

Reconstruction of 12 MV bremsstrahlung spectra from measured transmission data

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Abstract Transmission data for 12 MV bremsstrahlung beams produced by the process for continuous-pulsed 12 MV electrons coming from a linear accelerator hitting a 1.2mm thick plane tantalum target have been acquired with a combination of iron and Telfon attenuators. Two solid state dosimeters with LiF-TLD material surrounded Telfon were used as detectors. It has been checked that the experimental system achieves reasonably narrow-beam geometry by Monte Carlo simulations. From these transmission data, the original energy photon spectrum has been reconstructed using the iterative least-squares technique and compared with the spectrum calculated with Monte Carlo code system EGSnrc. The comparison shows that the numerical technique for analysis of transmission data can represent 12 MV bremsstrahlung spectrum acceptably well. The purpose of our work is to provide an effective way to reconstruct an unknown photon spectrum with high energy component and prove the correctness of this way.

Keywords Electron bremsstrahlung spectra, Attenuation analysis, Least-squares method, Monte Carlo simulation, EGSnrc system

CLC numbers TL543, TL84

1 Introduction

The “quality of radiation” for a high energy X-ray beam can be specified by its attenuation curve in some selected materials.^[1] However, an ideal solution for the photon energy distribution determination would be to measure the spectrum with a NaI(Tl) scintillator, a germanium detector, or by some other means. But such spectrometry of high energy X-rays is hardly suited for the confined space and time available in some situations. Additionally, high intensity during the short pulses presents significant difficulties.^[2]

The numerical technique to deduce an effective spectrum from transmission measurements was first applied by Twidell to determine 2 MV X-ray spectra using transmission data measured in lead.^[3] The numerical method is to iteratively calculate the X-ray spectrum with some physically sensible constraints until the calculated transmission values agree with the measured data within the experimental accuracy. If the measured transmission curve has a sufficient change

in its slope over a wide transmission range, this method may be sensitive enough for the determination of an X-ray energy spectrum. Huang P H *et al* had extended this technique further and, in particular, to study its feasibility for a 4 MV X-ray beam.^[4] To overcome the limitations imposed by using a high Z material, such as lead or tin, as attenuator, they had utilized combinations of high and low Z attenuating materials for transmission measurement. In addition, they used an iterative least-squares technique with simulated transmission data to evaluate its performance and concluded that it is reliable for spectral reconstruction and is sensitive to small spectral changes in the 4 MV X-ray beam.^[4] And Xiao K Y *et al* had determined experimentally the photon energy fluence spectrum of 6 MV X-ray.^[5] They used only one kind of attenuator material (Telfon) because of relatively low photon energy and the reconstructed spectrum has not been checked by any other method.

In this paper, the transmission data for 12 MV bremsstrahlung beams produced by the stopping

process for continuous-pulsed 12 MV electrons coming from a linear accelerator hitting a plane tantalum target with 1.2 mm thickness have been collected with a combination of iron and Teflon attenuators. Two solid dosimeters with LiF-TLD surrounded Teflon have been used as detectors. In order to construct a narrow-beam geometry, we have made an X-ray collimating apparatus. From these data, the self-consistent energy spectra have been reconstructed using the iterative least-squares technique and compared with the spectra calculated with Monte Carlo code system EGSnrc.^[6-8] That is to say, the initial X-ray spectra can be calculated with Monte Carlo method by simulating the stopping process of 12 MV electrons hitting the tantalum target. It has been checked if the experimental system achieves reasonably narrow-beam geometry by Monte Carlo simulations, too. Selections of both attenuation materials and detectors had been determined by the need for this kind of high energy X-rays.

