Calculations of absolute intensities of γ-rays associated with (ε+β$^+$) decay

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Abstract The calculation of absolute intensities of γ-rays arising from (ε+β$^+$) decay is introduced briefly. Some examples are given to illustrate their applications. And some discussions are also made in the text.

Key words (ε+β$^+$) decay, γ-ray normalization factor, Intensity calculation

CLC number O571.3

1 Introduction

The (ε+β$^+$) decay data and schemes are important basic data for basic research of nuclear physics, nuclear technology application, especially radioactive isotope application. In the (ε+β$^+$) decay process, the mother nuclide decays to different excited states and the ground state of the daughter nuclide, then γ-rays and their internal conversion electrons are emitted in the de-excited process from the higher excited states to the lower excited states and through to the ground state. In common uses, the users are interested in γ-ray emission probability (γ-ray absolute intensities per 100 decays of parent). In most radioactive decay measurements, γ-ray relative intensities are measured only, because measurements of γ-ray absolute intensities are very difficult and the measurement accuracy is quite low. Therefore, it is required to convert the γ-ray relative intensities to γ-ray absolute intensities (emission probabilities). This converting multiplier is called γ-ray normalization factor. Here, the calculation methods of normalization factor for (ε+β$^+$) decay are introduced briefly with some applications. The intensity balance checking and some discussions are also made.

2 Calculation methods

2.1 Calculation from β$^+$ decay intensity $I_{\beta^+,0}$ to ground state of daughter nuclide

Suppose that β$^+$ decay intensity $I_{\beta^+,0}$ to the ground state of the daughter nuclide has been measured, then we can write

$$b_0 = I_{(\varepsilon+\beta^+),0} = I_{\varepsilon,0} + I_{\beta^+,0} \tag{1}$$

where $I_{(\varepsilon+\beta^+),0}$ is the total (ε+β$^+$) decay intensity to the ground state of the daughter nuclide, $I_{\beta^+,0}$ is β$^+$ decay intensity and $I_{\varepsilon,0}$ is ε decay intensity to the ground state of the daughter nuclide, respectively.

Assume that $P_{\varepsilon,0}$ and $P_{\beta^+,0}$ are ε emission probability and β$^+$ emission probability decaying to the ground state of the daughter nuclide, respectively. Eqs. (2) and (3) can be gotten:

$$P_{\varepsilon,0} + P_{\beta^+,0} = 1 \tag{2}$$

$$P_{\varepsilon,0} = P_{\varepsilon K,0} + P_{\varepsilon L,0} + P_{\varepsilon M,0} + P_{\varepsilon N,0} \tag{3}$$

Received date: 2004-04-22

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where $P_{\varepsilon,0}$, $P_{\varepsilon K,0}$, $P_{\varepsilon L,0}$, $P_{\varepsilon M,0}$ and $P_{\varepsilon N,0}$ are the total capture probability and capture probabilities decaying to the ground state of the daughter nuclide from *K*, *L*, *M*, and *N* atomic electron shell, respectively. From Eq.(2), one can obtain

$$P_{\beta^+,0} = 1 - P_{\varepsilon,0} \tag{4}$$

From Eqs. (1) and (2), we can also get

$$b_0 = \frac{I_{\beta+,0}}{(1-P_{\varepsilon,0})} = \frac{I_{\beta^+,0}}{P_{\beta^+,0}} \tag{5}$$

It is supposed that the o-th γ-ray relative intensity to ground state from the *J*-th level of the daughter nuclide is $I_{\gamma J,0}$, and its total internal conversion coefficient is $\alpha_{J,0}$, in which $J(\geqslant 1)$ is known. From intensity balance, the γ-ray normalization factor of decay scheme, N_γ, can be calculated by Eq.(6) or Eq.(7):

$$(100 - b_0) = N_\gamma \sum_{J=1} I_{\gamma J,0}(1+\alpha_{J,0}) \tag{6}$$

or

$$N_\gamma = \frac{(100-b_0)}{\sum_{J=1} I_{\gamma J,0}(1+\alpha_{J,0})} \tag{7}$$

