorithm needs 1033 seconds while FDK algorithm needs 2453 s.

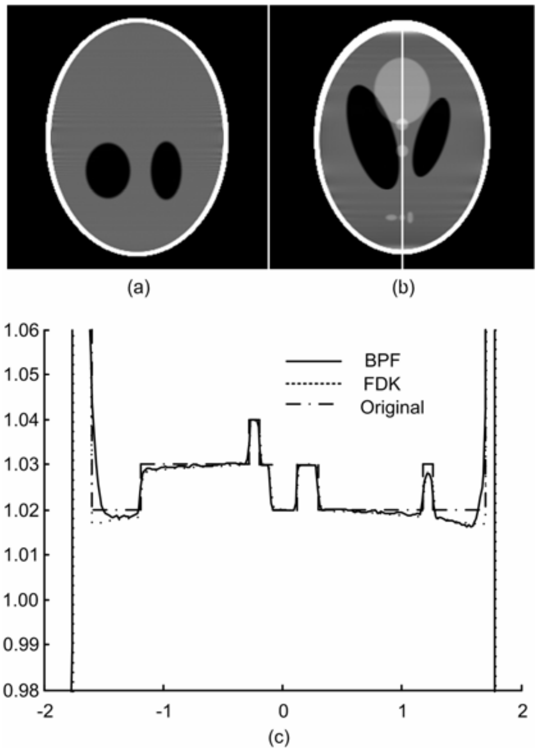

Fig.2 Numerical experimental results reconstructed by BPF algorithm with 3D Shepp--Logan phantom. (a) and (b) show two different slice images at $z=0\,\text{mm}$ and $y=-0.25\,\text{mm}$ with a display window [0.98, 1.06]; (c) shows two profiles reconstructed by BPF and FDK algorithm, respectively. The profiles are along the white line in (b).

In Fig. 3, we show the point-spread functions (PSF) of FDK and BPF reconstruction algorithm in the middle plane for cone-beam circular scans. The two curves respectively denote the PSFs of FDK and BPF algorithm with full scans. We notice that the spatial resolution of the BPF algorithm is a little worse than the FDK algorithm in the middle plane.

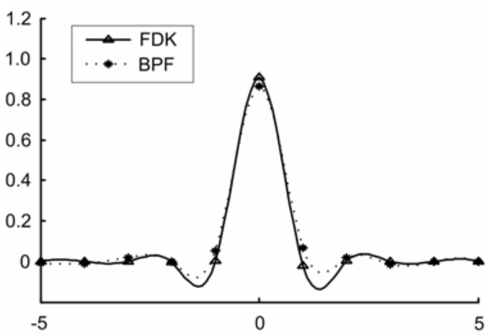

Fig. 3 The point-spread functions (PSF) of FDK and BPF algorithm in the middle plane for cone-beam full circular scans.

In Fig. 4, a linepair faceplate is reconstructed using the actual data on our 450 keV X-ray CT system. The Fig. 4 (a) and (b) are reconstructed by the FDK and BPF algorithm, respectively. As shown in these two figures, 2.4 lp/mm could both be easily distinguished. That means our proposed algorithm can have the same spatial resolution with the FDK algorithm in this CT application and, our CT system can achieve the spatial resolution of 2.4 lp/mm at least with the BPF algorithm.

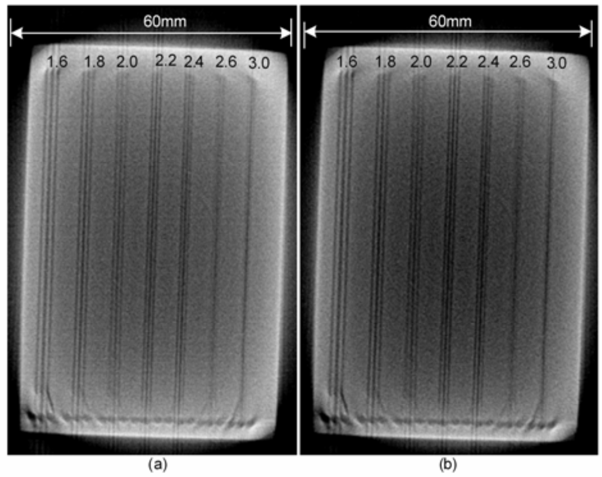

Fig. 4 Practical experiments are shown using a linepair faceplate on our 450 keV CT system with a flat-panel detector. (a) Reconstruction by FDK algorithm. (b) Reconstruction by BPF algorithm.

6 Discussions and conclusion

As we all know, the FDK algorithm is the most famous and widely used algorithm for cone-beam circular scans. But the FDK algorithm can only work in full scan or short scan. It means that it need backproject all the projection data in a full or short scan for reconstructing each point, which is different in the reconstructing process of the BPF algorithm. For different PI-line segments, the angles of the backprojection are different, which are almost less than the angle in a short scan. So, as the same backprojection-type algorithm, the BPF algorithm has a higher computational efficiency than the FDK algorithm. From the first computer simulation we notice that the computational time of FDK algorithm is 2.37 times of BPF algorithms. In Fig. 2(b), we find some “streak” artifacts due to the following two reasons. First, the last step in BPF algorithm is the inverse Hilbert transform which is very sensitive to the constant C. This constant is inequable among different PI-line segments. For the discrete sampling it is difficult to determine exact and coherent C in different PI-lines. The second reason is that there are discontinuous edges of the 3D Shepp-Logan phantom. When the derivatives are computed in Step 1 of the BPF algorithm, large errors occur with discrete sampling and interpolation operation. In practical experiments these “streak” artifacts are perhaps reduced as shown in Fig. 4. To reduce the “streak” artifacts, the detector resolution should be improved and the projection number increased. With the practical experiments, we verify that the above BPF reconstruction algorithm is effective. The qualities of reconstructed images are pretty good using the actual projection data from the 450 keV CT system. We achieve the spatial resolution of at least 2.4 lp/mm. In conclusion, the BPF algorithm can satisfy the requirement of the industrial CT inspection.

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