2 Experimental

2.1 Experimental setup

Transmission data were acquired with the experimental setup shown in Fig.1. The initial bremsstrahlung X-rays would emit when a thin tantalum plane was hit by parallel 12 MV electron beams from a linear accelerator. There was a collimating system in the experimental setup, which ensured that only the forward bremsstrahlung X-rays could pass the absorber materials. Meanwhile, there were two solid-state dosimeters in the collimating system. One was used for monitoring intensity of the X-rays entering the collimating part and located at entrance of the collimator. The other was used for measuring X-ray's attenuated intensity and located at the end of the collimator. The front wall thickness of these chambers, in principle, should be equal to or larger than the CSDA range of the maximum-energy secondary electrons produced by maximum-energy photons in Teflon. But an over-thick wall would reduce strongly the low-energy photon. It has been reported that the projected range of the parallel incident electrons on the incident direction is half of the CSDA range.^[9] In addition, it has been indicated that it is not necessary to have the

wall thickness equal to the maximum range of the secondary electrons, especially for small ionization chambers, and half of the maximum range may be considered quite sufficient for the equilibrium condition to be assumed.^[10] In our experimental system, the Teflon wall thickness of the solid chamber was 10.3 mm ($2.266 \text{ g} \cdot \text{cm}^{-2}$), which achieved the average CSDA range of secondary electrons produced by 5.8 MV photons.

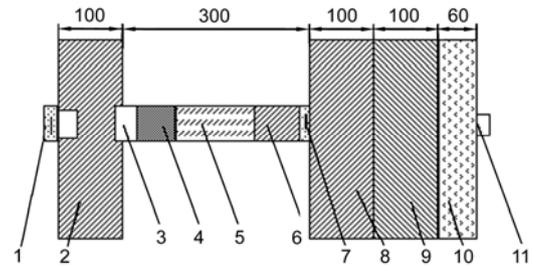


Fig.1 Experimental setup used for measuring transmission coefficients. 1.Solid state chamber ($\phi 34$, the front wall is about 10.3mm thick); 2. Rear shield (iron, 100mm thick); 3.Absorber bracket (inner diameter: 36mm); 4.Lead block plug (used to measure background); 5. Teflon absorber ($\phi 36$); 6. Iron absorber ($\phi 36$); 7.Solid state chamber used for measuring X-ray's intensity ($\phi 36$, total thickness of the former and rear walls is 20.8mm); 8. Middle shield (iron, 100mm thick); 9. Front shield (iron, 100mm thick); 10. Shield used for stopping 12MV electrons (perspex, 60mm thick); 11.Tantalum target (1.2mm thick).

With different length of the absorber materials in the collimating system, the transmission coefficients could be obtained from the LiF-TLD detector. The length of the collimator was constant. The spare sections not occupied by absorber materials in the collimator were filled with air.

2.2 Choice of absorber materials

A large value of $d\mu/dE$ for the attenuating material would be desirable over the relevant energy range to increase variation of the slope of the transmission curve for the spectral analysis.^[3] $\mu(E)$ here was the attenuation coefficient for photons with energy E in a material. It had been suggested that an optimum combination of two kinds of materials, usually a kind of high Z material and a kind of low Z material, enhance the sensitivity for the spectral analysis. We selected iron and Teflon as absorbers. The $d\mu/dE$ value at low energies was greater for Fe than for Teflon. But it would be zero for $E \geq 8 \text{ MV}$. However,

the relative large value of $d\mu/dE$ at high energies for Teflon made Teflon the preferred choice of absorbers. Because of the large energy range for our problem, this two kinds of absorber materials had been considered an optimum combination.

2.3 Test for narrow-beam geometry conditions

For a collimated bremsstrahlung X-ray beam (i.e. narrow beam geometry), the transmission coefficient $T(x)$ have been defined as follows:

$$T(x) = \frac{\left[\int_0^{E_{\max}} e^{-\mu(E)x} F(E) dE \right]}{\left[\int_0^{E_{\max}} F(E) dE \right]} \quad (1)$$

where $F(E)$ is the fraction of signal due to photons of energy E , per energy interval, in the unattenuated beam; $\mu(E)$ is the attenuation coefficient for photons of energy E in absorber; E_{\max} is the maximum energy of the spectrum; and x is thickness of absorber. It has been well known that the exponential attenuation for a monoenergetic photon beam only exists when the narrow beam geometry condition is kept. On the contrary, the effect of multi-scattering on the photon attenuation process must be considered in the practical setup to measure the transmission coefficients when the narrow beam geometry condition is not satisfied. The narrow beam geometry requires that diameter of the primary photon beam is made just large enough to cover the detector uniformly, and the detector is placed at a large enough distance from the attenuator so that the number of scattered or secondary particles that reach the detector is negligible as compared with the number of primary rays.

