PHASE CORRECTION AND BAND–PASS FILTERS IN CT IMAGING

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ABSTRACT

This paper reports the use of the fast Fourier transform (FFT) in the direct Fourier transform method (DFM) in Computerized Tomography (CT) reconstruction. Phase corrections are needed in the CT reconstruction. In order to eliminate the image distortion from the basic DFM, Padding and band–pass filters are used in the improved DFM. Finally, some reconstructions from simulated projections and several experimental results are given.

Keywords: CT imaging Phase correction Band–pass filters Fourier transformation Padding

1 INTRODUCTION

The methods of image reconstruction from projections have been studied in great detail in recent years because of the development of CT technique in medical diagnostic uses, and the application of CT technique has now been extended to industry field. A fast and effecient algorithm is worth searching in image reconstruction. The direct Fourier transform method (DFM), which had been proposed elsewhere[2–4], is studied in this paper, and padding, phase correction and filtering to eliminate image distortions such as aliasing and artifact[5,6] in an improved DFM, are emphasized in discussion. In section 2, an improved DFM by padding and lowpass filter is presented. In section 3, phase correction is emphasized, and some discussions of phase correction expression in DFM are deduced. In section 4, the different effects of filter functions to the image are shown, in order to get the better images, the function should be altered for different targets. At last, some simulated and experimental results are given.

2 DIRECT FOURIER METHOD IN CT

CT reconstruction by the direct Fourier techniques is based on the projection–slice theorem (as is BFP method). For the two dimension case the theorem states that the Fourier transform of projection is a "slice" through the Fourier transform of $f(x,y)$, where $f(x,y)$ stands for any two variable functions and in CT representing the linear attenuation coefficient distribution. The Fourier transform $F(wx,wy)$ of $f(x,y)$ can be obtained from projection $p(t,\theta)$ by the projection- slice theorem, and then can be $f(x,y)$ itself by 2- dimentional inversion transform of $F(wx,wy)$.

But in practical applications, the measured data are used to estimate $p(t,\theta)$ only in definiting discrete values of t and θ. For parallel- ray geometry, only M angles with interval θ $(0 \leqslant m \leqslant M-1)$ and N equispaced rays with interval d for each angle $(-N/2 \leqslant n \leqslant N/2)$ are measured. So, the DFT (discrete Fourier transform) will be adopted in place of Fourier transform in DFM. As one of faster and more convenient tools to complement DFT, FFT[7](the fast Fourier transform) will be used in CT reconstruction. Because the fast algorithm of 2d- IFFT does not exist for the polar raster, an efficient DFM in tomography requires a procedure for interpolation from a ploar raster to a Cartesian raster.

From the FFT algorithm,

$$X(k)=\sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad W_N^{kn}=e^{-j2\pi kn/N} \tag{1}$$

if expand sequence $x(n)$ with zero, for example

$$y(n)=\begin{cases} x(n) & 0 \leqslant n \leqslant N-1 \\ 0 & N \leqslant n \leqslant QN-1 \end{cases} \tag{2}$$

The Fourier coefficient of sequence $y(n)$ is

$$Y(k)=\sum_{n=0}^{QN-1} y(n)\, W_{QN}^{kn}=\sum_{n=0}^{N-1} y(n)\, W_{QN}^{kn}+\sum_{n=N}^{QN-1} y(n)\, W_{QN}^{kn} = \sum_{n=0}^{N-1} x(n)\, W_{QN}^{kn} \tag{3}$$

where $$W_{QN}^{kn}=\exp(-j2\pi kn)/QN=\exp[-j2\pi(k/Q)n]/N=W_N^{(k/Q)n} \tag{4}$$

so $$Y(k)=X(K/Q) \qquad 0 \leqslant k \leqslant QN-1 \tag{5}$$

More intermediate points (Q times as well as primary) for Fourier coefficient are obtained by expanding sequence with zero. The distance between the origin and the last frequency point and the maximum frequency kept no change, only the interval was altered to $1/Q$. This procedure is named padding.

