Theoretical study on the carbon nanotube used as hard X-radiation source

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Abstract Calculations and analyses are made on the interaction between the carbon nanotube and the incident positron of high energy. The results obtained show that it is possible to use carbon nanotube as hard X-radiation source with high intensity and good monochromaticity.
Keywords Carbon nanotube, High energy positron, Hard X-radiation

1 Introduction

The research work on the synthesis of so-called carbon nanotubes[1,2]—another important discovery after carbon 60, has achieved great success in recent years. These carbon nanotubes have a length of up to a few centimeters, while only a few nanometers in diameter. Hollowness is an important characteristic of nanotubes which is supposed to lead to their applications in various nanodevices for transporting both neutral and charged particles. On the other hand, high-energy electron or high-energy positron which can be generated by high-energy particle accelerator, when propagating in the carbon nanotube, interacts with the wall of the nanotube and can thus produce various electromagnetic radiations. It is mainly due to the Coulomb interaction between the charged particle and the charge of a nucleus partially screened by the electron shell of the atom. This kind of radiation emitted by a charged particle propagating in a carbon nanotube, just like the channel radiation due to charged particles channeling in crystal, has its particular properties and is different from the coherent bremsstrahlung, the transition radiation or synchrotron radiation; especially, it might have high intensity and good monochromaticity, therefore it can be of special interest in theoretical and experimental studies.

Recently, Klimov and Letokhov[3] have presented their study on the hard X-radiation emitted by a positively charged particle moving in a carbon nanotube, but the exploited potential between the charged particle and the carbon atom in their work is too simple to imitate the real potential. In the present paper, we make the similar calculation and analysis while exploiting Moliere potential which is based on Thomas-Fermi statistic model and is much closer to the real potential, and present the primary theoretical analyses and calculation results.

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2 Theoretical method

Studying the collision between the incident charged particle and the target atom, descriptions on either of the two objects are usually needed. There are two ways to describe the atomic system. One is Hartree-Fork self-consistent field method. It is pretty precise, but too complicated for dealing with multielectron system. The other is Thomas-Fermi statistic method, in which the electrons in the atom are regarded as degeneration gas. The electron density $n_e(r)$ and the total potential $V(r)$ at the point r are regarded as functions with the only variable of r, where r is the distance between the electron concerned in the atom and the centre of the atom (i.e., nucleus).[4]

According to the Thomas-Fermi statistic method mentioned above, the Coulomb interaction potential $V(r)$ between the incident ion with the charge Z_1e and the target atom with the nuclear charge Z_2e can be described by the following expression:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} u(\frac{r}{a}) \qquad (1)$$

where r is the distance between the propagating ion and the target atom, a is the screening parameter which can be given by the expression:

$$a = 0.8853 a_B (Z_1^{2/3} + Z_2^{2/3})^{-1/2} \quad (1a)$$

$$a_B = \hbar^2 (me^2)^{-1} = 0.529\text{Å}$$

a_B is the Bohr radius. And $u(r)$ is the Thomas-Fermi atom screening function which has only numerical solution instead of analytic solution.

Moliere provides a very good approximation expression[4] for $u(r)$:

$$u(\frac{r}{a}) = u_m(\frac{r}{a}) = \sum_{i=1}^{3} \alpha_i \exp(-\beta_i r/a) \quad (1b)$$

$$\{\alpha_i\} = \{0.1, 0.55, 0.35\};\ \{\beta_i\} = \{6.0, 1.2, 0.3\}$$

Propagating in the carbon nanotube, the incident ion interacts with not only the single atom in the wall, but also the continuous atom array, especially when its speed is close to that of light. With the simple potential superposition principle, continuous potential $U(\rho)$ on the ion can be obtained, in a first approximation, by summing Eq.(1) over a string of atoms:

$$U(\rho) = \frac{1}{d} \int_{-\infty}^{+\infty} V(z^2 + \rho^2) \mathrm{d}z$$

where d is the average inter-atomic distance; ρ is the perpendicular distance between the ion and the atom string.

In the case of a continuous tube, we can make simple extensions from the above expression, that is, the above expression for the interaction potential can be averaged over the periodic coordinates (z, φ) to obtain the expression:

$$U(r') = \frac{N}{2\pi d} \int_0^{2\pi} \int_{-\infty}^{+\infty} V(z^2 + r'^2) \mathrm{d}z \mathrm{d}\varphi \quad (2)$$

where N is the number of elementary periods present along the tube perimeter; and

$$r' = (R^2 + \rho^2 - 2R\rho\cos\varphi)^{1/2}$$

here R is the radius of the tube, i.e., the distance between the target atom located at the perimeter of the circular cross-section perpendicular to axis z and the center of the circle; ρ is the distance between the moving ion on the cross-section and the circle center.

