TRANSVERSE INSTABILITY OF A BUNCH OF HIGH CURRENT SHORT PULSE BEAM IN ELECTRON LINAC

Song Zhongheng (宋忠恒)

(China Institute of Atomic Energy, Beijing 102413, China)

(Received May 1990)

ABSTRACT

A set of equations for investigating transverse instability of a bunch of high current short pulse beam in electron linac is presented, and the result of numerical calculation is given in the paper.

Keywords: Field excited by beam Beam breakup

1 INTRODUCTION

The transverse motion instability of charged particles is one of main factors limiting beam current in electron linac. One of the instabilities is the phenomenon of beam breakup[1]. It is due to interaction between the beam and the hybrid electromagnetic wave excited by off–center beam with transverse velocity. Starting current of regenerative backward–wave oscillation at steady state in constant impedance structure was discussed in work[2], but how much pulse current can be travelled through for a special electron linac ? It is an important problem for high current linac, specially for short pulse high current one.

Few studies on the hybrid electromagnetic wave excited by beam was done due to the complexity of the problem, and a few papers refered to the discussion of transverse instability have been found[1,3]. In this paper, a set of equations for investigating transverse instability of a bunch of high current short pulse beam are presented, and numerical computation by computer on the developing process of the transverse instability for a bunch of short pulse high current beam is given.

2 BASIC SET OF EQUATIONS

According to work[4], the hybrid electromagnetic wave is excited by off–center beam with transverse velocity only. In order to discuss simply, the time part of electromagnetic wave excited by the beam as Fig.1, quoted from work[4], follows a following relation:

$$\ddot{q}(t)+2\alpha v_g\dot{q}(t)+\omega^2 q(t)=-i(a_{01}+a_{11})\exp(i\beta_z vt) \quad (1)$$

where $a_{01}=(Qv_z/N\omega)(A_{01}Kx_o/2)(\sin\beta_z L/\beta_z L)$, $a_{11}= i\, a_{01}(7/32)(v_r/v_z)(\beta_z a)(ax_o/R^2)$, and the electromagnetic field:

$$E=-\dot{q}(t)\psi(M) \qquad B=q(t)\nabla\times\psi(M) \tag{2}$$

where $\psi(M)$ is the space part of the wave excited by the beam in the waveguide, it follows the wave equation:

$$\nabla^2\psi+k^2\psi=0 \tag{3}$$

and normalized relation:

$$\int\psi\cdot\psi^*\,dV=N/\varepsilon_0 \tag{4}$$

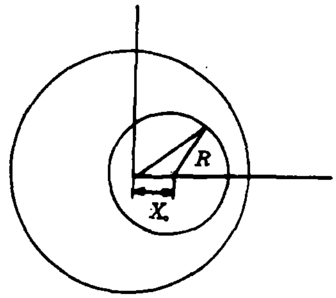

Fig.1 The sketch of the beam section in the waveguide

The analytic formula of coefficient A_{01} can be induced from Eq.(4):

$$A_{01}^2=2\omega^2/[\varepsilon_0\pi a^2 D J_0^2(\rho_{11})] \tag{5}$$

where ρ_{11} is the first root of first order Bessel function; α, V_g, ω, β_z are altenuation factor, group velocity, angular frequency and propagation constant of the wave, respectively; x_o is transverse displacement of the beam center in x–direction ($\varphi=0°$); R, $2L$, Q are radius, length and charge of the beam bunch, respectively; v_z, v_r are z–and r–direction velocity of the beam, respectively; a, D, N are radius of aperture, period and cell number of disc loaded waveguide, respectively.