Since only limited number of data can be got from experiments (in practice), the interpolation from polar raster to Cartesian grid mentioned above can not be accurate enough, this causes distortion in the final image. If padding is used before interpolation, the quality of the image will be improved, because of the Q times intermediate freqency points by padding. Furthermore, the values at intermediate points approximately equal to those values obtained using high interpolation from the analyses of the behavior of Fourier coefficients. The values of Q can be adjusted to meet the need of the experimental requirement. By padding, the image from DFM will be more acceptable.

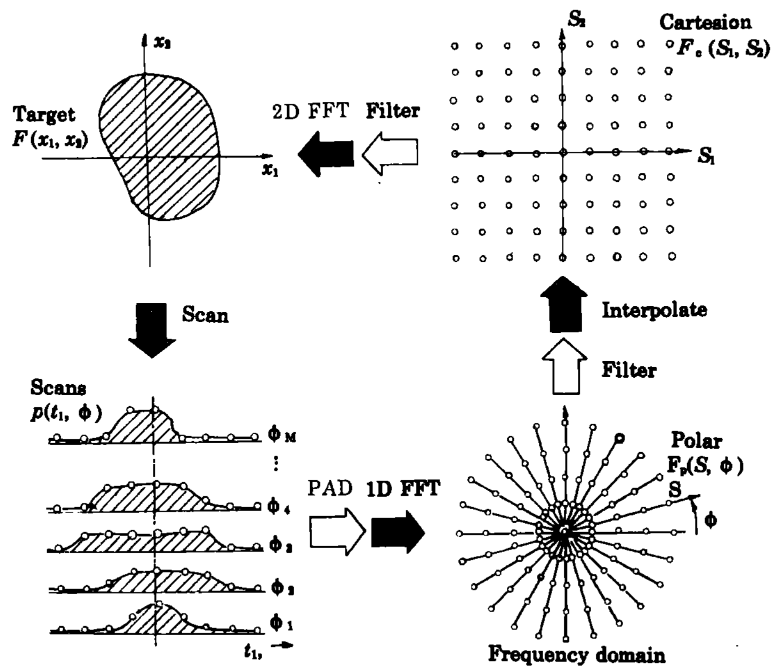

Fig.1 The whole steps of IDFM

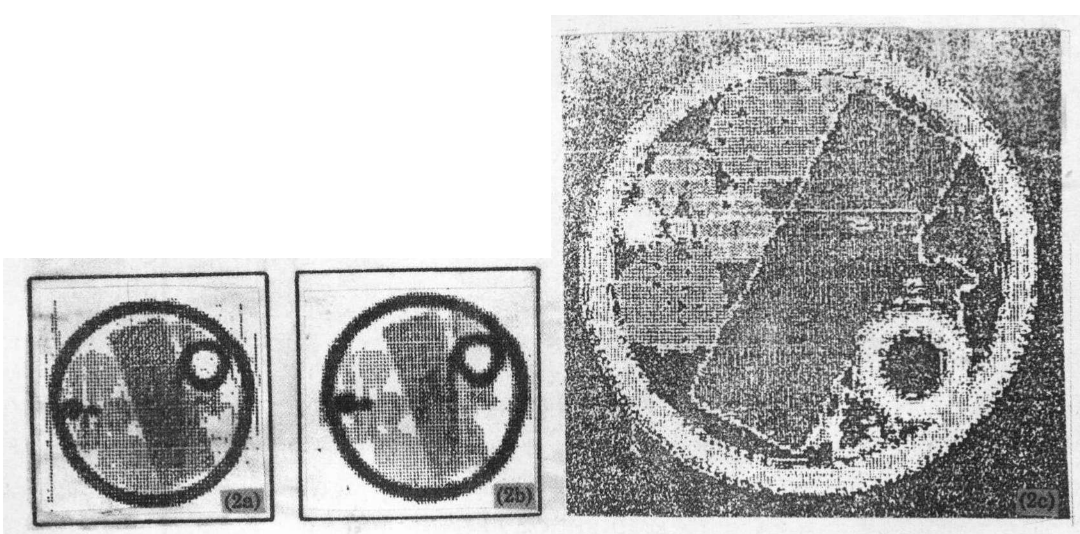

Fig.2 The images from DFM (a), IDFM (b) and BFP method

Artifacts will be decreased by padding as well, but there are still aliasings in the reconstructed image. Filter can be used to eliminate these aliasings. So by padding and filtering, the quality of the image from DFM will be improved. It can be comparable with images from BFP. This method is named as an improved DFM (IDFM).