In order to test if our experimental setup satisfied the narrow beam geometry or not, we simulated the responses of the solid state detector (part 1 in Fig.1) to various monoenergetic photons with Monte Carlo code system EGSnrc.^[8] The EGS (Electron-Gamma-Shower) system of computer codes is a general purpose package for the Monte Carlo simulation of the coupled transport of electrons and photons in an arbitrary geometry for particles with energies above a few keV up to several hundreds of GeV. There have been well over 1000 papers published concerning it since the original version of EGS was released. Being a new,

enhanced version of EGS4 distributed in 2000 by NRC (CANADA) and based on strong experimental benchmarks, EGSnrc works on Linux operating system. The statistical error of calculation is below 2%.

The detail simulating geometry was exactly in accordance with the experimental setup. It was supposed that the initial particles were parallel photons with certain mono-energy, and came into the collimator from part 11 in Fig.1. There were two parameters that could be varied, one was thickness of the absorber materials x , the other was energy of the incident photons E . The final simulation results had been compared with the calculations according to the formula $e^{-\mu(E)x}$. Fig.2 showed an example of these comparisons. In Fig.2, thickness of the iron attenuator was $23.5 \text{ g} \cdot \text{cm}^{-2}$, corresponding to its maximum thickness. Thickness of the Teflon attenuator was $88 \text{ g} \cdot \text{cm}^{-2}$, also corresponding to its maximum thickness. The statistical error of calculation is below 2%. The Monte Carlo simulation results showed that the exponential attenuation for a monoenergetic beam of the identical photons passing our experimental setup indeed existed. So our experimental setup satisfied the narrow geometry condition and Eq.1 could be used to calculate the transmission coefficient $T(x)$ with our experimental setup.

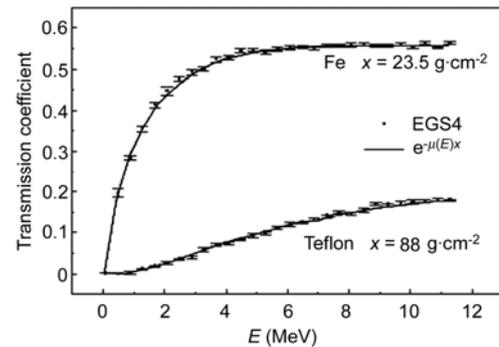


Fig.2 Comparison of transmission coefficients for monoenergetic photons passing the experimental setup by Monte Carlo simulations and analytic calculations. Thickness of the iron and Teflon attenuators was $23.5 \text{ g} \cdot \text{cm}^{-2}$ and $88 \text{ g} \cdot \text{cm}^{-2}$ respectively.

3 Iterative least-squares technique for spectrum reconstruction

If $\{F_j (j = 1, 2, \dots, n)\}$ were the fractional signals corresponding to equally spaced photon energies $\{E_j\}$ in the unattenuated beam and $\{\mu_j\}$ were the

attenuation coefficients of the attenuator for photons of energies $\{E_j\}$, Eq.1 could be approximated by Simpson's rule to describe the relative transmission $\{T_i (i=1, 2, \dots, m)\}$ for the attenuator thicknesses $\{x_i\}$,^[11]

$$T_i = (\Delta E/3) \sum_{j=1}^n A_{ij} F_j / (\sum F_j) \quad (2)$$

$$A_j = a_j e^{-\mu_j \cdot x_i}, \quad a_j = \begin{cases} 1 & j=1 \& j=n \\ 4 & j=3, 5, \dots, n-1 \\ 2 & j=2, 4, 6, \dots, n-2 \end{cases} \quad (3)$$

where $\Delta E = (E_1 - E_n)/(n-1)$ and n is an odd number. The first element A_{11} in Eq3 was assigned to the smallest transmission coefficient value (corresponding to the thickest attenuator) and the highest energy component, and the last element A_{mn} to the largest transmission coefficient value (no attenuator) and the lowest energy component. There was $T_m = 1$. In this paper, the energy range had been divided into 29 groups from 0.1 MeV to 11.3 MeV. It meant the energy grid was 0.4 MeV.

