# RADIATION FIELD FROM A RECTANGULAR PLAQUE SOURCE FOR RADIATION PROCESSING

Feng Yongxiang (冯永祥), Li Zhiliang (李志良), Li Panlin (李盘林)*
and Xu Mengjie (许梦杰)
*(Shanghai University of Science and Technology, Shanghai 201800, China.*
** Institute of Nuclear Research, Academia Sinica, Shanghai 201800, China )*

(Received November 1992)

## ABSTRACT

Evaluation of the Hubbell integral for an omnidirectional detector response to a rectangular plaque gamma emitting radiation source using a different numerical computation method on a personal computer was made. For a plaque consisting of several strip gamma emitters, exposure rates at different positions above the source were calculated using 'exact' evaluation and an approximate method. Optimization method is used to find source activity of each strip, so as to acquire desirable dose uniformity of the irradiated products. In uses of the Hubbell integral, by introducing appropriate programming, computation time can be saved.

**Keywords:** Numerical solution $^{60}$Co source Radioactivity Hubbell integral

## 1 INTRODUCTION

Various applications of ionizing radiation for processing of materials and products have been industrialized[1−3], such as radiation sterilization of medical products. In this application, gamma ray sources and electron accelerators (bremsstrahlung generating machines) are used. Most of the gamma ray facilities are implemented with rectangular plaque gamma emitters with source activity in PBq. Products to be irradiated are carried to the source by conveyors. In design and operation of the irradiator, there exists a requirement for assessment of the exposure rate from the gamma emitter.

## 2 DETECTOR RESPONSE TO A RECTANGULAR PLAQUE SOURCE

To make the assessment of exposure rate from a gamma emitter of rectangular shape, consider the case where an omnidirectional radiation detector is used and the emitter is isotropic. Assume its width be $w$ and lenth $l$, the detector is placed at a position with distance $h$ above one of the corners of the rectangle. For simplifying the computation, all linear dimensions are expressed in terms of dimensionless parameters: $a = w/h$, $b = l/h$, where $a > 0$, $b > 0$.

The response of the detector $I(a, b)$ is:

$$I(a, b)/(\sigma/4\pi) = \int\int [(\cos\theta)/r^2]\mathrm{d}s \tag{1}$$

where $r$ is the distance of the detector from a point of coordinate $(x,y)$ in the plane, i.e. $r^2 = 1 + x^2 + y^2$, $\mathrm{d}s = \mathrm{d}x \cdot \mathrm{d}y$, $\cos\theta = 1/[1 + x^2]^{1/2}$

In Eq(1), for simplification purpose $\sigma/4\pi$ is taken as 1, then

$$I(a, b) = \int\int [(1 + x^2 + y^2)(1 + x^2)^{1/2}]^{-1}\mathrm{d}x\mathrm{d}y$$
$$= \int_o^b \{\arctan [a/(1 + x^2)^{1/2}]\} \cdot (1 + x^2)^{-1/2}\mathrm{d}x \tag{1'}$$

This is the integral for calculating the radiation field from a rectangular plaque source derived by Hubbell[4]. And it has been evaluated by Hubbel *et al* using a rapid convergent series of the form:

$$f(a, b) = (\pi/2)\sinh^{-1}a - b/a \sum_{i=0}^{\infty} [1/(2i+1)]\,[a^2/(a^2+1)]^{i+1}$$
$$\times \{[(a^2+1)^{1/2}/b]\tan^{-1}[(a^2+1)^{1/2}/b] \tag{2}$$
$$- \Sigma_{j=0}^{i-1}\,[2^{2i}(j!)^2]/(2j+1)! \bullet [(a^2+1)/(a^2 + b^2 + 1)]^{j+1}\}$$

Numerical evaluation of the integral was made by Hanak and Cechak[5]. By using a different computation program here, the values of $f(a, b)$ calculated are found to be

**Tabel 1**

**Values of detector response**

| | Formula 1 | Formula 2 | Errors |
|---|---|---|---|
| 1 | 0.737742 | 0.726373 | 0.011369 |
| 2 | 0.711068 | 0.692114 | 0.018954 |
| 3 | 0.639510 | 0.599066 | 0.045450 |
| 4 | 0.544005 | 0.517348 | 0.026658 |
| 5 | 0.446598 | 0.433269 | 0.013329 |
| 6 | 0.360898 | 0.365568 | -0.004670 |
| 7 | 0.291206 | 0.321571 | -0.030545 |
| 8 | 0.863989 | 0.809245 | 0.054743 |
| 9 | 0.829541 | 0.808341 | 0.021201 |
| 10 | 0.737742 | 0.726373 | 0.011369 |
| 11 | 0.617075 | 0.613166 | 0.003909 |
| 12 | 0.496901 | 0.519146 | -0.022245 |
| 13 | 0.394163 | 0.408149 | -0.013986 |
| 14 | 0.312987 | 0.321751 | -0.008764 |