Obviously the beam would excite the field, and the particles move under the interaction of the summary fields. This is a alternate causation, so investigation for transverse instability of charged particle motion needs the particle motion equation also. Assuming that the particles move under the external magnetic field and the field excited by the beam, and considering acceleration progress, original equations for transverse motion of charged particle are

$$\begin{aligned} d(\gamma\dot{r})/dt &= \gamma r\dot{\varphi}^2+(e/m_o)r\dot{\varphi}(B_z+B_{zo})+(e/m_o)(E_r-\dot{z}B_\psi) \\ d(\gamma r^2\dot{\varphi})/dt &= (e/m_o)r\{E_\psi+[\dot{z}B_r-\dot{r}(B_z+B_{zo})]\} \end{aligned} \tag{6}$$

where $E_r=-\dot{q}(t)\psi_r(M)$, $\psi_r(M)=(A_{01}/8\omega)K\beta_z(a^2+r^2)\cos\varphi\ \exp(-i\beta_z z)$;
$E_\psi=-\dot{q}(t)\psi_\psi(M)$, $\psi_\psi(M)=(A_{01}/8\omega)K\beta_z(a^2-r^2)\sin\varphi\ \exp(-i\beta_z z)$;
$B_z=q(t)[\nabla\times\psi(M)]_z$, $[\nabla\times\psi(M)]_z=(A_{01}/2\omega)K\beta_z r\sin\varphi\ \exp(-i\beta_z z)$;
$B_\psi=q(t)[\nabla\times\psi(M)]_\psi$, $[\nabla\times\psi(M)]_\psi=-i(A_{01}/2\omega)K[1-\beta_z^2(a^2+r^2)/4]\cos\varphi\ \exp(-i\beta_z z)$;
$B_r=q(t)[\nabla\times\psi(M)]_r$ $[\nabla\times\psi(M)]_r=-i(A_{01}/2\omega)K[1-\beta_z(a^2+r^2)/4]\sin\varphi\ \exp(-i\beta_z z)$.

The Eq.(6) becomes following form:

$$\begin{aligned} &\ddot{r}+(\dot{\gamma}/\gamma)\dot{r}+\{e[B_{zo}+q(t)(\nabla\times\psi)_z]/2m_o\gamma\}r=-e[\dot{q}(t)\psi_r+v_z q(t)(\triangle\times\psi)_\psi]/m_o\gamma \\ &d(\gamma r^2\dot{\varphi})/dt=-(e/m_o)r\dot{q}(t)\psi_\psi(M)+(e/m_o)r\{q(t)(\dot{z}[\nabla\times\psi(M)]_r-\dot{r}[\nabla\times\psi(M)]_\psi)-\dot{r}B_{zo}\} \end{aligned} \tag{7}$$

In absentee of the external magnetic field, the equation becomes

$$\begin{aligned} &\ddot{r}+(\dot{\gamma}/\gamma)\dot{r}+[eq(t)(\nabla\times\psi)_z/2m_o\gamma]r=-e[\dot{q}(t)\psi_r+v_z q(t)(\nabla\times\psi)_\varphi]/m_o\gamma \\ &\mathrm{d}(\gamma r^2\dot{\varphi})/\mathrm{d}t=-(e/m_o)r\dot{q}(t)\psi_r(M)+(e/m_o)rq(t)\{\dot{z}[\nabla\times\psi(M)]_r-\dot{r}[\nabla\times\psi(M)]_\varphi\} \end{aligned} \tag{8}$$

Assuming $\varphi=0°$, i.e. the particles move in the offset direction, we can see that the longitudinal magnetic field of the electromagnetic wave excited by the beam does not produce focusing force in the direction, and need not take account of the coupling between $r-$ and φ -direction. That is why a set of relatative equasions in r- and φ - direction is not solved in studying the transverse instability of particle motion, because of polarization of the wave excited by the beam.