The iterative calculation began with the assumption of a "starting spectrum" $\{G_j\}$, which was an estimate of the true spectrum $\{F_j\}$. The choice of the starting spectrum was not completely arbitrary, but should be subjected to some physical constraints. The constraints applied included: (1) within a specified energy range, no G_j could be zero, although there was no restrictions on how close to zero it could be; (2) as a function of energy, G_j must be increased monotonically to a modal energy and then decrease monotonically. So the starting spectrum could be set as a simple triangle spectrum like Fig.3 (E_{mod} was called the modal energy). The purpose of the iterations was to successively develop spectra $\{G_j\}$ to which approached the real spectra $\{F_j\}$ by least-squares technique.

The photon energy fluence spectrum $\{\psi(E_j)\} = \psi_1, \psi_2, \dots, \psi_n$ could be derived from $\{F(E_j)\}$ as the following formula:

$$\psi_j = \frac{F_j}{(\mu_{\text{en}}/\rho)_j \cdot K_j \cdot K_{0j}}, \quad j=1 \sim n \quad (4)$$

where $(\mu_{\text{en}}/\rho)_j$ was Teflon's mass absorption coefficient for photons with energy E_j ; K_j and K_{0j} were given by the following equations:

$$k_j = \frac{[1 - \exp(\mu/\rho)_{j,\text{Tef}} \cdot R_{j,\text{Tef}}]}{\exp(\mu/\rho)_{j,\text{Tef}} \cdot R_{j,\text{Tef}}} \quad (5)$$

$$K_{0j} = \exp\left[(-\mu/\rho)_{j,\text{Tef}} \cdot (R - R_{j,\text{Tef}})\right]$$

In Eq.5, R_j was the CSDA range of the electron produced in Teflon medium by the photon with energy E_j ,

whose energy is $\frac{(\mu_{\text{en}}/\rho)_{E_j}}{(\mu/\rho)_{E_j}} E_j$ (i.e. secondary electrons' average energy), and R was the wall thickness of the solid state chamber.

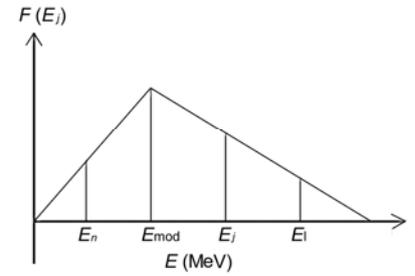


Fig.3 The assumed starting spectrum for iterative calculation.

4 Experimental results

Fig.4 showed the measured data of transmission coefficient $\{T_i\}$. The Teflon absorber was put behind the Fe absorber with the maximum thickness $23.5 \text{ g} \cdot \text{cm}^{-2}$. The bremsstrahlung spectrum produced by 12 MV beams of electrons hitting the tantalum target could be simulated by Monte Carlo method, too. It was supposed that the round parallel beam of electrons with 12 MeV energy was incident on the tantalum target surface vertically in the Monte Carlo simulation process by EGSnrc. The forward photons ($\pm 5^\circ$) behind the Ta plane were recorded. Fig.5 showed the comparison of the Monte Carlo calculated and reconstructed spectra from the measured transmission coefficients with iterative least-squares technique.

Based on the reconstructed energy fluence spectrum $\{\psi_j, j=1, 2, \dots, n\}$, we recalculated

the transmission coefficients presented with $\{T'_i, i=1, 2, \dots, n\}$ following Eq.1 and compared them with the measured results $\{T_i, i=1, 2, \dots, n\}$ by experiments. Table 1 showed this comparison.

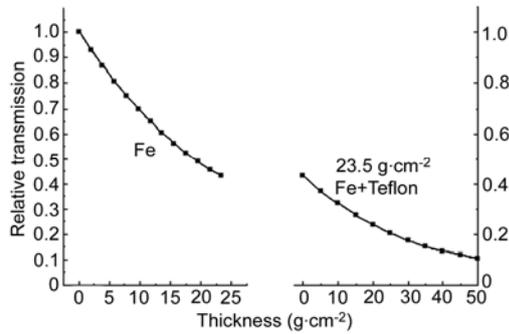


Fig.4 The measured relative transmission T vs attenuator thickness on the central axis.