In Eqs. (6) and (7), the summation is over all *J* (all γ-rays from the *J*-th levels to ground state), $I_{\gamma J,0}$ is o-th γ-ray relative intensity to the ground state from the *J*-levels of daughter nuclide, $\alpha_{J,0}$ is its total internal conversion coefficient. Therefore, when the normalization factor N_γ has been calculated from Eq.(7), the absolute intensity (emission probability) $P_{\gamma i}$ for *i*-th γ-ray can be calculated by using Eq. (8):

$$P_{\gamma i} = N_\gamma I_{\gamma i} \tag{8}$$

where $I_{\gamma i}$ is the relative intensity of the *i*-th γ-ray and $P_{\gamma i}$ is the absolute intensity (emission probability) of the *i*-th γ-ray.

2.2 Calculation from no (ε+β$^+$) decay to ground state of daughter nuclide

In the case of no (ε+β$^+$) decays to the ground state of the daughter nuclide, obviously, b_0=0. From Eq.(7), we can obtain

$$N_\gamma = \frac{100}{\sum_{J=1} I_{\gamma J,0}(1+\alpha_{J,0})} \tag{9}$$

In Eq.(9), the summation is also over all *J* (all γ-rays from the *J*-th levels to the ground state), $I_{\gamma J,0}$ is the o-th γ-ray relative intensity to the ground state from *J*-th levels of the daughter nuclide, and $\alpha_{J,0}$ is its total internal conversion coefficient.

2.3 Calculation from annihilation radiation intensity of β$^+$ decay

From Eq.(1), the intensity of (ε+β$^+$) decaying to the ground state of the daughter nuclide is b_0:

$$b_0 = I_{(\varepsilon+\beta^+),0} \tag{10}$$

It is supposed that the intensity of (ε+β$^+$) decaying to the *J*-th level of daughter nuclide is b_J:

$$b_J = I_{(\varepsilon+\beta^+),J} \tag{11}$$

where $J\geqslant 1$ (*J*=0 for ground state, *J*=1 for the 1-st excited state, *J*=2 for the 2-nd excited state, *J*=*J* for the *J*-th level state).

Meanwhile, it is supposed that the relative transition intensity imbalance X_J leaving and feeding the *J*-th level is as follows:

$$\begin{aligned} X_J &= [I_{\gamma+\mathrm{ce},J}(\mathrm{out}) - I_{\gamma+\mathrm{ce},J}(\mathrm{in})] \\ &= [\sum_k I_{\gamma J,k}(\mathrm{out})(1+\alpha_{J,k}) - \sum_n I_{\gamma n,J}(\mathrm{in})(1+\alpha_{n,J})] \end{aligned} \tag{12}$$

In Eq. (12), $I_{\gamma+\mathrm{ce},J}(\mathrm{out})$ and $I_{\gamma+\mathrm{ce},J}(\mathrm{in})$ are total relative transition intensities leaving and feeding the *J*-th level, respectively. $I_{\gamma J,k}(\mathrm{out})$ and $I_{\gamma n,J}(\mathrm{in})$ are the relative intensities leaving the *J*-th level to the *k*-th level lower than the *J*-th level and the feeding to the *J*-th level from the *n*-th level higher than the *J*-th level, $\alpha_{J,k}$ and $\alpha_{n,J}$ are their total internal conversion coefficients, respectively. The total annihilation radiation intensity $I_{\gamma^\pm}$ of β$^+$ decay can be calculated by using

$$I_{\gamma^{\pm}}=2\sum_{J}I_{\beta^{+},J} \tag{13}$$

where $I_{\beta^{+},J}$ is the β^{+} decay intensity to the J-th level of daughter nuclides.

