Hypernucleus-^{16}O in the density-dependent Hartree approach based on the chiral-σ model*

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Abstract A relativistic density-dependent interaction has been used to study hypernucleus ^{16}O. The density-dependent coupling constants of the relativistic effective Hartree-Lagrangian are obtained from the relativistic Brueckner-Bethe-Goldstone results of nuclear matter in the chiral-σ model. With these density-dependent coupling constants, the bound states and the single-particle energy spectra of the hypernuclei $^{16}_{\Lambda}$O and $^{16}_{\Sigma}$O are obtained. The theoretical results of $^{16}_{\Lambda}$O are in agreement with the experimental data fairly well.

Keywords Hartree approach, Chiral-σ model, Hypernucleus-^{16}O, Density-dependent

1 Introduction

The studies of the hypernucleus structure are very beneficial for investigating properties of the baryon-baryon interaction in nuclear medium and the nucleus structure since the hyperon in the hypernuclei is not constrained by the Pauli exclusive principle. In the past two decades, a number of $A(K^-,\pi^-)$[1~5] and $A(K^+,\pi^+)$[6~8] experiments had been performed to investigate the hypernuclei and the experimental data had been accumulated to be capable of providing more accurate details for hypernuclei. In the past years, much theoretical successes[9~15] had been achieved to analyze the spectroscopy of the hypernucleus. With the development of the experiments, nuclear theorists also try to find some new approaches to check the properties of hypernuclei.

The relativistic Hartree approximation (RHA) is an effective method to investigate the properties of nuclei as well as hypernuclei[9~20]. Nowadays, enthusiasm has been aroused to study the properties of nuclei by means of the relativistic density-dependent Hartree (RDDH) approach[21] or the relativistic density-dependent Hartree-Fock (DDHF)[22,23] approach. In the RDDH approach of Brockmann and Toki's work[21] the scalar and vector single-particle potentials for nucleon map the two-body correlation contributions from the exchanges of six non-strange mesons: π, η, ρ, ω, δ, σ.[24] In this work, we make an attempt to avail the RDDH approach, where the density-dependent effect is incorporated with the two-body correlation contributions by solving the relativistic Brueckner-Bethe-Goldstone (RBBG) equations in the chiral-σ model[25], to study properties of hypernuclei.

2 RDDH approach

The RDDH approach is interpreted in terms of the density-dependent coupling constants in the framework of the Walecka model. In addition, we add the tensor coupling of the vector meson to the Lagrangian. The effective Lagrangian is written as follows

$$\begin{aligned}\mathcal{L}_{\mathrm{RDDH}} &= \overline{\psi}[i\gamma_\mu\partial^\mu - M_{\mathrm{B}} - g_{\mathrm{BB}\sigma}(\rho)\sigma - g_{\mathrm{BB}\omega}(\rho)\gamma_\mu\omega^\mu + \frac{f_{\mathrm{BB}\omega}}{2M_{\mathrm{N}}}\sigma_{\mu\nu}\partial^\nu\omega^\mu]\psi \\ &+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}(\partial_\mu\omega^\nu - \partial_\nu\omega^\mu)^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \qquad (1)\end{aligned}$$

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Here, $g_{BB\sigma}(\rho)$ and $g_{BB\omega}(\rho)$ are the density-dependent coupling constants for $g_{NN\sigma}(\rho)$ and $g_{NN\omega}(\rho)$, they are obtained from the incorporation of the two-body correlation in the chiral-σ model, while for $g_{YY\sigma}(\rho)$, $g_{YY\omega}(\rho)$, we discuss them below.

It is known, the nucleon self-energy may be expressed by the single-particle potentials, which are the attractive scalar potential U_S and the repulsive vector potential U_V.

$$\Sigma = U_S + \gamma_0 U_V \tag{2}$$

They are expressed in terms of the density-dependent coupling constants $g_S^2(\rho)$ and $g_V^2(\rho)$ through:

$$U_S(\rho) = -\frac{g_{NN\sigma}^2(\rho)}{m_S^2}\rho_S, \quad U_V(\rho) = \frac{g_{NN\omega}^2(\rho)}{m_V^2}\rho_V \tag{3}$$

where ρ_S and ρ_V are scalar and vector densities, respectively. Now, equations of motion for baryons are straightforwardly reached[15]

$$(i\gamma_\mu\partial^\mu - M_B - g_{BB\sigma}(\rho)\sigma - g_{BB\omega}(\rho)\gamma_\mu\omega^\mu + \frac{f_{BB\omega}}{2M_N}\sigma_{\mu\nu}\partial^\nu\omega^\mu\psi = 0 \tag{4}$$

By solving the equations of nucleons and the hyperon in hypernuclei, the properties of hypernuclei are obtained for the given parameters related to the hyperon.