The whole steps of the IDFM is shown in Fig.1. Fig.2 is experimental reconstructions by the DFM, IDFM and FBP respectively. From these images, it can be seen that the image from IDFM is better than that from DFM, and be compariable with that from FBP.

3 PHASE CORRECTION IN DFM

During the DFM process, one of the most important points should be cared is the phase correction. Since the input, intermediate and output data of FFT are complex, the Fourier coefficients depend on the coordinate origin for the spatial measurement. Different choice of origin leads to progressive phase shifts.

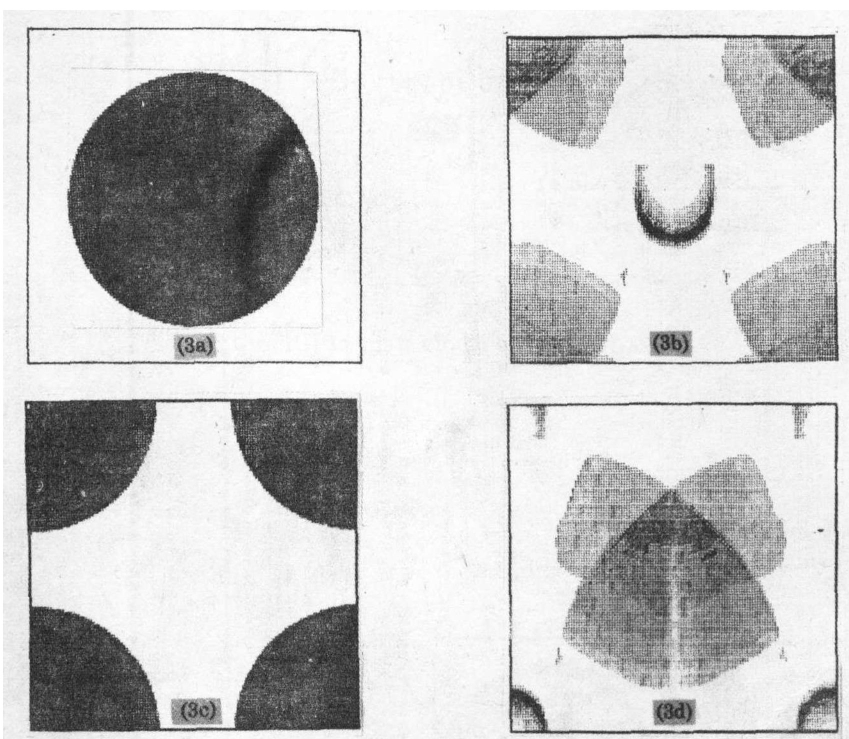

Fig.3 Phase correction

(a) With phase corrections, a circle reconstructed image from DFM (b) Without phase correction of FFT coefficient of projections (c) Without phase correction before 2D-IFFT (d) Without phase corrections both of 1D-FFT and 2D-IFFT

For example, corresponding to sequence $y(n)$ $(-N/2 \leqslant n \leqslant N/2)$, in order to call FFT function, argument n has to be changed from $(-N/2 \leqslant n \leqslant N/2)$ to $(0 \leqslant n \leqslant N)$, it produces a phase shift to the Fourier coefficients.

$$y(k) = \Sigma_{n=-N/2}^{N/2} y(n)\, W_N^{kn} = \Sigma_{n'=0}^{N-1} y(n'+N/2)\, W_N^{k(n'+N/2)}$$
$$= (-1)^k \Sigma_{n=0}^{N-1} y(n+N/2)\, W_N^{kn} = (-1)^k X(k) \quad (6)$$