From Eq. (13), we can write

$$\begin{aligned} I_{\gamma\pm}&=2\left[b_{0}P_{\beta^{+},0}+\sum_{J=1}I_{\varepsilon+\beta^{+},J}P_{\beta^{+},J}\right] \\ &=2\left[b_{0}P_{\beta^{+},0}+b_{1}P_{\beta^{+},1}+b_{2}P_{\beta^{+},2}+\cdots\right] \end{aligned} \tag{14}$$

where $P_{\beta^{+},J}$ is β^{+} emission probability of ($\varepsilon+\beta^{+}$) decay to the J-th level, b_J is ($\varepsilon+\beta^{+}$) intensity of ($\varepsilon+\beta^{+}$) decay to the J-th level. Therefore, we can get b_1, b_2, …, b_J, for the 1-st level, the 2-nd level, …, the J-th level:

$$b_{1}=X_{1}N_{\gamma}=X_{1}\frac{(100-b_{0})}{\sum_{J=1}I_{\gamma J,0}(1+\alpha_{J,0})} \tag{15}$$

$$b_{2}=X_{2}N_{\gamma}=X_{2}\frac{(100-b_{0})}{\sum_{J=1}I_{\gamma J,0}(1+\alpha_{J,0})} \tag{16}$$

$$\cdots$$

$$b_{J}=X_{J}N_{\gamma}=X_{J}\frac{(100-b_{0})}{\sum_{J=1}I_{\gamma J,0}(1+\alpha_{J,0})} \tag{17}$$

From Eq. (12), we can obtain

$$X_{1}=I_{\gamma1,0}(1+\alpha_{1,0})-\sum_{n>1}I_{\gamma n,1}(1+\alpha_{n,1}) \tag{18}$$

$$X_{2}=\sum_{k<2}I_{\gamma2,k}(1+\alpha_{2,k})-\sum_{n>2}I_{\gamma n,2}(1+\alpha_{n,2}) \tag{19}$$

After the physical quantitative data of Eqs. (12), (15), (16), (17), (18), (19) and (7), which are calculated from decay scheme, are substituted for ones in Eq.(14), b_0 is the only unknown number in Eq.(14). Therefore, when the solution of b_0 has been found, the γ-ray normalization factor of decay scheme, N_γ, can be calculated by Eq.(6) or Eq.(7). Obviously, the γ-ray absolute intensities can also be obtained by using γ-ray normalization factor N_γ of decay scheme in Eq.(8).

2.4 Calculation from *K*-X ray intensity of (ε+β⁺) decay to daughter nuclide

The *K*-X rays are produced from nuclear processes of γ-ray internal conversion and electron capture. These processes leave vacancies in the inner electron orbits. These vacancies will then be filled by atomic processes. This will result in the emission of X-rays. The intensity calculation method of *K*-X rays can be found in Ref.[1]. Assume the vacancy from ε decay in (ε+β⁺) decay to *K*-electron shell is $V(\varepsilon)_K$, we can write

$$\begin{aligned} V(\varepsilon)_{K}&=\sum_{J}I_{\varepsilon+\beta^{+},J}P_{\varepsilon K,J} \\ &=b_{0}P_{\varepsilon K,0}+N_{\gamma}\sum_{J=1}X_{J}P_{\varepsilon K,J} \end{aligned} \tag{20}$$

where $P_{\varepsilon K,0}$ and $P_{\varepsilon K,J}$ are electron capture probabilities of the ground state and the J-th state(level) of the daughter nuclide, respectively. The relation of electron capture probabilities of different electron shells is as follows:

$$P_{\varepsilon,J}+P_{\beta^{+},J}=1 \tag{21}$$

where $P_{\varepsilon,J}$ and $P_{\beta^{+},J}$ are electron capture probability and β^{+} emission probability of the J-th state(level) of the daughter nuclide, respectively. As for $P_{\varepsilon,J}$, we have

$$P_{\varepsilon,J}=P_{\varepsilon K,J}+P_{\varepsilon L,J}+P_{\varepsilon M,J} \tag{22}$$

where $P_{\varepsilon,J}$, $P_{\varepsilon K,J}$, $P_{\varepsilon L,J}$ and $P_{\varepsilon M,J}$ are electron capture probabilities of *K*, *L* and *M* shells of the *J*-th state(level) of the daughter nuclide, respectively.