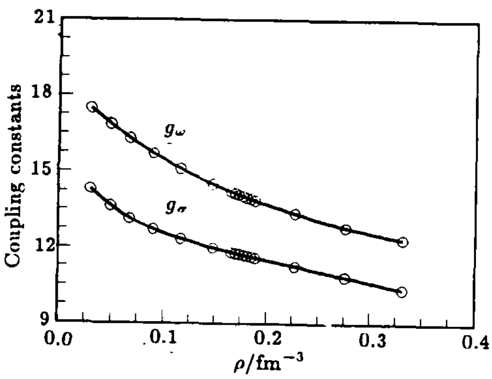

Fig.1 Density-dependent coupling constants deduced from the RBBG results, which are made to fit properties of the finite nuclei in the RDDH approach

The density-dependent coupling constants $g_{NN\sigma}(\rho)$ and $g_{NN\omega}(\rho)$ as shown in Fig.1 are fitted to give the description of nuclear matter and used to investigate the properties of spherical nuclei by choosing the suitable σ-meson mass (m_σ = 570 MeV). Owing to the uncertainty of the RBBG calculation at very low densities, the extrapolation of the density-dependent coupling constants is suitably needed to the calculation for the finite nuclei. Since the scalar and vector potentials are obtained through the integrations from zero to the Fermi momentum k_F, the potentials are taken as zeros at the zero density. Therefore, the density-dependent coupling constants at very low densities are calculated by virtue of Eqs.(2,3) where the potentials are interpolated between zero and the certain truncated density. The calculated binding energies, r.m.s. radii and single-particle spectra for the spherical nuclei with the density-dependent coupling constants are in agreement with the experimental data quite well. Here we tabulate the results of ^{16}O in Table 1 as an example.

Table 1 Comparison between calculated and experimental data of the binding-energy per nucleon, r.m.s. charge radius and single-particle energy spectra for ^{16}O

	Calculated data	Experimental data
$E\cdot A^{-1}$/MeV	-7.70	-7.98
r_c/fm	2.769	2.70
$\epsilon(1s_{\frac{1}{2}})$/MeV	-44.24	-40±8
$\epsilon(1p_{\frac{3}{2}})$/MeV	-22.25	-18.4
$\epsilon(1p_{\frac{1}{2}})$/MeV	-13.98	-12.1

In the following, the determination of the ratios of the meson-hyperon couplings to the meson-nucleon couplings is briefly focused on. For the ratio of vector meson's, it can be determined by the symmetric structure of the flavor SU(3)[11,20] in the free space and the Pauli exclusive principle in the nuclear medium: $g_{YY\omega}/g_{NN\omega}$ =8/9. However, large value of the ratio can not describe the properties of the ground state of hypernuclei well in case the ratio of the scalar meson's is determined by the correlated $\pi\pi$ and $K\overline{K}$ exchange[20], which is

0.490 for Λ and 0.405 for Σ.[20] Therefore, the ratio of $g_{\Lambda\Lambda\omega}/g_{NN\omega}$ is reduced, and it is taken as 0.504 for Λ and 0.405 for Σ in the present work. As for the tensor coupling $f_{BB\omega}$, which is simply considered as a density-independent constant in our work, it is illustrated in two aspects. For $f_{NN\omega}$, it contributes only a small part to the results and hence is neglected in this work; while for $f_{YY\omega}$, it plays the important role on the spin-orbit(SO) splittings of hypernuclei. Here, the ratios of $f_{\Lambda\Lambda\omega}/g_{\Lambda\Lambda\omega}$ and $f_{\Sigma\Sigma\omega}/g_{\Lambda\Lambda\omega}$ are -0.616, 1.132, respectively, same as in Ref.[20]

Table 2 Ratios of the meson couplings of hyperons to the meson couplings of nucleons, used in the RDDH approach

RDDH	$g_{YY\sigma}/g_{NN\sigma}$ *	$g_{YY\omega}/g_{NN\omega}$	$f_{YY\omega}/g_{YY\omega}$ **
Λ	0.49	0.504	-0.616
Σ	0.405	0.405	1.132

*$g_{YY\sigma}/g_{NN\sigma}$ is determined by the correlated $\pi\pi$ and $K\overline{K}$ exchange. **$f_{YY\omega}/g_{YY\omega}$ is obtained from the SU(3) symmetric structure

3 Calculations for $^{16}_{\Lambda}$O and $^{16}_{\Sigma}$O hypernucleus

3.1 $^{16}_{\Lambda}$O

In the following, we are going to calculate $^{16}_{\Lambda}$O and $^{16}_{\Sigma}$O. The numerical results with parameters for $^{16}_{\Lambda}$O are listed in Table 3. It is shown that the density-dependent interaction plays larger roles on the reduce of the SO splittings and the shifts of single-particle spectra in the hypernucleus compared to the RHA result[12~14]. Table 3 shows that tensor coupling of ω meson has large effect on the reduce of SO force. The single-particle levels for $^{16}_{\Lambda}$O, where the tensor coupling is included, are plotted out in Fig.2 along with the experimental data.[3,7,26] It indicates that the results with the density-dependent couplings agree very well with the experimental data.