Using Cartesian coordinate is appropriate, since assuming the beam offsets in the x-direction without y- direction velocity. For convenience, we study the motion of charge center in the x-direction, and assume there is a focusing (or steering) magnetic field, then the motion equation is

$$\ddot{x}_o+(\dot{\gamma}/\gamma)\dot{x}_o+(ev_z/m_o\gamma)(\mathrm{d}B_y/\mathrm{d}x)x_o=-e[\dot{q}(t)\psi_x+v_z q(t)(\nabla\times\psi)_y]/m_o\gamma \tag{9}$$

Obviously, longitudinal magnetic field does not supply force for the motion. One can observe that ψ_x, $(\nabla\times\psi)_y$ of eq. (9) is in accordance with ψ_r, $(\nabla\times\psi)_\varphi$ for $\varphi=0°$ in Eq. (6).

Eq (1 and 9) is the set of the equations for studying transverse instability of the particle motion.

3 NUMERICAL COMPUTATION

Combining Eq. (1) with Eq. (9), the numerical investigation for the transverse instability of particle motion can be done. One can let $q(0)=\dot{q}(0)=0$, since there is no electromagnetic field excited by the beam in waveguide when the beam is not yet in the waveguide. In the case of certain parameters, such as Q, L, ... and so on, $x_o(0)$, $\dot{x}_o(0)$ being given at first, $q(t)$, $\dot{q}(t)$ can be computed with Eq.(1); and then put $q(t)$ and $q(t)$ into Eq.(9), compute $x_o(t)$ and $\dot{x}_o(t)$; put the new x_o and $\dot{x}_o$ into Eq.(1) again, repeat above procedure, it does not continues until particles collide with the waveguide wall. If others keep constant, changing the quantity and the state of the beam only, and repeating the above, one can obtain the quantity and quality of the beam current resulting in the beam breakup.

Differential equations are solved by the Runge–Kutta method with variable or constant energy and with or without external magnetic field. For convenience, assume that the energy gains linearly. For the example of a short pulse high current protype of electron linac: its characteristics are $2\pi/3$–mode, constant impedence accelerator, 2856 MHz in frequence for TM_{01} wave; quality factor 6000, 4153 MHz in frequence for HEM_{11} wave; radius of aperture $a=0.0125963$ m, period of waveguide $D=0.035$ m, number of cell $N=69$ for disc loaded waveguide. Assume that the energy factor changes from 3 to 28 linearly. The bunches with phase width 9°, 18°, 27° and 36° are investigated respectively, the influence from the phase width of bunch is not a decisive factor on the topic in moderate charge of bunch. Now typical data of computation result for 9° in phase width and 6 mm in beam radius show as curves in Fig.2 and 3.