Assume the vacancy from γ-ray internal conversion process for *K*-electron shell arising from (ε+β⁺) decay is $V(\mathrm{ce})_K$, we can obtain

$$V(\mathrm{ce})_{K}=N_{\gamma}\sum_{i}I_{\gamma i}\alpha_{K,i} \tag{23}$$

where $I_{\gamma i}$ and $\alpha_{K,i}$ are relative intensity and its internal conversion coefficient of *K*-shell for the *i*-th γ-ray, respectively. The relationship of internal conversion coefficients of different electron shells for the γ-rays is as follows:

$$\alpha_{i}=\alpha_{K,i}+\alpha_{L,i}+\alpha_{M,i}+\alpha_{N,i}\,(i=1,2,\cdots) \tag{24}$$

where α_i , $\alpha_{K,i}$, $\alpha_{L,i}$, $\alpha_{M,i}$ and $\alpha_{N,i}$ are total internal conversion coefficient and ones of *K*, *L*, *M*, and *N*-electron shells for the *i*-th γ-ray, respectively.

Assume the vacancy from ε decay and γ-ray internal conversion process for *K*-electron shell in $(\varepsilon+\beta^+)$ decay is $V(\varepsilon+\mathrm{ce})_K$. From Eqs. (20) and (23), we can write

$$V(\varepsilon+\mathrm{ce})_K = b_0 P_{\varepsilon K,0} + N_\gamma \sum_{J=1} X_J P_{\varepsilon K,J} + N_\gamma \sum_i I_{\gamma i}\alpha_{K,i} = b_0 P_{\varepsilon K,0} + N_\gamma [\sum_{J=1} X_J P_{\varepsilon K,J} + \sum_i I_{\gamma i}\alpha_{K,i}] \tag{25}$$

Assume the *K*-X ray intensity from ε decay and γ-ray internal conversion process for *K*-electron shell in $(\varepsilon+\beta^+)$ decay is I_{KX}. From Eq.(25), we can get

$$I_{KX} = \omega_K \{b_0 P_{\varepsilon K,0} + N_\gamma [\sum_{J=1} X_J P_{\varepsilon K,J} + \sum_i I_{\gamma i}\alpha_{K,i}]\} \tag{26}$$

where ω_K is the X-ray fluorescence yield of *K*-electron shell for the daughter nuclide. They have been evaluated and tabulated in Ref.[2]. After the physical quantitative data are substituted for ones of Eq.(26), b_0 is the only unknown number in Eq.(26). Therefore, when the solution b_0 has been found, the γ-ray normalization factor of decay scheme, N_γ, can be calculated by Eq.(6) or Eq.(7). Obviously, the γ-ray absolute intensities are also obtained by using γ-ray normalization factor N_γ of decay scheme in Eq.(8).

3 Applications of the calculation methods

3.1 Calculation of γ–ray intensities of ^{65}Zn $(\varepsilon+\beta^+)$ decay

In ^{65}Zn $(\varepsilon+\beta^+)$ decay[3], $I_{\beta^+,0}$=1.420% of $(\varepsilon+\beta^+)$ decay to ground state of daughter nuclide is known. Its ε decay probability to the ground state, $P_{\varepsilon,0}$=0.9713, can be calculated by using LOGFT code. From Eq.(5), we can get b_0 =(49.40±0.24)%. From Eq.(7), one can calculate the γ-ray normalization factor of decay scheme, N_γ=0.506±0.002, of the γ-ray relative intensity for I_γ (1115.546 keV)=100. In Table 1, the radiation data of ^{65}Zn $(\varepsilon+\beta^+)$ decay are listed, where EC means electron capture, β^+ means positron emission, XL and XK are X-ray emission for the *L*- and *K*-shell, γ means γ-ray emission, $\gamma^\pm$ means positron annihilation radiation. The calculation method of X-ray data has been given in Ref.[2].The scheme for ^{65}Zn $(\varepsilon+\beta^+)$ decay is shown in Fig.1, where the energies, multipolarities, absolute intensities, and place for γ-rays are given. Besides, the ε and β^+ decay intensities to each levels of daughter nuclide and other physical data are also presented.