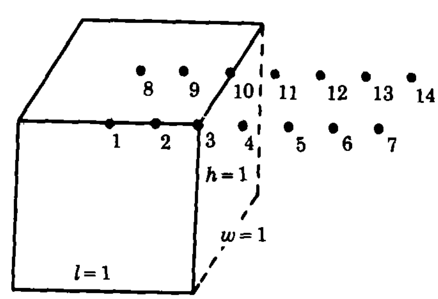


**Fig.1 Points of detector response**

consistent with the exact evaluations of Hubbell and Hanak and Cechak (see Table 1). The points of detector response are illustrated in Fig.1. In radiation processing, the exposure rate measured at 1 m from the center of the rectangular plaque is an important quantity for dose monitoring and controlling. In general, for an overlapping source irradiator, the height of the plaque is usually not greater than 2 m and the length may be much greater than the height.

If an accuracy for the assessment of the exposure rate needed is 0.1 (for radiation processing, it is sufficient), for $a$ and $b$ in the above range, the series can be truncated at $i=6$. The evaluation of the exposure rate at the center point of the projection of the rectangular plaque can be easily done using the equation (see Fig.2(a)):

$$I_{center}=4I(a/2,\ b/2) \tag{3}$$

and the response at a distance $h$ above the mid points of the four sides of the rectangle are:

$$I_{mid}=2I(a/2,\ b) \tag{4}$$

or

$$I_{mid}=2I(a,\ b/2) \tag{5}$$

Fig.2b shows the scheme for determination of the detector response at an arbitrary position, a distance $h$ from the rectangle. The response is:

$$I=I_1+I_2+I_3+I_4 \tag{6}$$

Using the similar scheme, response at points of the projection plane of the rectangular plaque source can be evaluated by adding and/or subtracting of fractional responses. The field to be calculated is schemetically illustrated in Fig.2c.

$$I=I_{1-2}-I_2 \tag{7}$$

Here $I$ is the detector response at $Q$ position to planar source 1 only.

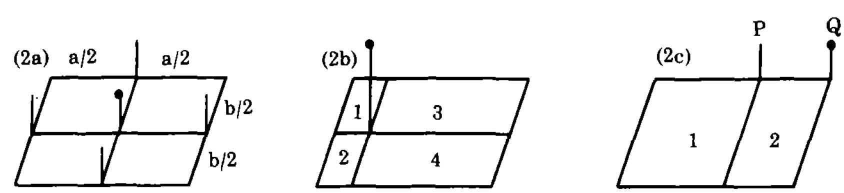


**Fig.2 Points of detector response**

## 3 STRIP SOURCES

In practice, a rectangular gamma emitter is made up of many gamma pencils assembled together to form a rectangular plaque. If a single pencil of cobalt 60 weighes several ten grams, and the specific activity of the cobalt is 200 Ci/g, the strength of a pencil is about several thousand curies. a plaque of million curies then has to be built by putting together a great number of pencils. A practical rectangular source may be considered as an array of strip gamma sources. For an infinitely long strip, where $a < l$ and $b \to \infty$.

$$f(0,\infty) = \pi/2 \sin h^{-1} a \tag{8}$$

the series part of equation vanishes. For a strip of finite length, and a<<b approximately:

$$f(a, b) = (\pi/2) \cdot \sinh^{-1} a - a \cdot \tan^{-1}(1/b) \tag{9}$$

This is the approximation of an expansion of the detector response.

$$f(a, b) = (\pi/2) \arcsin ha - (a/b)\sum_{k=0}^{\infty}[(-1)^k a^{2k}]/[(1+b^2)^k(2k+1)^2 \times F(1/2, 1, k+3/2, -1/b^2) \tag{10}$$

given by Gabutti *et al*[6], for $a< b$, only the first term of the series is significant, if error of 0.1 % is permitted, where $a$ is much less than $b$, this approximation applies well in assessment of the exposure rate of plaque sources.