Where $X(k)$ is the Fourier coefficients of sequence $x(n)$, $x(n) = y(n+N/2)$, $(0 \leqslant n \leqslant N-1)$. In DFM, 1D-FFT and 2D-IFFT are used in the procedure, inevitably phase corrections are needed. In order to achieve the computational advantage offered by FFT, the FFT coefficient of projections should have phase correction. However, to apply results of the projection-slice theorem, especially in the interpolation step below, it is necessary to shift all transforms so that the origin corresponds to a center point in the target. Complementing 1D FFT of projections, the coefficients should be multiplied by a phase correction $(-1)^k$; if padding is used to the projections before their Fourier coefficients, the correction will be changed.

$$X(k) = \Sigma_{n=-N/2}^{(Q-1)N+N/2} x(n)\, W_{QN}^{kn} = \Sigma_{n=0}^{QN-1} x(n+N/2)\, W_{QN}^{k(n+N/2)}$$
$$= W_{QN}^{kN/2} X(k/Q) = e^{-j\pi k/Q} X(k/Q) \quad (7)$$

the Fourier coefficients have to be multiplied by $e^{-j\pi k/Q}$, which varied with the Q.

Like 1D-FFT, in the case of 2D-IFFT, it is also needed to have phase correction. The phase correction will be $(-1)^{i+j}$, where i and j are the arguments in the Cartesian raster.

Making it clear, Fig.3 shows the difference between the phase correction and without phase correction.

4 FILTERS IN THE IMPROVED DFM

As mentioned in the section 2, there are some aliasings in the DFM reconstructed image. The oscillational artifacts encountered particularly near the objects where the linear attenuations vary violently. These oscillations deteriorate the resolution of the low contrast images comprising high contrast of image objects, the only possible way to suppress such artifacts is to use filtering. Lowpass filter can elimate the effect of the false high frequency (from the inconsistence of the attenuation), but the resolution of the image will be decreased. The lowpass filters can be both used in the spatial and frequency domains. There are some lowpass filters can be used in DFM in CT reconstruction[8], such as, the band-limiting window (rectangular window), the lowpass *sine* window, the lowpass cosine window, the generalized Hamming lowpass window, the Blackman window, the Papoulis window and the triangular window. Fig.4 shows the effects of the lowpass filter to the image.

A certain compromise between the oscillational suppression and smoothing effect on the images has led us to use the Hanning function to the frequencies in the polar raster and Lanczon function to smooth the frequencies in the Cartesian raster.

The Hanning function is

$$W(w) = 1/2[1+\cos(\pi w / W_c)] \tag{8}$$

where W_c is the cutoff frequency and Lanczos function is

$$W(w_1, w_2) = [\sin(\pi w_1 / W_{cx}) \quad \sin(\pi w_2 / W_{cy})] / [(\pi w_1 / W_{cx})(\pi w_2 / W_{cy})] \tag{9}$$

where W_{cx}, W_{cy} are the cutoff frequencies in x, y orientations respectively.