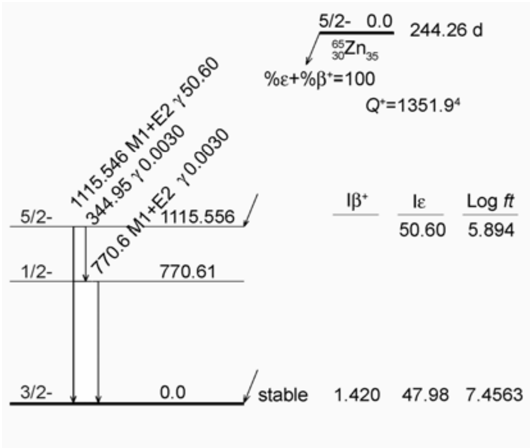

Fig.1 ^{65}Zn $(\varepsilon+\beta^+)$ decay scheme.

Table 1 Radiation data for ^{65}Zn $(\varepsilon+\beta^+)$ decay

Radiation type	Energy (keV)	Absolute intensity (%)
EC_1#		50.60 24
EC_2#		47.98 23
β^+_1 max	329.9@ 4	
avg	143.01 17	1.420 17
XL*	0.9300	0.574 11
$XK_{\alpha2}$Δ	8.027830 10	11.5 3
$XK_{\alpha1}$Δ	8.047780 10	22.6 5
XK_βΔ	8.910	4.61 13
γ_1	344.95 20	0.0030 3
γ_2	770.60 20	0.0030 3
γ_3	1115.546 4	50.60 24
$\gamma^\pm$	510.99906 15	2.84 4

Notes: # Electron capture; @ Uncertainty (error): the uncertainty in any number is given space after the number itself: for example, 329.9 4 means 329.9±0.4 (all the same below);
* X-ray of *L* shell; Δ X-ray of *K* shell in different energies.

3.2 Calculation of γ–ray intensities of ^{58}Co (ε+β$^+$) decay

In ^{58}Co (ε+β$^+$) decay[4], the (ε+β$^+$) decay intensity of (ε+β$^+$) decay to ground state of daughter nuclide is known as $I_{(\varepsilon+\beta^+),0}$ =0. From Eq.(7), one can calculate the γ-ray normalization factor of decay scheme, N_γ=0.9945±0.0001, of the γ-ray relative intensity for I_γ (810.775 keV)=100. In Table 2, the radiation data for ^{58}Co (ε+β$^+$) decay are listed.

Table 2 Radiation data for ^{58}Co (ε+β$^+$) decay

Radiation type	Energy (keV)	Absolute intensity (%)
EC_1		1.20 1
EC_2		83.90 16
β^+_1 max	474.6 11	
avg	201.1 5	14.90 16
e AuL*	0.6700	116.6 4
e AuK*	5.620	49.4 7
XL	0.7000	0.3637 11
$XK_{\alpha2}$	6.39084 3	7.79 21
$XK_{\alpha1}$	6.40384 3	15.4 4
XK_β	7.060	3.10 10
γ_1	810.775 9	99.450 10
eC_{e1K}^{Δ}	803.663 9	0.0298 9
γ_2	863.959 9	0.687 11
γ_3	1674.730 10	0.521 8
$\gamma^\pm$	510.99906 15	29.80 32

Notes: * Auger electrons of *L* and *K* shell, respectively. Δ Internal conversion electron of *K* shell from γ_1 ray.

The scheme of ^{58}Co (ε+β$^+$) decay is shown in Fig.2, where the energies, multipolarities, absolute intensities, and places for γ-rays are given. Besides, the ε and β$^+$ decay intensities to each levels of daughter nuclide and other physical data are also shown.

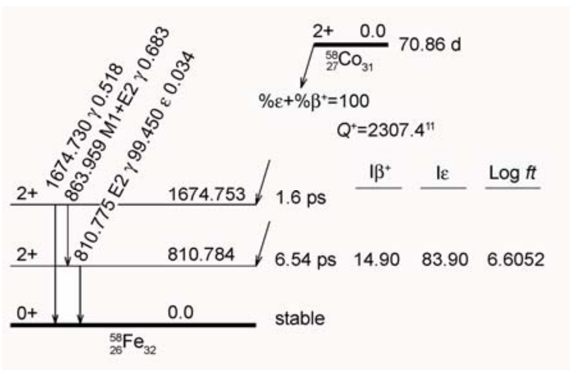

Fig.2 Scheme of ^{58}Co (ε+β$^+$) decay.