For source configuration of $a<<b$, approximate calculation has been made and the exact integration was also undertaken in order to find the deviation of approximate calculation for $a=1$, $b=100$. The exact integration for the detector response is

$$F_1 = \int_0^{100} \text{arctg}\{[1/(1+x^2)^{1/2}] \cdot (1+x^2)^{-1/2} dx\} = 1.374459003$$

and the approximate calculation gives

$$F_2 = \pi/2 \text{ arsh}1 - \text{arctg}(1/100) = 1.374458725$$

the error is $F_1 - F_2 = 2.78 \times 10^{-7}$.

## 4 STRIP SOURCE DESIGN

Simplified calculation for design of a source made up of strips of varying strip source strengths is undertaken here as an example.

Take a source consisting of three strips of same strengths. The configuration of the source is shown in Fig.(3).

Using simplified calculation, the detector responses at three points on the projected plane are different. To acquire uniform exposure, the gamma strengths of strips have to be different.

Detector responses are calculated using the equations as follows:

$$[(\pi/2)\text{arsh}1 - \text{arctg}(1/0.2)]x_1 + [\text{arctg}(1/0.4) - \text{arctg}(1/0.6)]x_2 + [\text{arctg}(1/0.8) - \text{arctg}1]x_3 = 1$$

$$[\text{arctg}(1/0.3) - \text{arctg}(1/0.5)]x_1 + 2[(\pi/2)\text{arsh}1 - \text{arctg}(1/0.1)]x_2 + [\text{arctg}(1/0.3) - \text{arctg}(1/0.5)]x_3 = 1$$

$$[\text{arctg}(1/0.8) - \text{arctg}1]x_1 + [\text{arctg}(1/0.4) - \text{arctg}(1/0.6)]x_2 + [(\pi/2)\text{arsh}1 - \text{arctg}(1/0.2)]x_3 = 1$$

According to the method mentioned above the equations can be simplified by using symbolic representation.

$$ax_1 + bx_2 + cx_3 = 1$$
$$dx_1 + ex_2 + fx_3 = 1$$
$$cx_1 + bx_2 + ax_3 = 1$$

where
$a = (\pi/2)\mathrm{arsh}1 - \mathrm{arctg}(1/0.2)$
$b = \mathrm{arctg}(1/0.4) - \mathrm{arctg}(1/0.6)$
$c = \mathrm{arctg}(1/0.8) - \mathrm{arctg}1$
$d = \mathrm{arctg}(1/0.3) - \mathrm{arctg}(1/0.5)$
$e = 2[(\pi/2)\mathrm{arsh}1 - \mathrm{arctg}(1/0.1)]$
$f = \mathrm{arctg}(1/0.3) - \mathrm{arctg}(1/0.5)$

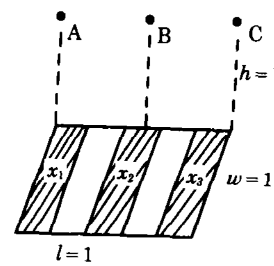

**Fig.3 The configuration of source**

By applying symbolic operations aid by our computer program the activities of the three strips are 4.375160 for $x_1$, 2.923324 for $x_2$, 4.375160 for $x_3$. Detector response at A, B, C are equal.

## 5 CONCLUSION

Hubbell integral for detector response of a rectanguler plane source is useful for design of industrial size cobalt sources. Taking the geometrical configuration of a source being a structure of arrays of pencils, to enable economic use of computer time, some simplified methods are introduced by applying symbol algebraic represantation of linear equations. Optimization computation can be made with a pc-computer. Varying the source strengths of strips forming the planar plaque, uniformity of detector response is easily acquired in design aided by the computer programs introduced here.

## REFERENCES

1 Moote F G. Nucleonics, 1961; 19:102
2 Margulis U Ya, Khrustalev A V. At Energ, 1957; 3:109
3 Manowitz B, Kuhl O A, Galanter L *et al.* Experimental parametric study of large-scale $^{60}$Co, $^{24}$Na and $^{137}$Cs slab irradiators. In: Large radiation sources in industry. Vienna: International Atomic Energy Agency, 1960; 65-91
4 Ghose A M, Bradley D A, Hubbell J H. Appl Radiat Isot, 1988; 39(5):421
5 Hanak V, Cechak T. Jaderna Euergie, 1978; 24:94
6 Gabutti B, Kalla S L, Hubbel J H. J Computational & Applied Mathematics, 1991