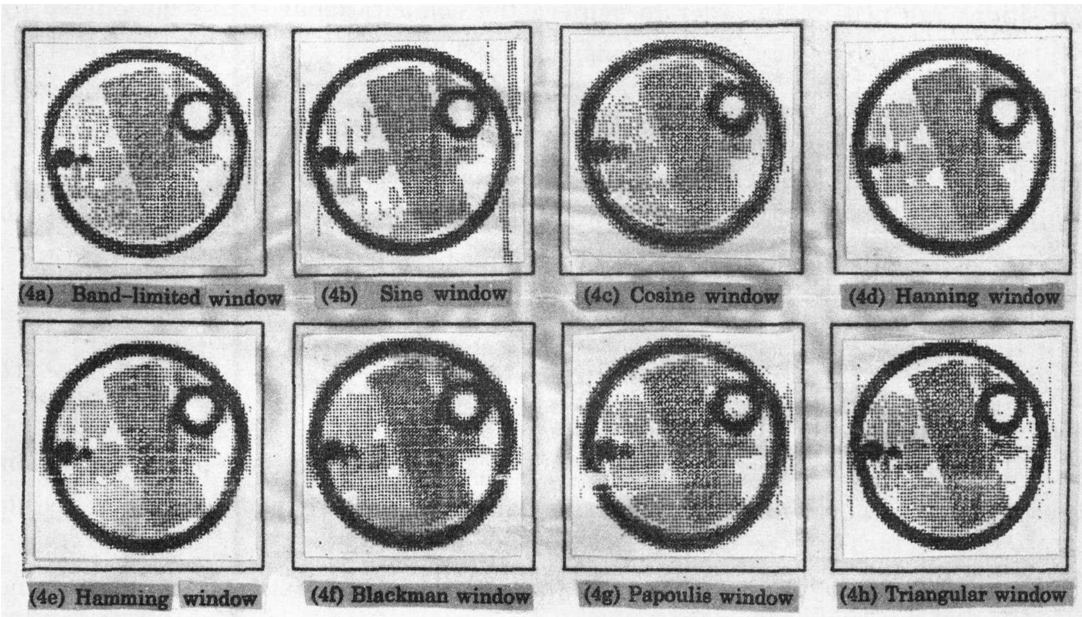

Fig.4 The effects of filters to the image

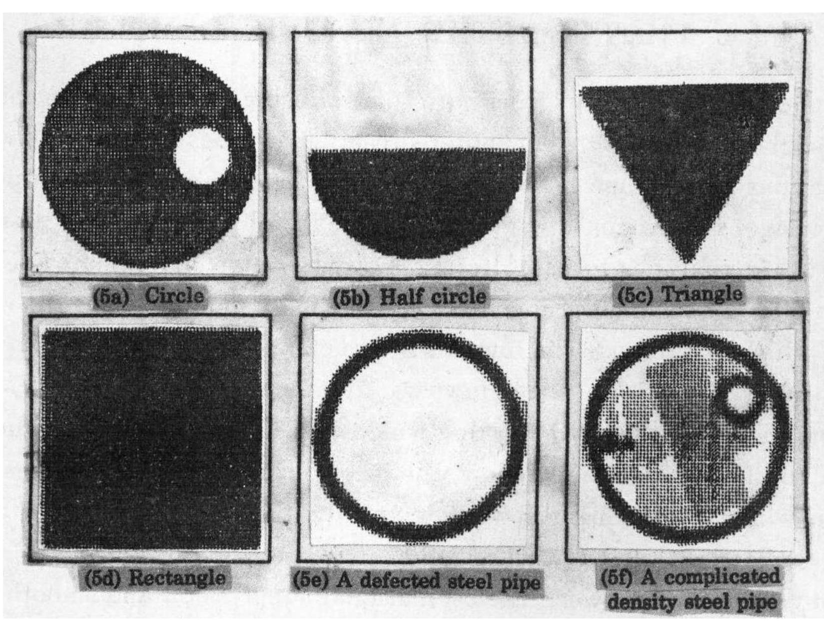

Fig.5 The reconstructions from experimental and simulated data

5 EXPERIMENTAL RESULTS

The simulated objects are shown in Fig.5. In the simulation, parallel-ray geometry was used to scan 40 views spanning 180°. The projection data associated with each view consisted of 128 discrete samples. The main purpose of the simulation was to see how well the Discrete Fourier Transform techniques work. The IDFM was used also to reconstruct two experimental projections which used parallel-ray to scan 90 views spanning 180°, each view consisted of 128 samples. It can test how much improvement in image quality by using padding and filter. One steel pipe has some defects in the ringe, the other steel pipe has lots of different linear attenuation objects in the interior, such as, a little copper pipe, two little aluminium pots, a rectangular marble stone. The results are displayed in Fig.5.

6 CONCLUSIONS

Using FFT algorithm is described in DFM, and an IDFM derived by using padding and lowpass filters. From this paper, some conclusions are drawn: a. Using DFM method in CT reconstruction, it is needed to have phase corrections; b. Lowpass filters eliminate some distortions of reconstruction image, it also decreases the resolution of the image, which filter function used is based on the targets; c. More high quality image can be achieved with the Q getting large, it takes more time in reconstruction; d. If both padding and lowpass filters are used in reconstructing, relatively better image can be gotten from DFM. At last, the experimental reconstruction shows that the image from the improved method is better than that from DFM; and it can be comparable to the reconstruction from BFP.

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