3.3 Calculation of γ–ray intensities of ^{186}Re ε decay

In ^{186}Re ε decay [5], the *K* X ray intensity, I_{KX} = (6.02±0.09)%, is known. The relative intensity and internal conversion coefficients for the γ-ray of ^{186}Re ε decay are listed in Table 3. The electron capture probabilities of *K*, *L* and *M* electron shells of decay to different levels of daughter nuclide for ^{186}Re ε decay are listed in Table 4. When ω_k =0.958 is adopted, and relative data are put into Eq.(26), one can calculate the ε decay intensity to ground state of the daughter nuclide, b_0=(5.78±0.10)%. Meanwhile, b_0 value and %ε=7.47±0.10 (see Fig.3) are put into Eq.(7), one can obtain the γ-ray normalization factor of decay scheme, N_γ=(0.603±0.010)%, of the γ-ray relative intensity for I_γ (122.30 keV)=100. In Table 5, the radiation data for ^{186}Re ε decay are listed, where the data on energies, multipolarities, absolute intensities, and places for γ-ray are given. The scheme of ^{186}Re ε decay is shown in Fig.3, where ε decay intensities to different levels of daughter nuclide, and other physical data are also given (for example, %ε=7.47 10 means ε decays of (7.47±0.10) branching per 100 decays of parent nuclide).

Table 3 γ-ray relative intensity and its internal conversion coefficients for ^{186}Re ε decay

E_γ (keV)	I_γ*	α_K	α_L	α_M	α
122.30 7	100	0.592	0.908	0.228	1.79

Notes: * Relative intensity

Table 4 Electron capture probabilities to different levels of daughter nuclide for ^{186}Re ε decay

E_L (keV)	$P_{\varepsilon K}$	$P_{\varepsilon L}$	$P_{\varepsilon M}$
0.0	0.7946	0.1559	0.0495
122.30	0.7838	0.1638	0.0524

Table 5 Radiation data of ^{186}Re ε decay

Radiation type	Energy (keV)	Absolute intensity (%)
EC_1		1.69 3
EC_2		5.78 10
e Au_L	6.530	4.8 3
e Au_K	45.70	0.269 8
XL	8.400	2.06 23
$XK_{\alpha2}$	57.9817 5	1.73 4
$XK_{\alpha1}$	59.31820 10	3.00 6

XK_{β}	67.20		1.28	4
γ_1	122.30	7	0.603	10
e Ce_{1K}	52.77	7	0.359	13
e Ce_{1L}	110.20	7	0.55	4
e Ce_{1M}	119.48	7	0.139	6
e Ce_{1N}	121.70	7	0.0409	15

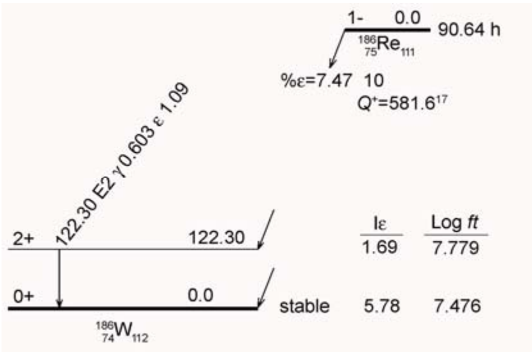

Fig.3 Scheme of ^{186}Re ε decay.

4 Discussion

4.1 Calculation of γ–ray intensities of only ε decay

When ε decay energy Q_{ε}<1.02 MeV, no β^{+} decay will be produced. In this case, β^{+} decay probability is zero, $P_{\beta+}$ =0, and the annihilation radiation intensity of β^{+} decay is also zero, i.e. $I_{\gamma^{\pm}}$ =0. For calculating the γ-ray normalization factor of decay scheme, all formulas as indicated above are suitable, except for the annihilation radiation intensity of β^{+} decay.

4.2 Calculation of annihilation radiation intensity of β^{+} decay

When γ-transitions undergo significant pair conversions, their contribution to annihilation radiation intensity should be subtracted from $I_{\gamma^{\pm}}$.

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