Table 3 The numerical results of the single-particle spectra for $^{16}_{\Lambda}$O in the RDDH approach

Single-particle spectra	With tensor coupling	Without tensor coupling
$\epsilon(1s\frac{1}{2})$/MeV	-12.45	-13.12
$\epsilon(1p\frac{3}{2})$/MeV	-2.76	- 3.29
$\epsilon(1p\frac{1}{2})$/MeV	- 2.48	- 2.01

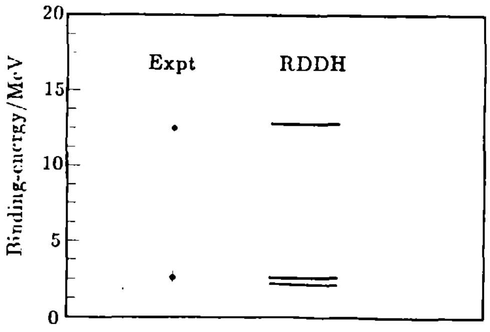

Fig.2 Single-particle energy level for $^{16}_{\Lambda}$O. The empty diamonds are calculated in the RDDH approach where the tensor coupling is contained. The solid disks are the experimental data taken from Refs.[3,7,26]

3.2 $^{16}_{\Sigma}$O

As for the Σ-hypernuclei, it is known that there are no experimental data yet. The numerical results depend on the parameters. As pointed out in Ref.[20], the Σ hypernuclei may not lead to the bound state if the ratio of the scalar meson-hyperon couplings to the scalar meson-nucleon ones is determined by the $\pi\pi$ and $K\overline{K}$ exchange while the ratio of the vector couplings is kept to be the same as the Λ-hypernuclei. However, there are many choices of parameters to lead to the Σ-hypernuclei bound state. (a): As an attempt, we first take the ratios of the Λ row in Table 2 except for the tensor coupling ratio (which is 1.132 for Σ-hypernuclei listed in Table 2) to simulate the properties for $^{16}_{\Sigma}$O. Table 4 tabulates the results for cases with and without tensor couplings. (b): As a comparison, we then use the parameters listed in the row for Σ-hypernuclei in Table 2 to calculate $^{16}_{\Sigma}$O. The results are also listed in the Table 4. It is found that the SO splittings are larger than that for $^{16}_{\Lambda}$O if the $^{16}_{\Sigma}$O and other Σ-hypernuclei exist. It is also seen that the positive tensor coupling enlarges the SO splittings, which is opposite to the situation for Λ-hypernuclei. So far, the properties for the Σ-hypernuclei need the experimental proof.

As for the sensitivity of the results on the scalar meson mass, we do not intend to make further consideration in the present work since the scalar meson mass is suitably chosen in the calculation for finite nuclei and correspondingly, the good description for finite nuclei is achieved.

Table 4 The single-particle spectra for $^{16}_{\Sigma}$O for different cases in the RDDH approach

Single-particle spectra	Different choices of parameters used in RDDH approach			
	$a(1)^*$	$a(2)^*$	$b(1)^*$	$b(2)^*$
$\epsilon(1s_{\frac{1}{2}})$/MeV	-15.18	-13.85	-14.67	-13.71
$\epsilon(1p_{\frac{3}{2}})$/MeV	-5.27	-4.02	-4.66	-3.73
$\epsilon(1p_{\frac{1}{2}})$/MeV	-1.98	-2.90	-2.19	-2.88

$^*a$ and b are illustrated as text above; the indications (1) and (2) represent the cases with and without the tensor coupling, respectively

4 Summary

In this work, the density-dependent interaction is successfully applied to investigate the properties of hypernucleus ^{16}O. We calculate out the single-particle spectra for $^{16}_{\Lambda}$O and $^{16}_{\Sigma}$O in the RDDH approach based on the chiral-σ model. The density-dependent interaction has large effect on the energy level and the SO splittings. In addition, the tensor coupling for the ω meson in the RDDH description is included, which, as shown from the result, is important for the hypernuclei, and different from the nuclei. To sum up, the numerical results for $^{16}_{\Lambda}$O agree quite well with the experimental data, while theoretical results for $^{16}_{\Sigma}$O need to be examined by the experiments.

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