Substituting the above Eqs.(1), (1a) and (1b) into Eq.(2) and integrating it over z, we obtain the expression:

$$U(r') = \frac{Z_1 Z_2 e^2 N}{d} \sum \alpha_i K_o(\beta_i \frac{r'}{a}) \quad (3)$$

where K_0 is the second modified Bessel function.

The further integration of Eq.(3) over φ is hard to be obtained analytically. In this case, Eq.(2) is numerically integrated to obtain $U(\rho)$, and thus further to get the U_2, here U_2 is defined as:

$$U_2 = \frac{1}{2} \frac{\partial^2 U(\rho)}{\partial \rho^2} |_{\rho=0} \quad (4)$$

3 Results

In the present paper, the propagating ion concerned is a positron with high energy. The positron is more likely to be moving close to the axis of the nanotube than to the wall because a positron coming close to the surface of the nanotube is acted upon by a repulsive force due to the incomplete screening of the positively charged nuclei. Generally, the positron develops a fast spiral motion in a direction parallel to the axis of the nanotube. The equation of motion of the positron in the transverse plane may be taken as the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m v_\rho) = -\frac{\partial U}{\partial \rho} \quad (5)$$

where m is the mass of the positron and

$$\gamma = (1 - \beta^2)^{-1/2};\ \beta = \frac{V_z}{C}$$

Here we consider the case where the positron enters the nanotube close to its axis, i.e., $R \gg \rho$. In this case, the potential $U(\rho)$ can be described by the harmonic approximation:

$$U(\rho) = U_0(\rho = 0) + U_2 \rho^2 \quad (6)$$

and then, we can easily solve the equation of motion (Eq.(5)) in the radial direction:

$$\rho = \rho_0 \cos(\Omega t) \quad (7)$$

where ρ_0 is the radius of the point of entry of the positron coming into the tube and Ω is the radial oscillation frequency defined as follows:

$$\Omega^2 = \frac{2U_2}{m\gamma} \quad (8)$$

Supposing a typical nanotube with 11Å in diameter and substituting the U_2 into Eq.(8), which is obtained by numerical integration mentioned above, we may have the following expression:

$$\Omega \approx \frac{1.7 \times 10^{16}}{\sqrt{\gamma}}[\mathrm{s}^{-1}] \tag{9}$$

For positron with an energy of 1 GeV, we have:

$$\frac{\Omega}{2\pi} \approx 0.62 \times 10^{14}\mathrm{Hz} \tag{10}$$

This result is almost one order greater than that estimated by exploiting the simple Linhard potential.[3]

It is well known that a charged particle moving with the acceleration may emit electromagnetic radiation. Obviously, the positron in a fast spiral motion as mentioned above does emit the electromagnetic radiation. For the spatialtemporal structure of the radiation, according to the theoretical analysis, the radiation maximum is located at $\omega(\theta)$, while the relation between the radiation frequency ω and the angle θ (the angle between the nanotube axis and the observation direction) is showed as follows:

$$\omega(\theta) = \frac{\Omega}{1 - \beta\cos\theta} \tag{11}$$

One can see from Eq.(11) that with the angle θ fixed, the radiation is emitted at a perfectly definite frequency. The radiation emitted forward ($\theta = 0$) has the maximum frequency $\omega_{\max}$ expressed as:

$$\omega_{\max} = \frac{\Omega}{1 - \beta} \tag{12}$$

In the case of the nanotube with a diameter of 11Å and the positron entering at $\rho_0 \approx 4.3$Å, we have:

$$\omega_{\max} \approx 3.3 \times 10^{16}\gamma^{3/2}[\mathrm{s}^{-1}] \tag{13}$$

For the positron with an energy of 1 GeV, one has γ=2000, the energy of the quanta with the maximum-frequency can be obtained to be $\hbar\omega_{\max} \approx 2.0$ MeV. Obviously, hard X-radiation is emitted.

As for the total radiation power P, it can be defined by the following well known formula:

$$P = \frac{2e^2}{3c^3}\gamma^4\dot{v}^2 = \frac{e^2}{3c^3}\gamma^4\Omega^4\rho_0^2 \tag{14}$$

In consequence, for the loss of energy per unit length by the radiation, we may have:

$$\Delta\varepsilon = P/c \approx 1.5\overline{\rho}_0^2\gamma^2 10^{-11}[\mathrm{erg/cm}] \tag{15}$$

where $\overline{\rho}_0 = \rho_0/3d$. For the positron with an energy of E=1 GeV, γ=2000 and the point of entry of the positron into the nanotube at $\overline{\rho}_0 = 1$ (i.e., $\rho_0 \approx 4.3$Å), we can obtain:

$$\Delta\varepsilon \approx 3.6 \times 10^1[\mathrm{MeV/cm}] \tag{16}$$

The results obtained above show that the positron with high energy propagating in a carbon nanotube does emit the hard X-radiation with the high intensity and the good monochromaticity. It is bound to be a bright future in the study of this area.

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