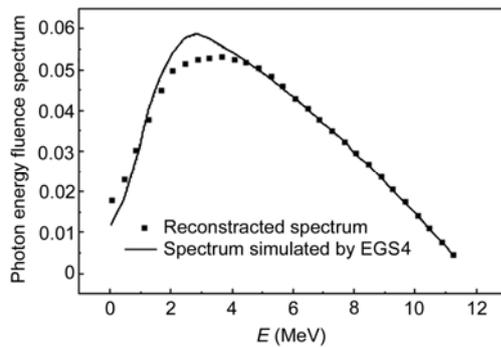


Fig.5 Comparison of the Monte Carlo calculated and reconstructed spectra $\{\psi_j\}$ from the measured transmission coefficients.

5 Conclusion

There are three important parts in this paper. The first is the measurement of transmission data for 12 MV bremsstrahlung beams produced by the process for continuous-pulsed 12 MV electrons coming from a linear accelerator hitting a plane tantalum target with 1.2 mm thickness. How to reconstruct the bremsstrahlung spectra by these transmission data is the second step. So the iterative least-squares technique has been used to complete the reconstruction. But there must be some other methods to test if the reconstructed spectra are correct or not. This is the third step. Monte Carlo simulation by EGSnrc can be a very effective way to make this comparison because it can simulate the real radiation process accurately.

Table 1 Comparison of the measured transmission coefficients $\{T_i, i=1, 2, \dots, n\}$ and the calculated transmission coefficients $\{T'_i, i=1, 2, \dots, n\}$ based on the reconstructed energy fluence spectrum $\{\psi_j, j=1, 2, \dots, n\}$

Serial No.	Thickness of absorbers ($\text{g} \cdot \text{cm}^{-2}$)		T_i	T'_i	$\frac{(T'_i - T_i)}{T_i}$
	Fe	Teflon			
1	23.50	50.0	0.1053	0.1019	-3.2%
2	23.50	45.0	0.1184	0.1168	-1.4%
3	23.50	40.0	0.1343	0.1339	-0.3%
4	23.50	35.0	0.1536	0.1539	+0.2%
5	23.50	30.0	0.1768	0.1770	+0.1%
6	23.50	25.0	0.2046	0.2040	-0.3%
7	23.50	20.0	0.2374	0.2355	-0.8%
8	23.50	15.0	0.2758	0.2725	-1.2%
9	23.50	10.0	0.3204	0.3159	-1.4%
10	23.50	5.0	0.3716	0.3668	-1.3%
11	23.50	0.0	0.4301	0.4275	-0.6%
12	21.45	0.0	0.4577	0.4522	-1.2%
13	19.50	0.0	0.4890	0.4905	+0.3%
14	17.55	0.0	0.5238	0.5259	+0.4%
15	15.60	0.0	0.5623	0.5634	+0.2%
16	13.65	0.0	0.6043	0.6001	-0.7%
17	11.70	0.0	0.6500	0.6455	-0.7%
18	9.75	0.0	0.6993	0.6923	-1.0%
19	7.80	0.0	0.7522	0.7469	-0.7%
20	5.85	0.0	0.8088	0.8023	-0.8%
21	3.90	0.0	0.8689	0.8628	-0.7%
22	1.95	0.0	0.9326	0.9278	-0.5%
23	0.00	0.0	1.0000	1.0000	—

The purpose of this paper is to test the iterative least-squares technique for 12 MV spectral reconstruction and give a way to determine the photon spectra at some situations when the photon spectra can not be determined by ordinary photon spectrometers. Based on the narrow beam geometry condition and the optimal selection of absorber materials, the result of this experiment shows that the numerical technique for analysis of transmission data can represent 12 MV bremsstrahlung spectra acceptably well.

Acknowledgments

We thank Professor CUI Gao-Xian of Institute of Southwest Nuclear Physics and Chemistry for many helpful discussions about the experimental method and reconstruction technique.

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