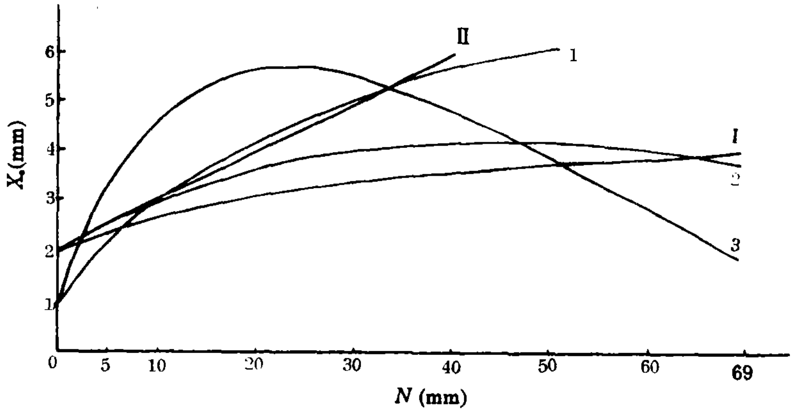

Fig.2 The curves of the transverse motion of the particles

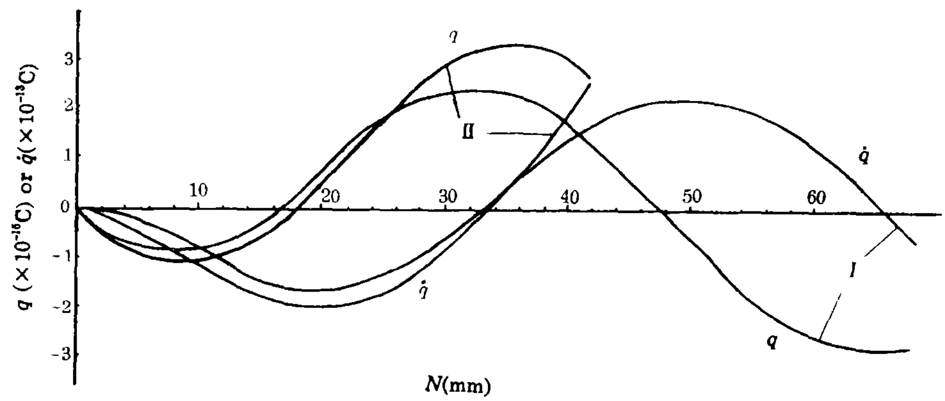

Fig.3 The curves of $q(t)$ and $q(t)$

In Fig.2, the curve I is for 30 A in beam current, and initial displacement and divergence of the beam (0.2×10^{-2}m, 0.29×10^{-2}), variable energy and without the external magnetic field. The curve II is according to curve I with constant energy. The curve 1 is for 30 A in beam current, and initial displacement and divergence of the beam (0.1×10^{-2} m, 0.87×10^{-2}), variable energy and without the external magnetic field. The curve 2 is according to curve 1 with y-direction magnetic field gradient 0.01 (T/m). The curve 3 is the trace of 30 A in the beam current, and initial displacement and divergence of the beam (0.1×10^{-2} m, 0.175×10^{-2}), variable energy and with y-direction magnetic field gradient 0.025 (T/m).

The curves for $q(t)$ and $\dot{q}(t)$ in Fig.3 are according to curve I and II in Fig.2.

It can be seen from the calculation that, when the initial transverse velocity of the beam is small, more than 50 A of beam current with an offset of 3 mm, can pass through the waveguide well without external magnetic field. The curve I in Fig.2 is a very good example. Comparison between curve I and II in Fig.2 shows that, the conclusion is over-exacting when constant energy is used for treatment of the question. When other conditions are kept constant except for the increasing of initial transverse velocity, even if offset of the beam is so small to 1 mm, it passes hardly through the waveguide; if set a *y*- direction magnetic field, it can steer the beam current passing through the waveguide; the curves 1, 2, 3 are the examples.

Above discusions show that, so long as the offset of the beam is not too large, and the beam travels fair parallelly, it does not lead to serious result in the system being alignment, but a fatal factor is the inclination of the beam. Of course, the range of the beam current accelerated generally in the electron linac of short pulse high current, the quantity of the pulse current is not most important, and the quality of the beam and the waveguide system is important factor for controlling the beam breakup.

In fact, coils are winded on the waveguide in the electron linac with high current short pulse, to supply uniform longitudinal magnetic field in the waveguide. But the case of particle motion is not so ideal like above discussion due to the longitudinal magnetic field being not very uniform practically. In order to control the beam breakup, modulating B_{20} can also lay part a little, but the problem can not be solved at all. The set of steering coil is very important. It is difficult to know when and where is the beam offseted and to set steering coil on the waveguide, to set steering coil on drift section is suitable for modulating mechanically convenient, besides need for a certain beam quality and alignment of the accelerating system.

REFERENCES

[1] Lapostolle P M, Septier A L eds. Linear accelerators. Amsterdam: North Holland Publ Co, 1970. 173.

[2] Song Zhongheng. *Atomic Energy Science and Technology* (in Chinese), 1983, 17(1):68.

[3] Ajzakij N I. *Zh Eksp & Teor Fiz* (USSR), 1982, 52(9):1315.

[4] Song Zhongheng. *Atomic Energy Science and Technology* (in Chinese), 1985, 19(2): 140.